Name:
ID. No.

Eskişehir Osmangazi University - Electrical Engineering Department
Differential Equations- Resit Examination

Duration: 60 minutes; **Directions**: All answers must be written below the respective questions. Anything written elsewhere won't be graded.

Question 1.

Solve the Riccati differential equation

$$\dot{y} = (y-x)^2 + 1,\ y(0) = 0.5$$

given that one of its solution is $y_1(x) = x$.

Using change of variables $y = v + x \rightarrow \dot{y} = \dot{v} + 1$ *leads to*

$$\dot{v} + 1 = (v + x - x)^2 + 1 \rightarrow \dot{v} = v^2, \quad \textit{Bernouilli diff. eqn.}$$

$$v = w^{-1} \rightarrow \dot{v} = -w^{-2}\dot{w} \rightarrow -w^{-2}\dot{w} = w^{-1} \rightarrow \quad \dot{w} = -1$$

$$w = -x + c \rightarrow v = \frac{1}{c-x} \rightarrow \quad y = \frac{1}{c-x} + x$$

Thus, $y = x$ *and* $y = \frac{1}{c-x} + x$ *are solutions to the Riccati equation. The solution* $y = x$ *does not satisfy the d.e., so we use the other one*

$$y(x) = \frac{1}{c-x} + x \rightarrow 0.5 = \frac{1}{c-0} + 0 \rightarrow c = 2$$

The solution is therefore

$$y(x) = \frac{1}{2-x} + x$$

Question 2.

Find a particular solution of

$$x^2\ddot{y} - 2x\dot{y} + 2y = e^{\frac{9}{2}}$$

given that the corresponding homogeneous equation

$$x^2\ddot{y} - 2x\dot{y} + 2y = 0$$

has solutions x *and* x^2.

The solution has the form

$$y_p = u_1 x + u_2 x^2$$

with $\dot{u}_1 x + \dot{u}_2 x^2 = 0 \ldots (*)$ *Substitute this in the given equation:*

$$x^2(\dot{u}_1 + 2u_2 + 2x\dot{u}_2) - 2x(u_1 + 2xu_2) + 2(u_1 x + u_2 x^2) = x^{\frac{9}{2}}$$

$$\dot{u}_1 x^2 + 2x^3\dot{u}_2 = x^{\frac{9}{2}} \rightarrow \quad \dot{u}_1 + 2x\dot{u}_2 = x^{\frac{5}{2}} \ldots (**)$$

Equation (*) implies $\dot{u}_1 = -\dot{u}_2 x$. *Substitute this into Equation (**):* $\dot{u}_2 = x^{\frac{3}{2}}$. *This leads to* $\dot{u}_1 = -x^{\frac{5}{2}}$. *Last two expressions yield*

$$u_1 = -\frac{2}{7}x^{\frac{7}{2}} \quad \textit{and} \quad u_2 = \frac{2}{5}x^{\frac{5}{2}}$$

Thus

$$y_p = u_1 x + u_2 x^2 = -\frac{2}{7}x^{\frac{7}{2}}x + \frac{2}{5}x^{\frac{5}{2}}x^2 = \frac{4}{35}x^{\frac{9}{2}}$$

Question 3.

[20 pts.] Sketch the phase portrait of

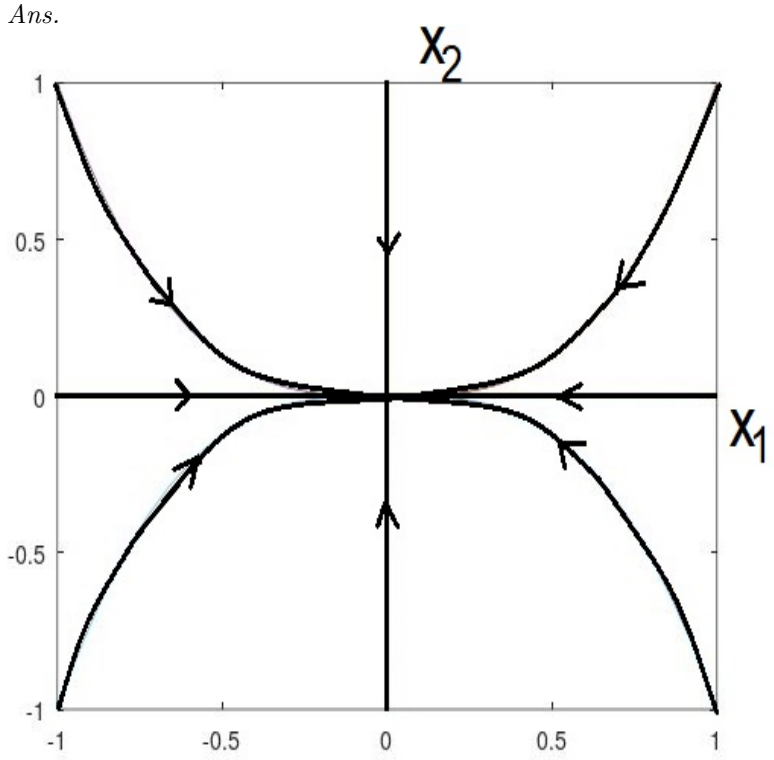
$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

In particular, on the phase portrait show the trajectories corresponding to the initial conditions

$$\begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ -1 \end{bmatrix}, \begin{bmatrix} 0 \\ -1 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

Use arrows to indicate the trajectory directions. Also, trajectories must exhibit the directions of their slow and fast changes.

Ans.



Good Luck
A. Karamancıoğlu