Name:
ID No.

Eskişehir Osmangazi University - Electrical Engineering Department
Differential Equations- Midterm Examination - Fall 2023

Duration: 45 minutes; **Directions**: All answers should be positioned beside or below their corresponding questions. Anything written elsewhere won't be graded. Up to one percent error in the results is tolerated.

Question 1.

Let $v = y^{-6}$ *transform the differential equation*

$$\frac{dy}{dx} + x^5 y = x^5 y^7$$

into a first order linear linear differential equation:

$$\frac{dv}{dx} + S(x)v = R(x).$$

Find the function $S(x)$ *and* $R(x)$*. Show your work.*

It is a Bernouilli d.e. Let $v = y^{1-n}$ *with* $n = 7$. $\therefore v = y^{-6} \to y = v^{-\frac{1}{6}} \to \frac{dy}{dx} = -\frac{1}{6}v^{-\frac{7}{6}}\frac{dv}{dx}$

Substitute in the given equation:

$$-\frac{1}{6}v^{-\frac{7}{6}}\frac{dv}{dx} + x^5 v^{-\frac{1}{6}} = x^5 v^{-\frac{7}{6}}$$

Multiply throughout by $-6v^{\frac{7}{6}}$

$$\frac{dv}{dx} - 6x^5 v = -6x^5$$

$$S(x) = -6x^5; \quad R(x) = -6x^5$$

Question 2.

(Y/N) Are the functions $\sin(x)$, $\sin(2x)$, *and* $\sin(4x)$ *linearly independent on*

(a) $0 \leq x \leq \frac{\pi}{2}$ *Y*

(b) $0 \leq x \leq \pi$ *Y*

(c) $0 \leq x \leq \frac{\pi}{4}$ *Y*

(d) $0 \leq x \leq 2\pi$ *Y*

(e) $-\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$ *Y*

(For this question, there is no partial credits!)

Question 3.

(a) 10 pts. Find the c values so that $y = c/x$ satisfies the Riccati differential equation

$$\frac{dy}{dx} + y^2 = \frac{2}{x^2}. \qquad \textit{Eq. (1)}$$

Show your work.

$$y = \frac{c}{x} \to \dot{y} = -\frac{c}{x^2}$$

Substitute in the d.e.

$$-\frac{c}{x^2} + \frac{c^2}{x^2} = \frac{2}{x^2} \to c^2 - c - 2 = 0$$

$$c = 2, \ \textit{and } c = -1$$

(b) 15 pts. Let c_1 be the largest number such that $y = c_1/x$ satisfies Eq. (1). Use $y = c_1/x$ as a known solution to transform Eq. (1) into a first order linear differential equation. Write this first order linear differential equation below. Show your work.

$$y = z + \frac{2}{x} \to \dot{y} = \dot{z} - \frac{2}{x^2}$$

Substitute in the Riccati Equation:

$$\dot{z} - \frac{2}{x^2} + (z + \frac{2}{x})^2 = \frac{2}{x^2}$$

$$\dot{z} \cancelto{0}{-\frac{2}{x^2}} + z^2 + \cancelto{0}{\frac{4}{x^2}} + 4\frac{z}{x} = \cancelto{0}{\frac{2}{x^2}}$$

$$\dot{z} + z^2 + 4\frac{z}{x} = 0 \to \dot{z} + 4\frac{z}{x} = -z^2 \ \textit{ it is a Bernouilli d.e.!}$$

This can be written as: $z^{-2}\dot{z} + \frac{4}{x}z^{-1} = -1$ *Use* $v = z^{1-2} = z^{-1} \to \dot{v} = -\frac{\dot{z}}{z^2}$ *in the Bernouilli d.e.:* $-\dot{v} + \frac{4}{x}v = -1$ *or equivalently:* $\dot{v} - \frac{4}{x}v = 1$

Question 4.

Find a general solution for

$$\frac{d^4y}{dx^4} - 4y = \cos(2x)$$

Show your work.

Aux. Eqn.: $m^4 - 4 = 0 \to (m^2-2)(m^2+2) = 0 \to (m-\sqrt{2})(m+\sqrt{2})(m-i\sqrt{2})(m+i\sqrt{2}) = 0$

Homogeneous part's general solution: $y_c(x) = c_1e^{\sqrt{2}x} + c_2e^{-\sqrt{2}x} + c_3\cos(\sqrt{2}x) + c_4\sin(\sqrt{2}x)$
Particular solution candidate: $y_p = A\sin(2x) + B\cos(2x)$
Substitute in the d.e.:
$16A\sin(2x) + 16B\cos(2x) - 4(A\sin(2x) + B\cos(2x)) = \cos(2x) \to 12A\sin(2x) + 12B\cos(2x) = \cos(2x)$
$B = 1/12 \to y_p(x) = 1/12\cos(2x)$ *Thus general solution is:*

$$y(x) = c_1e^{\sqrt{2}x} + c_2e^{-\sqrt{2}x} + c_3\cos(\sqrt{2}x) + c_4\sin(\sqrt{2}x) + \frac{1}{12}\cos(2x)$$

Good Luck
A. Karamancıoğlu