Name:
ID No.

Eskişehir Osmangazi University - Electrical Engineering Department
Differential Equations- Midterm Examination - Fall 2024

Duration: 70 minutes; **Directions**: All answers should be positioned beside or below their corresponding questions. Anything written elsewhere won't be graded. Up to one percent error in the results is tolerated.

Question 1.

Find all c *values so that* cx^2 *solves the differential equation*

$$x^3\frac{dy}{dx}+x^2y-y^2=2x^4,\quad x>0$$

Show your work.

$$\frac{dy}{dx}+\frac{y}{x}-\frac{y^2}{x^3}=2x$$

$$2cx+\frac{cx^2}{x}-\frac{c^2x^4}{x^3}=2x$$

$$c^2-3c+2=0$$

Thus $c=1$ *and* $c=2$.

Question 2.

Find an explicit solution to the differential equation

$$\frac{dy}{dx}=\frac{y^2}{x^2}+\frac{y}{x}+1,\ x>0$$

Show your work.

Homogeneous d.e.! Use $y=vx$ *and* $\dot{y}=\dot{v}x+v$

$$\dot{v}x+v=v^2+v+1$$

$$x\frac{dv}{dx}=v^2+1\to\frac{dv}{v^2+1}=\frac{dx}{x}\to\tan^{-1}v=\ln x+c\to v=\tan(\ln x+c)$$

$$\frac{y}{x}=\tan(\ln x+c)\to y=x\tan(\ln x+c)$$

Question 3.

Solve the initial value problem

$$\frac{d^4y}{dx^4} - y = 0; \quad y(0) = \dot{y}(0) = \ddot{y}(0) = \dddot{y}(0) = 1$$

Show your work.

Auxiliary equation: $m^4 - 1 \Longrightarrow (m^2+1)(m^2-1) = 0 \rightarrow m = 1, -1, i, -i$
General solution $y(x) = c_1e^x + c_2e^{-x} + c_3 \sin x + c_4 \cos x$. *Use initial conditions:*

$$y(0) = 1 \rightarrow c1 + c_2 + c_4 = 1, \quad (a)$$

$$\dot{y}(0) = 1 \rightarrow c1 - c_2 + c_3 = 1, \quad (b)$$

$$\ddot{y}(0) = 1 \rightarrow c1 + c_2 - c_4 = 1, \quad (c)$$

$$\dddot{y}(0) = 1 \rightarrow c1 - c_2 - c_3 = 1, \quad (d)$$

(a) and (c) imply $c_4 = 0$, *(b) and (d) imply* $c_3 = 0$. *Thus, (a) and (b) becomes*

$$c_1 + c_2 = 1, \; c_1 - c_2 = 1$$

These eaquations imply $c_1 = 1$ *and* $c_2 = 0$. *Thus the solution is*

$$y(x) = e^x$$

Question 4.

A solution to the differential equation

$$2t^2\frac{d^2y}{dt^2} + t\frac{dy}{dt} - 3y = 0, \; t > 0$$

is $y_1(t) = t^{-1}$. *Find another solution* y_2 *so that the set* $\{y_1, y_2\}$ *is linearly independent. Show your work.*

Use the order reduction technique.
$y_2 = t^{-1}v, \; \dot{y}_2 = -t^{-2}v + t^{-1}\dot{v}, \; \ddot{y}_2 = 2t^{-3}v - 2t^{-2}\dot{v} + t^{-1}\ddot{v}$
Substitute in the d.e.

$$2t^2(2t^{-3}v - 2t^{-2}\dot{v} + t^{-1}\ddot{v}) + t(-t^{-2}v + t^{-1}\dot{v}) - 3(t^{-1}v) = 0$$

$$2t\ddot{v} - 3\dot{v} = 0$$

Change of variable: $w = \dot{v}$. *We have* $2t\dot{w} - 3w =$, *a 1st order linear d.e.*
Its solution $w = ce^{\int \frac{3}{2t}dt} = ct^{\frac{3}{2}}$
Since $w = \dot{v}$ *we have* $v = \int wdt = ct^{\frac{3}{2}}dt = \frac{3c}{2}t^{\frac{5}{2}}$
The other solution is $y_2(t) = t^{-1}\frac{3c}{2}t^{\frac{5}{2}}, \; y_2(t) = kt^{\frac{3}{2}}$ *where* k *is an arbitrary constant*

Good Luck
A. Karamancıoğlu