Name:
ID No.

Eskişehir Osmangazi University - Electrical Engineering Department
Differential Equations- Midterm Examination - Fall 2025

Duration: 70 minutes; **Directions**: All answers should be positioned beside or below their corresponding questions. Anything written elsewhere won't be graded.

Question 1.

Find a family of solutions for

$$\frac{dy}{dx} = y^2 + y - 2$$

when a known solution $y = 1$ *is given. Show your work. Other solutions have the form:*

$$y = 1 + \frac{1}{z}$$

This leads to $\dot{y} = -\frac{\dot{z}}{z^2}$. *Substitute in the d.e.:*

$$-\frac{\dot{z}}{z^2} = (1 + \frac{1}{z})^2 + (1 + \frac{1}{z}) - 2$$

This simplifies to

$$-\frac{\dot{z}}{z^2} = \frac{1}{z^2} + \frac{3}{z}$$

Multiply throughout by z^2*:*

$$-\dot{z} = 1 + 3z \text{ or } \dot{z} + 3z = -1$$

It is a first order linear d.e.:

$$\frac{d}{dx}(ze^{3x}) = -e^{3x} \implies ze^{3x} = \frac{-1}{3}e^{3x} + c \implies z = -\frac{1}{3} + ce^{-3x}$$

Family of other solutions are, therefore,

$$y(x) = 1 + \frac{1}{ce^{-3x} - \frac{1}{3}}$$

Question 2.

Find the power series solution of the differential equation

$$\dot{y} + x^3 y = 0$$

about $x = 0$ by:
Assuming a solution of the form $y(x) = \sum_{n=0}^{\infty} c_n x^n$,
Determining the recurrence relation for the coefficients c_n, and
Finding, at least, the first four nonzero terms of the series.
Show your work. Assumed solution:

$$y(x) = \sum_{n=0}^{\infty} c_n x^n$$

Substitute in the differential equation:

$$\sum_{n=1}^{\infty} n c_n x^{n-1} + x^3 \sum_{n=0}^{\infty} c_n x^n = 0, \ \textit{or} \ \sum_{n=1}^{\infty} n c_n x^{n-1} + \sum_{n=0}^{\infty} c_n x^{n+3} = 0$$

This can be written as:

$$\sum_{n=1}^{\infty} n c_n x^{n-1} + \sum_{n=4}^{\infty} c_{n-4} x^{n-1} = 0$$

Combining the Series

$$(1 \cdot c_1 x^0 + 2 \cdot c_2 x^1 + 3 \cdot c_3 x^2) + \sum_{n=4}^{\infty} n c_n x^{n-1} + \sum_{n=4}^{\infty} c_{n-4} x^{n-1} = 0 \ \textit{or} \ (c_1 + 2c_2 x + 3c_3 x^2) + \sum_{n=4}^{\infty} (n c_n + c_{n-4}) x^{n-1} = 0$$

The coefficient of each power of x must be zero.
Coefficient of x^0: $c_1 = 0$; Coefficient of x^1: $2c_2 = 0 \implies c_2 = 0$; Coefficient of x^2: $3c_3 = 0 \implies c_3 = 0$
Coefficient of x^{n-1} for $n \geq 4$:

$$n c_n + c_{n-4} = 0$$

This gives the recurrence relation:

$$c_n = -\frac{c_{n-4}}{n} \quad \textit{for } n \geq 4$$

Calculate the coefficients:
$n = 4$: $c_4 = -\frac{c_{4-4}}{4} = -\frac{c_0}{4}$
$n = 5$: $c_5 = -\frac{c_1}{5} = -\frac{0}{5} = 0$
$n = 6$: $c_6 = -\frac{c_2}{6} = -\frac{0}{6} = 0$
$n = 7$: $c_7 = -\frac{c_3}{7} = -\frac{0}{7} = 0$
$n = 8$: $c_8 = -\frac{c_4}{8} = -\frac{1}{8}\left(-\frac{c_0}{4}\right) = \frac{c_0}{8 \cdot 4}$
$n = 9$: $c_9 = -\frac{c_5}{9} = 0$
$n = 10$: $c_{10} = -\frac{c_6}{10} = 0$
$n = 11$: $c_{11} = -\frac{c_7}{11} = 0$
$n = 12$: $c_{12} = -\frac{c_8}{12} = -\frac{1}{12}\left(\frac{c_0}{8 \cdot 4}\right) = -\frac{c_0}{12 \cdot 8 \cdot 4}$
The Power Series Solution Substituting the coefficients back into $y(x) = \sum_{n=0}^{\infty} c_n x^n$:

$$y(x) = c_0 + c_4 x^4 + c_8 x^8 + c_{12} x^{12} + \ldots$$

$$y(x) = c_0 - \frac{c_0}{4} x^4 + \frac{c_0}{8 \times 4} x^8 - \frac{c_0}{12 \times 8 \times 4} x^{12} + \ldots$$

$$y(x) = c_0 \left(1 - \frac{1}{4} x^4 + \frac{1}{8 \times 4} x^8 - \frac{1}{12 \times 8 \times 4} x^{12} + \ldots\right)$$

Good Luck
A. Karamancıoğlu