Name:
ID No.

Eskişehir Osmangazi University - Electrical Engineering Department
Differential Equations- Midterm Examination - Fall 2026

Duration: 65 minutes;
Directions: No calculators, no class notes. All answers should be positioned beside or below their corresponding questions. Anything written elsewhere won't be graded.
Recommendation: To avoid mistakes, always verify your solutions. Verification is for your own benefit and is not part of the exam submission. Therefore, do not include your verification calculations in your exam paper.

Question 1.

Solve the following differential equation:

$$\boxed{\frac{dy}{dx} + \frac{7}{x}y = x^2}$$

subject to the initial condition

$$y(1) = 4.$$

This is a first-order linear equation. The integrating factor is

$$\mu(x) = e^{\int \frac{7}{x}\,dx} = e^{7\ln x} = x^7.$$

Multiplying the equation by $\mu(x)$ gives

$$\frac{d}{dx}\left(x^7 y\right) = x^9 \implies x^7 y = \frac{x^{10}}{10} + C \implies y = \frac{x^3}{10} + \frac{C}{x^7}.$$

Using $y(1) = 4$:

$$4 = \frac{1}{10} + C \implies C = \frac{39}{10}.$$

$$\boxed{y(x) = \frac{x^3}{10} + \frac{39}{10\,x^7}}$$

Question 2.

Solve the following differential equation:

$$\boxed{\frac{dy}{dx} = 2x(y+1)}$$

subject to the initial condition

$$y(0) = 2.$$

The equation is separable:

$$\frac{dy}{y+1} = 2x\,dx \implies \ln|y+1| = x^2 + c_1 \implies y + 1 = Ce^{x^2}.$$

Using $y(0) = 2$:

$$3 = C \cdot e^0 \implies C = 3.$$

$$\boxed{y(x) = 3e^{x^2} - 1}$$

Question 3.

Find a general solution for the following differential equation:

$$\boxed{y'' + 4y = 8\cos(x)}$$

Consider the homogeneous equation:

$$y'' + 4y = 0$$

The corresponding characteristic equation is:

$$r^2 + 4 = 0 \implies r = \pm 2i$$

Since the roots are purely imaginary complex conjugates ($r = 0 \pm 2i$), the complementary solution is:

$$y_c(x) = C_1 \cos(2x) + C_2 \sin(2x)$$

where C_1 and C_2 are arbitrary constants. Particular Solution (y_p)
Assume a solution of the form:

$$y_p(x) = A\cos(x) + B\sin(x) \rightarrow$$
$$y_p'(x) = -A\sin(x) + B\cos(x)$$
$$y_p''(x) = -A\cos(x) - B\sin(x)$$

Substitute $y_p(x)$ and $y_p''(x)$ into the original equation:

$$(-A\cos(x) - B\sin(x)) + 4(A\cos(x) + B\sin(x)) = 8\cos(x) \rightarrow$$

$$3A\cos(x) + 3B\sin(x) = 8\cos(x) \rightarrow$$

$$3A = 8 \implies A = \frac{8}{3}$$
$$3B = 0 \implies B = 0$$

The particular solution is:

$$y_p(x) = \frac{8}{3}\cos(x)$$

The general solution is:

$$y(x) = C_1 \cos(2x) + C_2 \sin(2x) + \frac{8}{3}\cos(x)$$

Question 4.

Find a general solution for the following differential equation:

$$\boxed{y'' + 4y = 8\cos(2x)}$$

Homogeneous solution. *The characteristic equation is $r^2 + 4 = 0$, so $r = \pm 2i$ and*

$$y_h = c_1 \cos(2x) + c_2 \sin(2x).$$

Particular solution. *Since $\cos(2x)$ already solves the homogeneous equation, we multiply by x:*

$$y_p = x\big(A\cos(2x) + B\sin(2x)\big).$$

Differentiating twice,

$$y_p'' = -4A\sin(2x) + 4B\cos(2x) - 4x\big(A\cos(2x) + B\sin(2x)\big),$$

so

$$y_p'' + 4y_p = -4A\sin(2x) + 4B\cos(2x) = 8\cos(2x).$$

Matching coefficients gives $A = 0$ and $B = 2$, hence $y_p = 2x\sin(2x)$.

General solution

$$\boxed{y(x) = c_1 \cos(2x) + c_2 \sin(2x) + 2x\sin(2x)}$$

Good Luck!
A. Karamancıoğlu
