Name:
ID No.

Eskişehir Osmangazi University - Electrical Engineering Department
Differential Equations- Final Examination - Fall 2023

Duration: 50 minutes; **Directions**: All answers should be positioned beside or below their corresponding questions. Anything written elsewhere won't be graded. In the decimal parts of the results, up to one percent error is tolerated.

Question 1.

30 points. Find a general solution to

$$\dot{y} = -2xy$$

Show your work.

$$\dot{y} + 2xy = 0, \text{ linear, 1st order d.e. } P(x) = 2x \rightarrow \mu(x) = e^{\int P(x)dx} = e^{x^2}$$

$$\left(ye^{x^2}\right)' = 0 \rightarrow y(x) = ce^{-x^2}$$

where c is arbitrary constant.

Question 2.

30 points. Solve

$$x^2\frac{d^2y}{dx^2} = -7x\frac{dy}{dx} - 9y, \;\; y(1) = 5, \; \dot{y}(1) = 10$$

Show your work.

$$x^2\frac{d^2y}{dx^2} + 7x\frac{dy}{dx} + 9y = 0$$

Substitute $y(x) = x^r$ in the d.e., this results in

$$x^r(r^2 + 6r + 9) = 0 \rightarrow r_1 = r_2 = -3$$

General solution

$$y(x) = c_1x^{-3} + c_2x^{-3}\ln x$$

where c_1, c_2 are arbitrary constants. Use the initial conditions:

$$y(1) = 5 \rightarrow c_1 = 5$$

$$y'(x) = -3c_1x^{-4} + c_2(-3x^{-4}\ln x + \frac{1}{x}x^{-3})$$

$$y'(1) = 10 \rightarrow 10 = -15 + c_2 \rightarrow c_2 = 25$$

$$y(x) = 5x^{-3} + 25x^{-3}\ln x$$

Question 3.

40 points. Find V_1, V_2, V_3 *so that* $c_1 V_1 e^{3t} + c_2 V_2 e^{2t} + c_3 V_3 e^{2t}$ *is a general solution to*

$$\dot{x} = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 2 & 2 \\ 0 & 0 & 2 \end{bmatrix} x$$

where c_1, c_2, c_3 *are arbitrary constant scalars. Show your work.*

Characteristic vector for 3 results in V_1*:*

$$V_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

Characteristic vector for 2 results in V_2*:*

$$V_2 = \begin{bmatrix} 0 \\ 2 \\ 0 \end{bmatrix}$$

One more linearly independent solution is needed. This solution has the form $(V_2 t + W)e^{2t}$*. Substituting this in the d.e., we obtain* V_3 *as*

$$V_3 = \begin{bmatrix} 0 \\ 2t \\ 1 \end{bmatrix}$$

Thus the general solution is

$$x(t) = c_1 \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} e^{3t} + c_2 \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} e^{2t} + c_3 \begin{bmatrix} 0 \\ 2t \\ 1 \end{bmatrix} e^{2t}$$

Note that the V_1, V_2, V_3 *representations are not unique.*

Good Luck
A. Karamancıoğlu