Name:
ID No.

Eskişehir Osmangazi University - Electrical Engineering Department
Differential Equations- Final Examination - Fall 2024

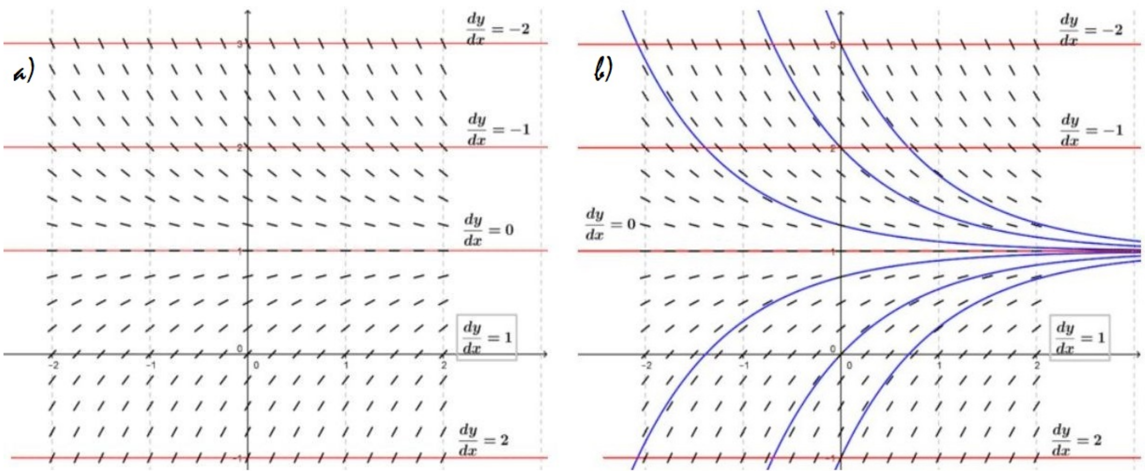
Duration: 70 minutes; **Directions**: All answers should be positioned beside or below their corresponding questions. Anything written elsewhere won't be graded. Numerical errors within a 1% range are allowed.

Question 1.

30 points. a) On the xy plane, sketch five or more isoclines for

$$\frac{dy}{dx} = 1 - y$$

and sketch the line elements on each isocline.
b) Use part a) to sketch three or more trajectories on the xy plane.



Question 2.

35 points. Solve

$$\frac{dx(t)}{dt} = \begin{bmatrix} 0 & 1 \\ 2 & 1 \end{bmatrix} x(t), \quad x(0) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

Show your work.
Characteristic equation:

$$|A - \lambda I| = \det \begin{bmatrix} -\lambda & 1 \\ 2 & 1-\lambda \end{bmatrix} = \lambda^2 - \lambda - 2 = 0, \quad \lambda_{1,2} = 2, -1$$

Characteristic vectors:

$$\begin{bmatrix} -\lambda & 1 \\ 2 & 1-\lambda \end{bmatrix}_{\lambda=2} V = \begin{bmatrix} 0 \\ 0 \end{bmatrix}; \quad \begin{bmatrix} -2 & 1 \\ 2 & -1 \end{bmatrix} V = \begin{bmatrix} 0 \\ 0 \end{bmatrix}; \quad \textit{a solution:} \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

$$\begin{bmatrix} -\lambda & 1 \\ 2 & 1-\lambda \end{bmatrix}_{\lambda=-1} V = \begin{bmatrix} 0 \\ 0 \end{bmatrix}; \quad \begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix} V = \begin{bmatrix} 0 \\ 0 \end{bmatrix}; \quad \textit{a solution:} \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

General solution:

$$x(t) = c_1 \begin{bmatrix} 1 \\ 2 \end{bmatrix} e^{2t} + c_2 \begin{bmatrix} 1 \\ -1 \end{bmatrix} e^{-t}$$

Use the initial conditions:

$$\begin{bmatrix} 1 \\ 0 \end{bmatrix} = c_1 \begin{bmatrix} 1 \\ 2 \end{bmatrix} e^{2\cdot 0} + c_2 \begin{bmatrix} 1 \\ -1 \end{bmatrix} e^{-0} \to \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} = \begin{bmatrix} \frac{1}{3} \\ \frac{2}{3} \end{bmatrix}$$

The solution:

$$x(t) = \frac{1}{3} \begin{bmatrix} 1 \\ 2 \end{bmatrix} e^{2t} + \frac{2}{3} \begin{bmatrix} 1 \\ -1 \end{bmatrix} e^{-t}$$

Question 3.

35 points. Consider the initial value problem

$$\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 3y = e^{-2x}, \quad y(0) = 1,\ \dot{y}(0) = 3$$

a) 10p. Let the Laplace transform of y *be* Y*. Obtain* $Y(s)$ *in the form* $Y(s) = \frac{N(s)}{D(s)}$ *where* $N(s)$ *and* $D(s)$ *are polynomials.*
b) 25p. Find $y(x)$ *by inverse transforming* $Y(s)$ *obtained in part a).*
Show your work.

a)

$$s^2Y(s) - s - 3 + 4sY(s) - 4 + 3Y(s) = \frac{1}{s+2}$$

$$s^2Y(s) + 4sY(s) + 3Y(s) = \frac{1}{s+2} + s + 7$$

$$(s^2 + 4s + 3)Y(s) = \frac{1}{s+2} + s + 7$$

$$Y(s) = \frac{1}{(s+1)(s+2)(s+3)} + \frac{s+7}{(s+1)(s+3)}$$

$$Y(s) = \frac{s^2 + 9s + 15}{(s+1)(s+2)(s+3)}$$

$$Y(s) = \frac{s^2 + 9s + 15}{s^3 + 6s^2 + 11s + 6}$$

b)

$$Y(s) = \frac{s^2 + 9s + 15}{(s+1)(s+2)(s+3)}$$

$$Y(s) = \frac{A}{s+1} + \frac{B}{s+2} + \frac{C}{s+3}$$

$$A = 7/2;\ B = -1;\ C = -3/2$$

$$y(t) = \frac{7}{2}e^{-x} - e^{-2x} - \frac{3}{2}e^{-3x}$$

Good Luck
A. Karamancıoğlu

This page won't be graded! Bu sayfa müsvedde içindir.