Name:
ID. No.

Eskişehir Osmangazi University - Electrical Engineering Department
Fundamentals of Control Systems- Midterm Examination - Spring 2024

Duration: 50 minutes; **Allowed**: A calculator; **Directions**: All answers must be written below the questions. Anything written elsewhere won't be graded. Up to 1% error in the answers are tolerable.

Question 1.

35 points. Consider the 2nd order LTI system whose output response to unit step input is

$$y(t) = 1 - \frac{e^{-1.5t}}{\sqrt{0.91}} \sin(\sqrt{22.75}\, t + 1.2661),\ t \geq 0$$

Find the transfer function of this system. Show your work.

$1 - \zeta^2 = 0.91 \rightarrow \zeta = 0.3,\ \ \zeta w_n = 1.5 \rightarrow w_n = 5$

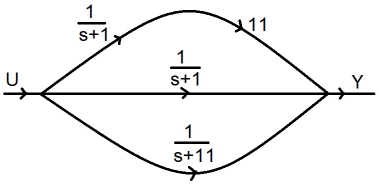
$w_d = 5\sqrt{1-0.3^2} = 4.7697 = \sqrt{22.75}$ ✓ $\tan^{-1}\dfrac{\sqrt{1-0.3^2}}{0.3} = 1.2661$✓

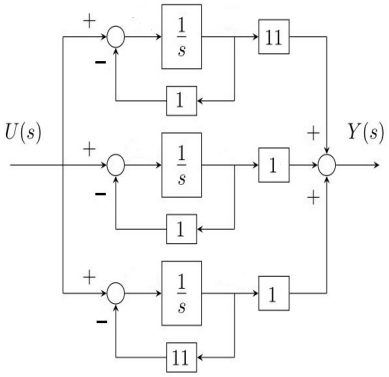
$$\rightarrow \frac{Y(s)}{U(s)} = \frac{w_n^2}{s^2 + 2\zeta w_n s + w_n^2} = \frac{25}{s^2 + 3s + 25}$$

Question 2.

35 points. Find the transfer function $\dfrac{Y(s)}{U(s)}$ *for the configuration given on the right in the form of*

$$\frac{a_m s^m + a_{m-1}s^{m-1} + \cdots + a_1 s + a_0}{b_n s^n + b_{n-1}s^{n-1} + \cdots + b_1 s + b_0}$$





Add the parallel branches:

$11\dfrac{1}{s+1} + \dfrac{1}{s+1} + \dfrac{1}{s+11} = \dfrac{13s+133}{s^2+12s+11}$ ✓

Mason's formula also gives an acceptable result:

$\frac{13s^2+146s+133}{s^3+13s^2+23s+11}$

which is just $\frac{13s+133}{s^2+12s+11} \times \frac{s+1}{s+1} = \frac{13s^2+146s+133}{s^3+13s^2+23s+11}$ ✓

Question 3.

30 points Find the transfer function $\dfrac{Y(s)}{U(s)}$ *for the state space model below*

$$\dot{x} = \begin{bmatrix} 0 & 1 \\ 3 & 4 \end{bmatrix} x + \begin{bmatrix} 1 \\ 1 \end{bmatrix} u,\ \ y = \begin{bmatrix} 2 & 1 \end{bmatrix} x + 4u$$

$$\begin{bmatrix} 2 & 1 \end{bmatrix} \left(s \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 0 & 1 \\ 3 & 4 \end{bmatrix} \right)^{-1} \begin{bmatrix} 1 \\ 1 \end{bmatrix} + 4 = \frac{4s^2 - 13s - 15}{s^2 - 4s - 3}$$

Good Luck
A. Karamancıoğlu