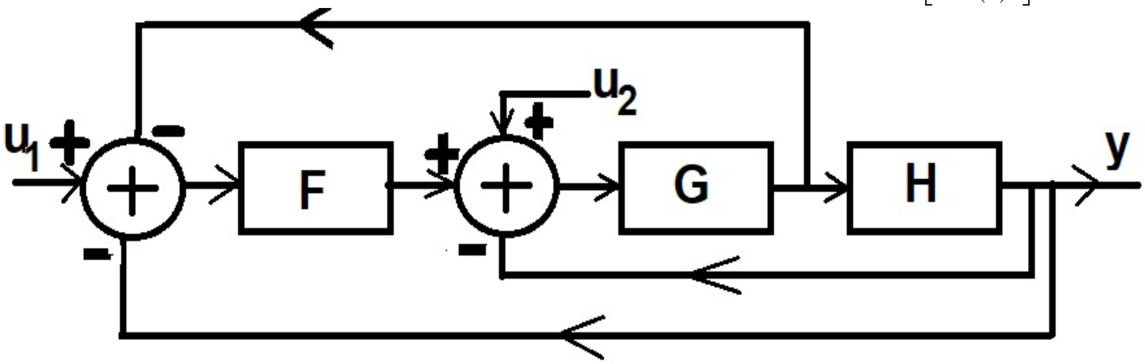
Name:
ID. No.

Eskişehir Osmangazi University - Electrical Engineering Department
Fundamentals of Control Systems- Midterm Examination - Spring 2025

Duration: 75 minutes; **Allowed**: A calculator; **Directions**: All answers must be written below the questions. Anything written elsewhere won't be graded. Up to 1% error in the answers are tolerable.

Question 1.

35 points. Consider the feedbeack system configuration below, where F, G, and H are transfer functions of LTI systems. Obtain the transfer function matrix $M(s)$ such that $Y(s) = M(s)\begin{bmatrix} U_1(s) \\ U_2(s) \end{bmatrix}$. Show your work.



Loop gains: $L_1 = -FG,\ L_2 = -GH,\ L_3 = -FGH$
Determinant $\Delta = 1 + FG + GH + FGH$
Path from u_1 *to y:* FGH*, Subdeterminant=1.*
Path from u_2 *to y:* GH*, Subdeterminant=1.*

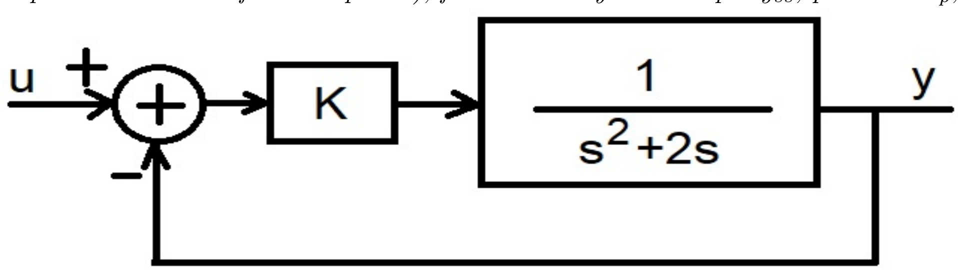
$$Y(s) = \begin{bmatrix} \frac{FGH}{1+FG+GH+FGH} & \frac{GH}{1+FG+GH+FGH} \end{bmatrix} \begin{bmatrix} U_1(s) \\ U_2(s) \end{bmatrix}$$

Thus, the transfer function matrix is

$$M(s) = \begin{bmatrix} \frac{FGH}{1+FG+GH+FGH} & \frac{GH}{1+FG+GH+FGH} \end{bmatrix}$$

Question 2.

A step input of magnitude 2 is applied to the system shown below.
a) 20 points. Determine the value of K for which the maximum overshoot $M_p = 0.2$*. Show your work.*
b) 15 points. For the K found in part a), find the steady state output y_{ss}*, peak time* t_p*, and peak value* $y(t_p)$*.*



a)

$$M_p = e^{\frac{-\zeta\pi}{\sqrt{1-\zeta^2}}} = 0.2,\ \frac{-\zeta\pi}{\sqrt{1-\zeta^2}} = \ln 0.2,\ (\zeta\pi)^2 = (1-\zeta^2)(\ln 0.2)^2,\ \zeta^2(\pi^2 + (\ln 0.2)^2) = (\ln 0.2)^2,\ \zeta = 0.4559$$

The closed loop system's transfer function is $\frac{K}{s^2+2s+K}$*. This implies*

$$\zeta = \sqrt{K},\ 2\zeta w_n = 2 \rightarrow \zeta = \frac{1}{\sqrt{K}} \rightarrow K = 4.81$$

b)

$$y_{ss} = 2,\ t_p = \frac{\pi}{w_d} = \frac{\pi}{2.19\sqrt{1-0.2079}} = 1.61,\ y(t_p) = (1+M_p)y_{ss} = (1+0.2)2 = 2.4$$

Question 3.

30 points. Consider the state space model below

$$\dot{x} = \begin{bmatrix} -4 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -1 \end{bmatrix} x + \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} u, \;\; x(0) = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \;\; y = \begin{bmatrix} 4 & 6 & 7 \end{bmatrix} x$$

where u is the unit step function.
a) Find the state transition matrix. Show your work.
b) Find $x(t),\ t \geq 0$ by using the answer of part a). Show your work.
c) Find $y(t),\ t \geq 0$. Show your work
a)

$$(sI - A)^{-1} = \begin{bmatrix} \frac{1}{s+4} & 0 & 0 \\ 0 & \frac{1}{s+2} & 0 \\ 0 & 0 & \frac{1}{s+1} \end{bmatrix} \longleftrightarrow e^{At} = \begin{bmatrix} e^{-4t} & 0 & 0 \\ 0 & e^{-2t} & 0 \\ 0 & 0 & e^{-t} \end{bmatrix}$$

b)

$$x(t) = e^{A(t-t_0)}x(t_0) + \int_{t_0}^{t} e^{A(t-\tau)}Bu(\tau)d\tau$$

$$= \begin{bmatrix} 0 \\ 0 \\ e^{-t} \end{bmatrix} + \int_0^t \begin{bmatrix} e^{-4\tau} \\ 0 \\ 0 \end{bmatrix} d\tau = \begin{bmatrix} 0 \\ 0 \\ e^{-t} \end{bmatrix} + \begin{bmatrix} -\frac{1}{4}e^{-4t} + \frac{1}{4} \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} -\frac{1}{4}e^{-4t} + \frac{1}{4} \\ 0 \\ e^{-t} \end{bmatrix}$$

c)

$$y = \begin{bmatrix} 4 & 6 & 7 \end{bmatrix} \begin{bmatrix} -\frac{1}{4}e^{-4t} + \frac{1}{4} \\ 0 \\ e^{-t} \end{bmatrix} = 1 - e^{-4t} + 7e^{-t}$$

Good Luck!
A. Karamancıoğlu