Name:
ID. No.

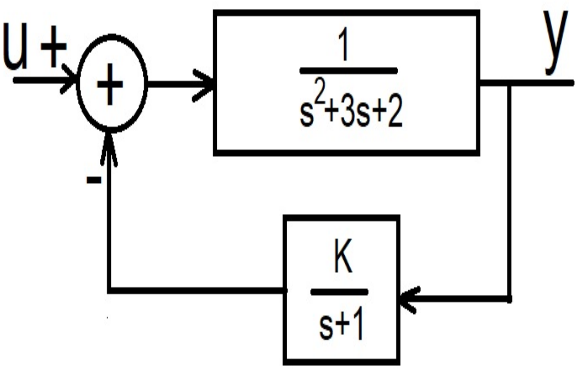
Eskişehir Osmangazi University - Electrical Engineering Department
Fundamentals of Control Systems- Final Examination - Spring 2025

Please set your cell phone to silent mode, place it at the far end of the desk, and leave it there for the duration of the exam.

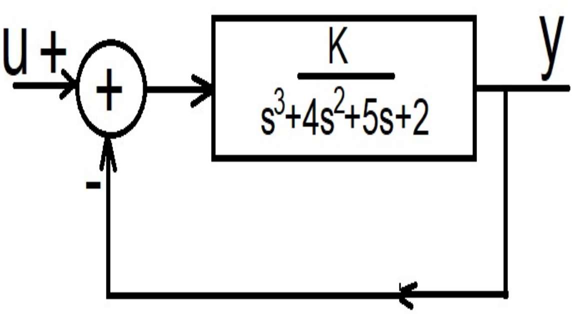
Duration: 70 minutes; **Allowed**: A calculator; **Directions**: All answers must be written below the questions. Anything written elsewhere won't be graded. Up to 1% error in the answers are tolerable.

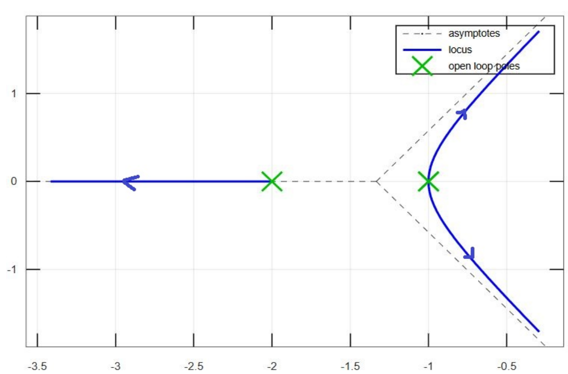
Question 1.

*40 p. Consider the feedback system configuration shown below. Use **root locus techniques** to analyze the system and determine the following:*
(a) Construct the root locus diagram, clearly indicating the trajectories with arrows to show their directions.
(b) Compute and display the asymptote(s).
(c) Determine and mark the imaginary axis crossing point(s).
(d) Identify the range of K values that ensure the stability of the closed-loop system.
Provide detailed calculations and show all necessary steps.



a)
Closed loop tf $\frac{s+1}{(s^2+3s+2)(s+1)+K} = \frac{1}{s^3+4s^2+5s+2+K}$
The zeros of $1 + K\frac{1}{s^3+4s^2+5s+2} = 0$ *are the poles of the closed loop system. Thus we analyze the following instead:*





b)Three asymptotes. Their intersection on the real axis: $\frac{\sum poles - \sum zeros}{P-Z} = \frac{-4}{3}$
c) Closed loop system poles on the imaginary axis satisfy: $(s^3+4s^2+5s+2+K)_{s=iw} = -iw^3 - 4w^2 + 5iw + 2 + K = 0$ *or* $-w^3+5w=0$; $-4w^2+2+K=0$. *This leads to* $w = \pm\sqrt{5} = \pm 2.24$ *and* $K = 18$. *So, crossings are at* $\pm i2.24$.
d) Routh table:

1	5
4	$2+K$
$\frac{18-K}{4}$	
$2+K$	

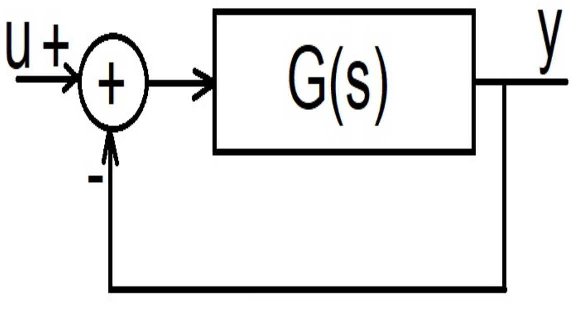
Thus, the system is stable for $K < 18$.

Question 2.

Consider the feedback system configuration shown below and Bode plots of minimum phase LTI system $G(s)$ in the last page.

a) 20 pts. Obtain the transfer function $G(s)$ from the Bode plots. Show your work.

b) 20 pts. Find the gain and phase margins from the Bode plots (For this part of the question: 10% error in the results is tolerable). Show your work.



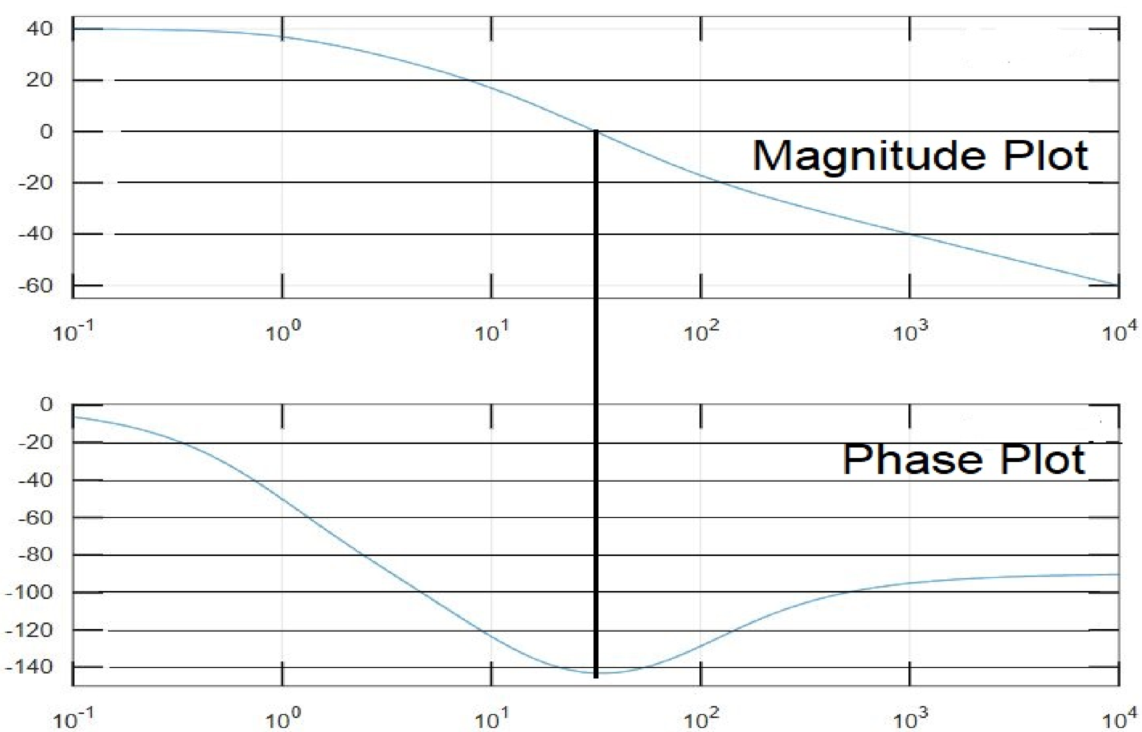
a) Bode magnitude plot has -20 dB/dec slope starting at w=1; this slope becomes -40 dB/dec at w=10; and it becomes -20 dB/dec at w=100. Transfer function has the form:

$$\frac{K(0.01s+1)}{(s+1)(0.1s+1)}$$

If K were 1, then we would have 0 dB at w=1. Since we get 40 dB at w=1, we must have K=100. Thus

$$\frac{10(s+100)}{(s+1)(s+10)}$$

b) When magnitude equals 0 dB, phase equals -145^0. Thus phase margin is $180^0 - 145^0 = 35^0$.
Phase never reaches -180^0, therefore, gain margin is infinity.



Question 3.

20 points. Consider the difference equation

$$y(k+2) + 0.5y(k+1) + 0.2y(k) = u(k),\ k = 0, 1, \ldots$$

where u is a unit step sequence.
a) Find the transfer function $\frac{Y(z)}{U(z)}$ and determine whether this system is stable. Show your work.
b) Find the steady state value of $y(k)$ for any initial condition. Show your work
a) In the z domain, when i.c. are all zero, we have:

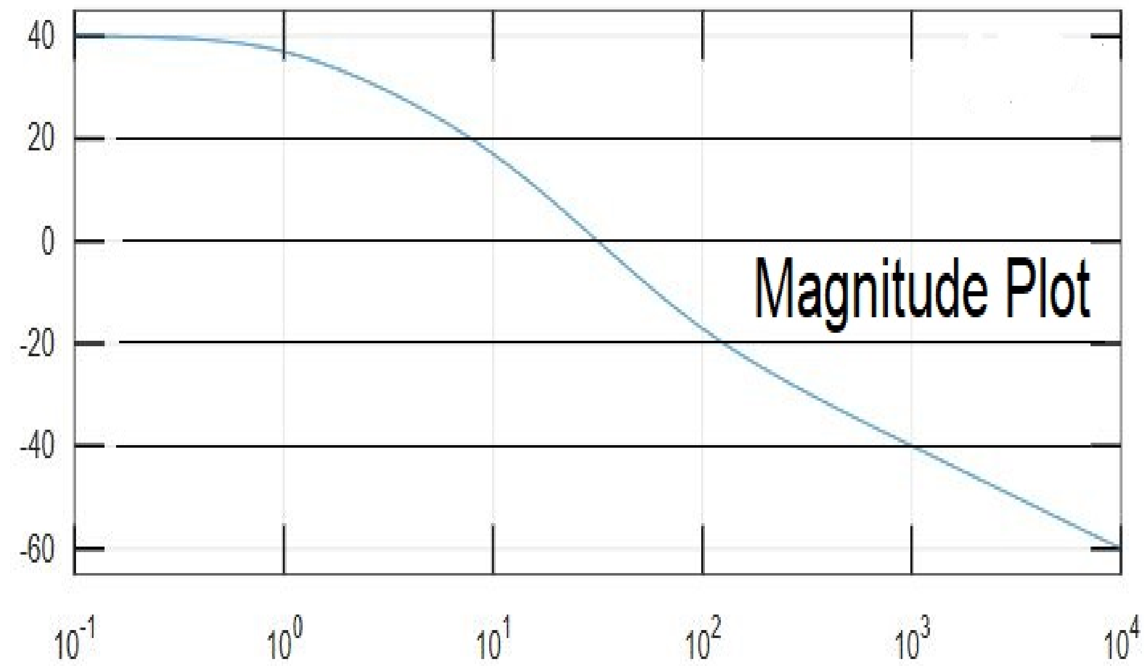
$$z^2Y(z) + 0.5zY(z) + 0.2Y(z) = U(z).$$

$$\frac{Y(z)}{U(z)} = \frac{1}{z^2 + 0.5z + 0.2} = \frac{1}{(z + 0.25 - i0.37)(z + 0.25 + i0.37)}$$

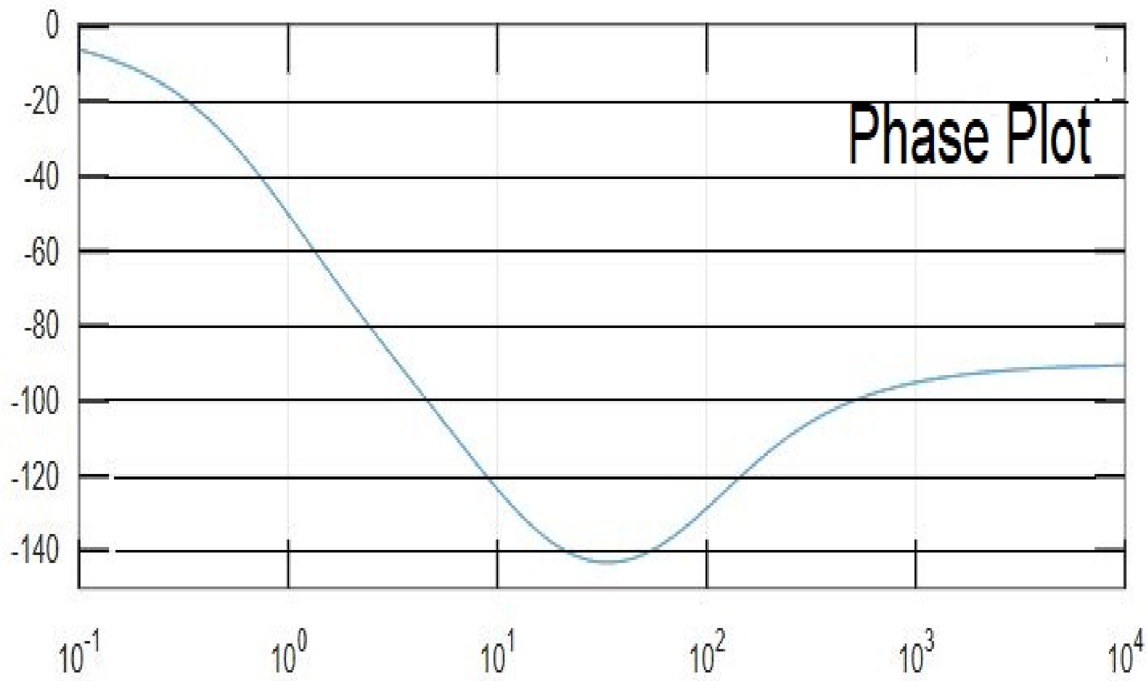
Since the poles are in the unit circle, the system is stable.
b) Use final value theorem:

$$\lim_{z\to 1}(z-1)\frac{1}{z^2 + 0.5z + 0.2}\frac{z}{z-1} = \frac{1}{1.7} = 0.588$$

Good Luck!
A. Karamancıoğlu



40
20
0
-20
-40
-60
Magnitude Plot
10^{-1}
10^{0}
10^{1}
10^{2}
10^{3}
10^{4}



0
-20
-40
-60
-80
-100
-120
-140
Phase Plot
10^{-1}
10^{0}
10^{1}
10^{2}
10^{3}
10^{4}