Name:
ID. No.

Eskişehir Osmangazi University - Electrical Engineering Department
Fundamentals of Control Systems- Midterm Examination - Spring 2026

Duration: 65 minutes; **Allowed**: A calculator; **Directions**: All answers must be written below the questions. Anything written elsewhere won't be graded. Up to 1% error in the answers are tolerable.

Question 1.

35 points. Propose a bounded input signal $x(t)$ that causes the output $|y(t)|$ of the system $G(s) = \frac{1}{s^2+100}$ to go to infinity as $t \to \infty$. Calculate the resulting output $y(t)$ for this specific input.
Let the input be:

$$x(t) = \cos(10t)u(t) \implies X(s) = \frac{s}{s^2+100}$$

The output in the s-domain is:

$$Y(s) = G(s)X(s) = \frac{s}{(s^2+100)^2}$$

Applying the inverse Laplace transform:

$$y(t) = \frac{t}{2(10)}\sin(10t) = \frac{t}{20}\sin(10t)$$

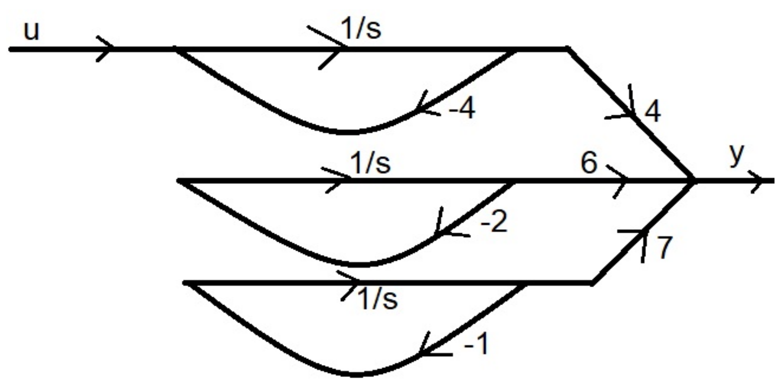
Question 2.

35 points. Consider a linear time-invariant (LTI) system defined by the following state-space representation:

$$\dot{\mathbf{x}}(t) = \begin{bmatrix} -4 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -1 \end{bmatrix} \mathbf{x}(t) + \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} u(t)$$

$$y(t) = \begin{bmatrix} 4 & 6 & 7 \end{bmatrix} \mathbf{x}(t)$$

where $u(t)$ denotes a scalar control input.
(a) Construct a realization of this system utilizing integrators, summation nodes, and constant gain elements.
(b) Apply Mason's Gain Formula to the resulting signal flow graph to derive the system transfer function $G(s) = \frac{Y(s)}{U(s)}$. Explicitly present the forward path gains (P_i), the system determinant (Δ), and the associated sub-determinants (Δ_i) used in your calculation.



- *Only one path exists from $u(t)$ to $y(t)$*

$$P_1 = (1) \cdot \left(\frac{1}{s}\right) \cdot (4) = \frac{4}{s}$$

- *There are three non-touching loops:*

$$L_1 = -\frac{4}{s}, \quad L_2 = -\frac{2}{s}, \quad L_3 = -\frac{1}{s}$$

-

$$\Delta = 1 + \frac{4}{s} + \frac{2}{s} + \frac{1}{s} + \frac{8}{s^2} + \frac{4}{s^2} + \frac{2}{s^2} + \frac{8}{s^3}$$

- $$\Delta_1 = 1 + \frac{2}{s} + \frac{1}{s} + \frac{2}{s^2}$$

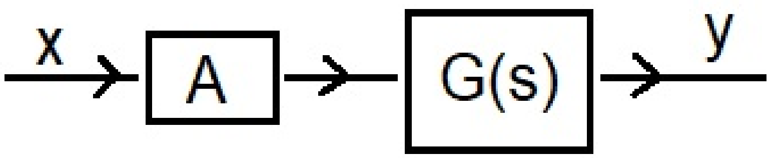
Applying Mason's Gain Formula:

$$G(s) = \frac{P_1\Delta_1}{\Delta}$$

$$G(s) = \frac{\frac{4}{s}\left(1 + \frac{2}{s} + \frac{1}{s} + \frac{2}{s^2}\right)}{1 + \frac{4}{s} + \frac{2}{s} + \frac{1}{s} + \frac{8}{s^2} + \frac{4}{s^2} + \frac{2}{s^2} + \frac{8}{s^3}} = \frac{4}{s+4}$$

Question 3.

30 points Consider the system below with $A = 4$ and $G(s) = \frac{1}{2s^2+6s+18}$.



Let the input x be a unit step function. Find
a) Steady state output $y_{ss}(t)$. Show your work.
b) Peak time of the signal $y(t)$. Show your work.
c) Peak value of the signal $y(t)$. Show your work.
Overall transfer function

$$H(s) = A \cdot G(s) = \frac{4}{2s^2 + 6s + 18} = \frac{2}{s^2 + 3s + 9}$$

$$s^2 + 3s + 9 = s^2 + 2\zeta\omega_n s + \omega_n^2$$

$$\omega_n = 3, \quad \zeta = \frac{3}{2 \cdot 3} = 0.5, \quad \omega_d = \omega_n\sqrt{1-\zeta^2} = 3\sqrt{1-0.25} = \frac{3\sqrt{3}}{2}$$

(a)

Output in Laplace domain

$$Y(s) = H(s)X(s) = \frac{2}{s(s^2 + 3s + 9)}$$

$$y_{ss} = \lim_{s\to 0} sY(s) = \lim_{s\to 0} \frac{2}{s^2 + 3s + 9} = \frac{2}{9}$$

(b)

$$t_p = \frac{\pi}{\omega_d} = \frac{2\pi}{3\sqrt{3}} \approx 1.21 \ s$$

(c)

$$y_{peak} = y_{ss}\left(1 + e^{-\frac{\pi\zeta}{\sqrt{1-\zeta^2}}}\right) = \frac{2}{9}\left(1 + e^{-\frac{\pi}{\sqrt{3}}}\right) \approx 0.258$$

Good Luck!
A. Karamancıoğlu