Spread Spectrum

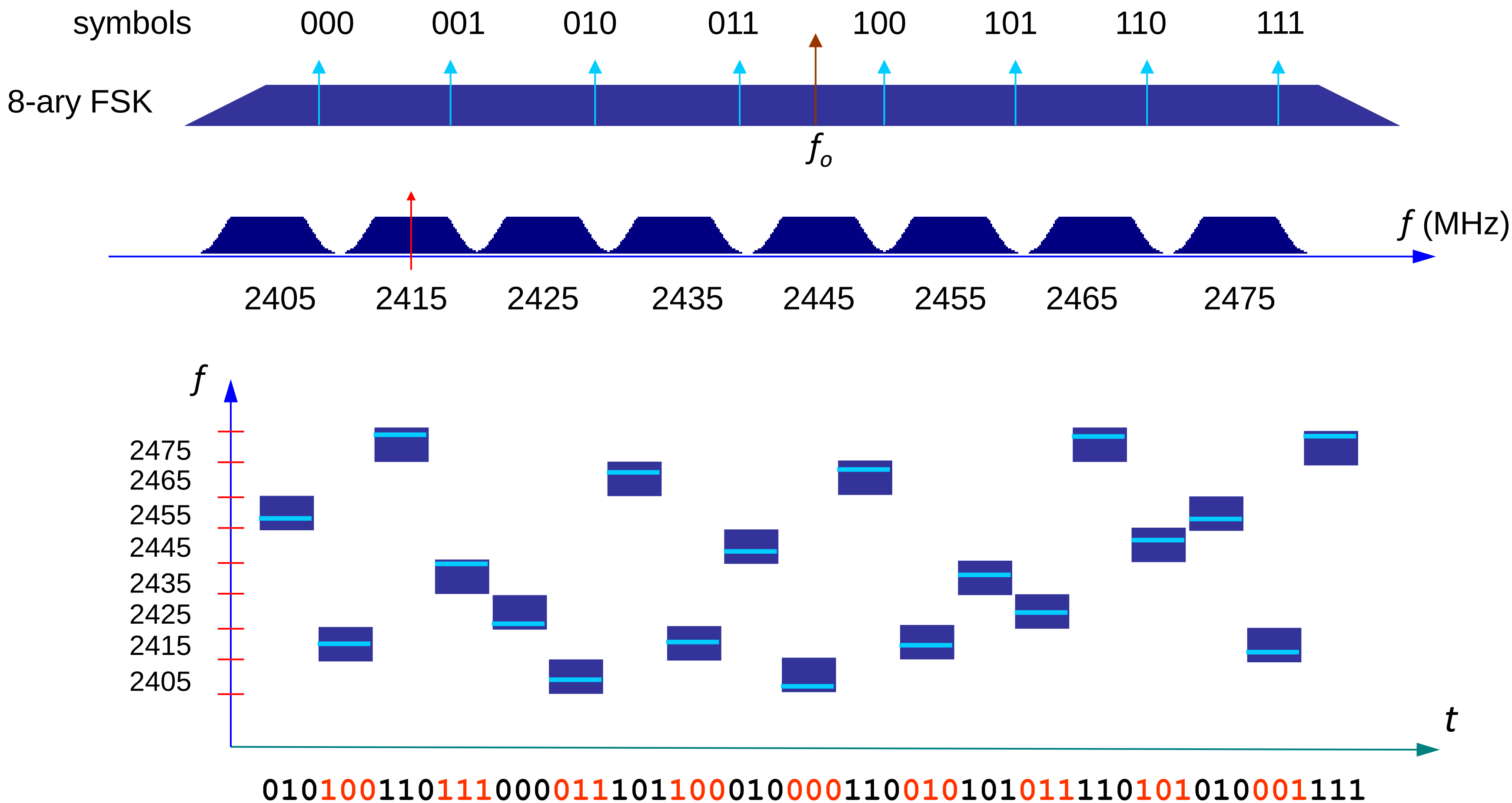
(Part 2)

by Erol Seke

For the course “**Communications**”

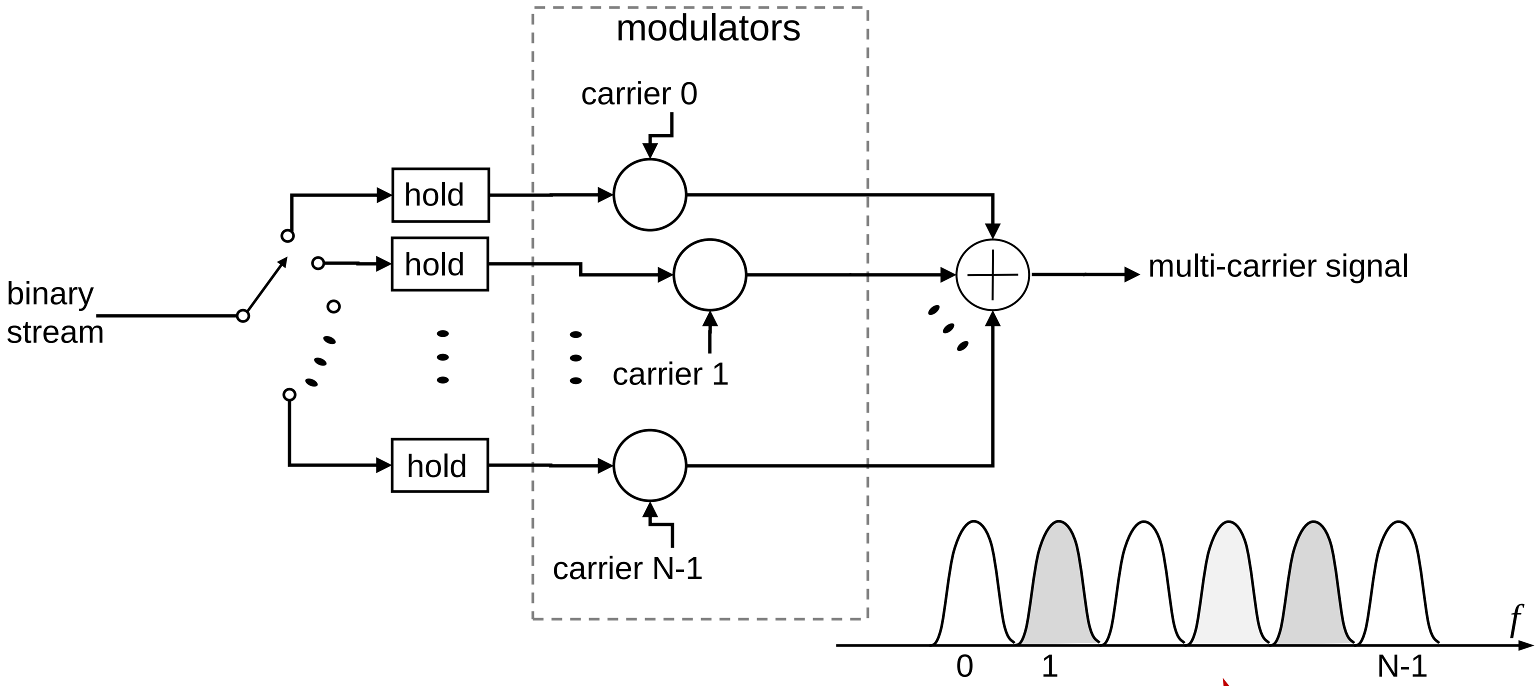
ESKİŞEHİR OSMANGAZİ UNIVERSITY

Remember the sub-channels in frequency hopping example



What if they are all used in parallel by the same single transmitter?

FDM for Single Binary Source



current-symbol

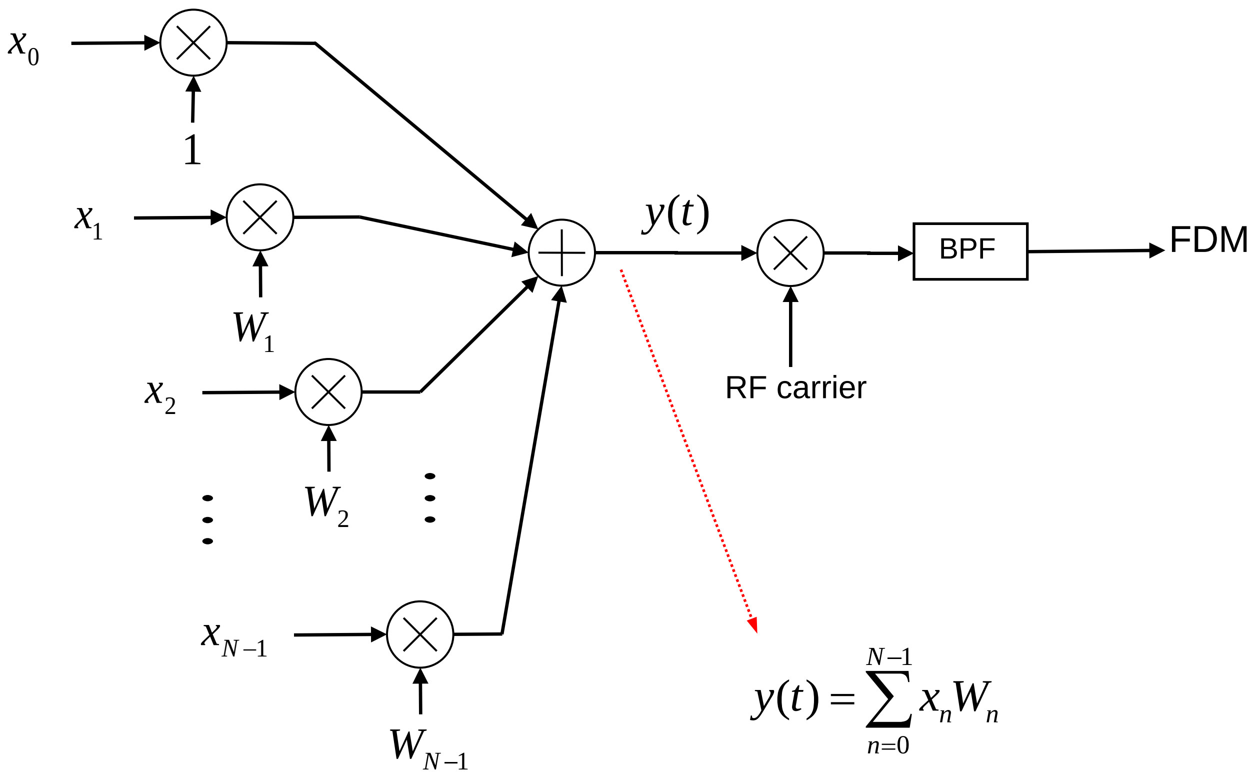
example: ...010111001011 111001000111 010000110110

sub-symbols 11 1001 000 1 11

each sub-symbol is transmitted using T_{si} symbol rate to have a non-overlapping sub-spectrums

(using M-PSK or M-QAM)

Frequency Upconversion of All FDM Channels



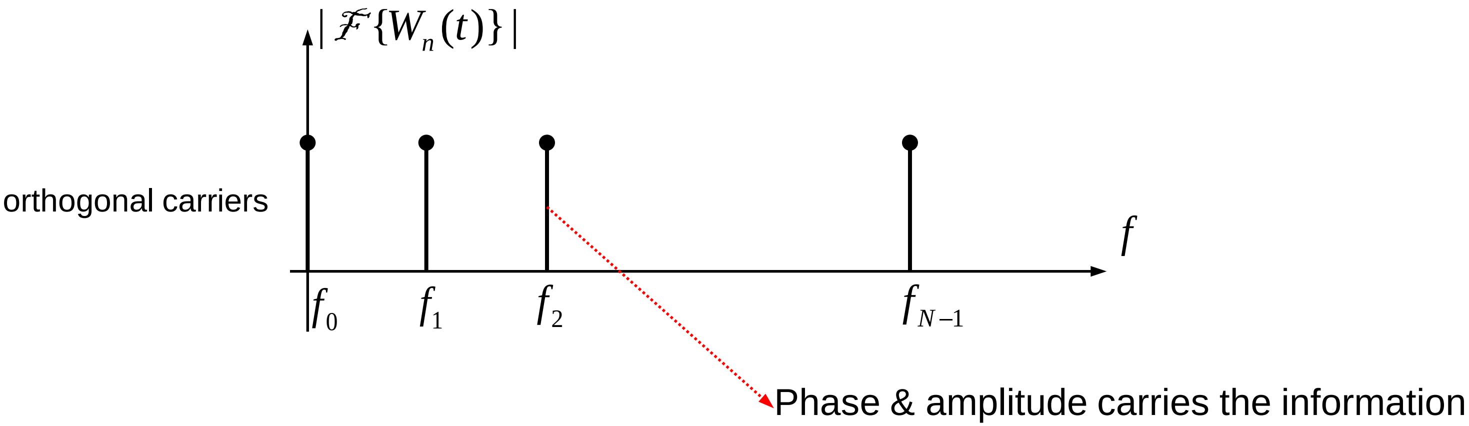
$$y(t) = \sum_{n=0}^{N-1} x_n W_n$$

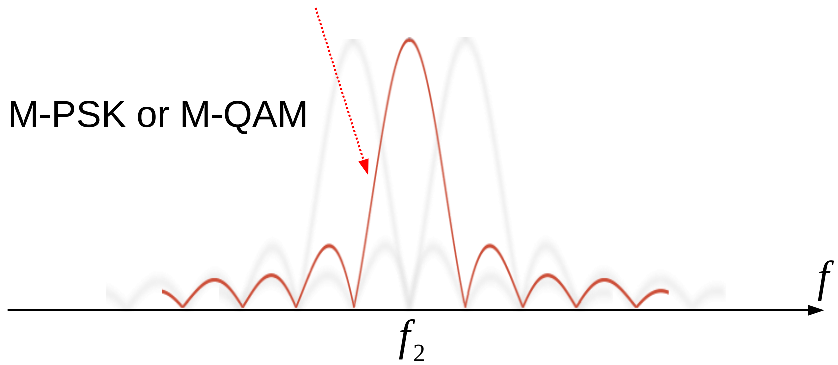
If W_n 's are orthogonal and x_n 's are constant for a symbol period T_s , then it is called OFDM

Orthogonality is maintained even when phase and/or amplitude changes between symbol periods (M-PSK, M-QAM)

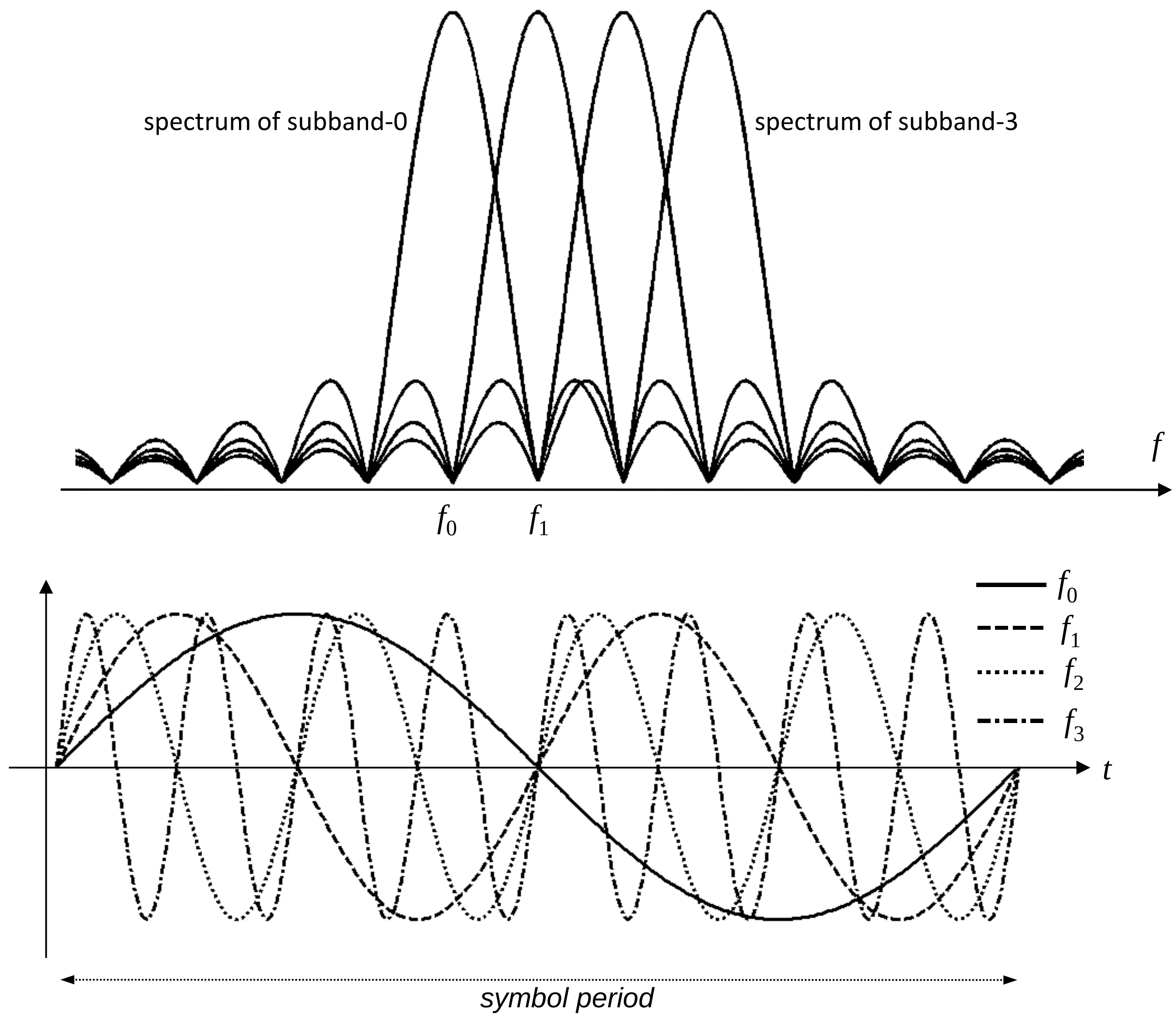
$W_n(t)$'s are orthogonal when $W_n(t) = \cos(2\pi f_n t)$ and $f_n = n / T_s$

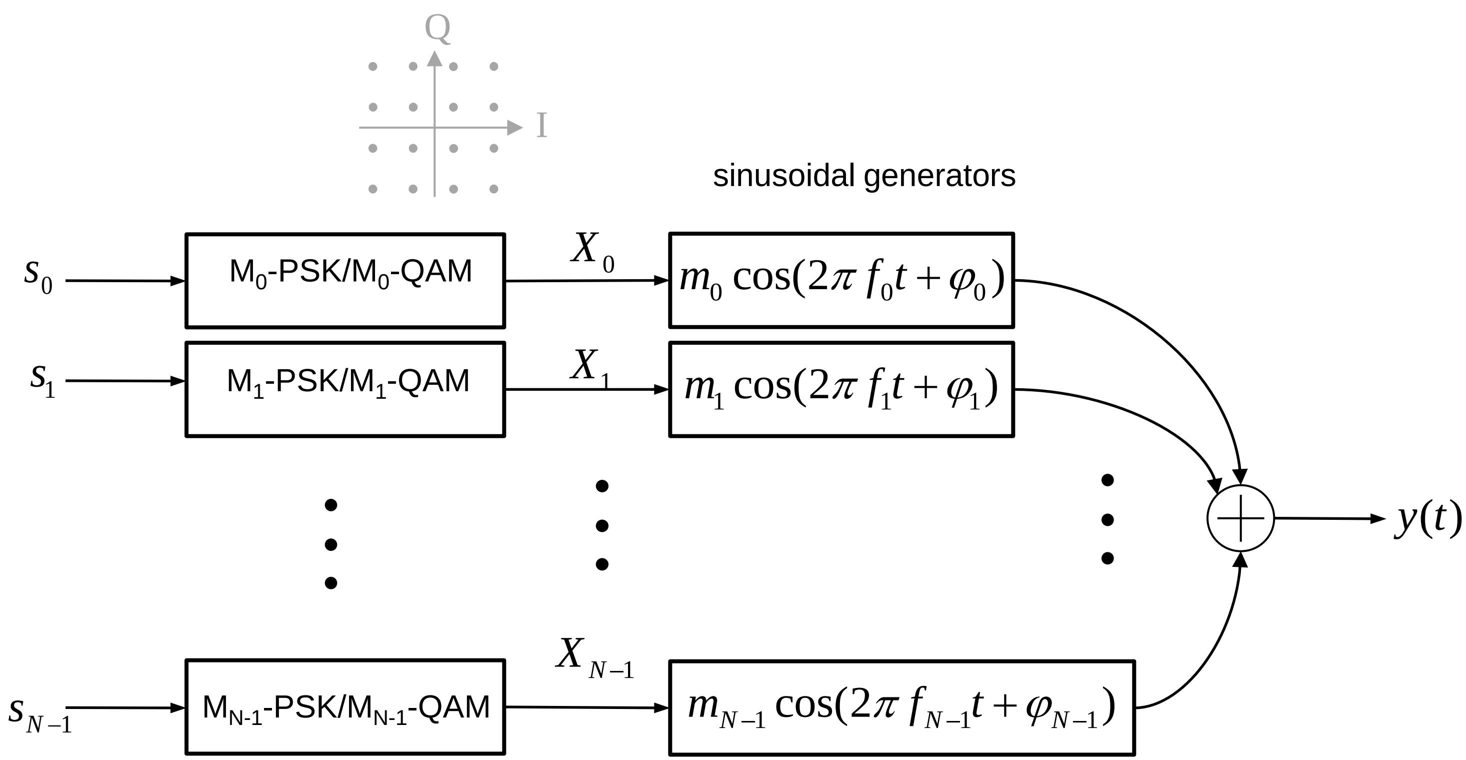
Information is carried by the carriers' phase and amplitude : M-PSK or M-QAM





Orthogonal Carriers for a Symbol Period





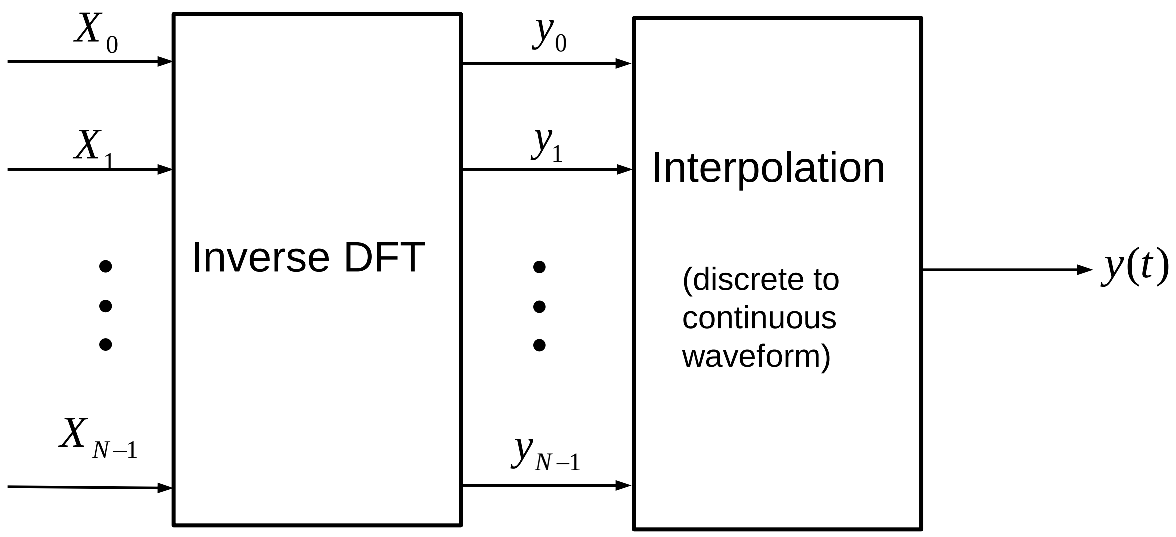
s_n : M_{N-1}-ary symbol

X_n : complex numbers for one symbol period

m_n : magnitudes

φ_n : phases

right block is equivalent to

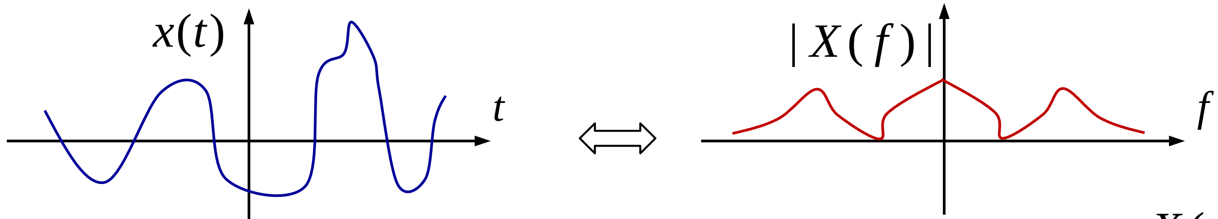


where y_n's are samples of $y(t)$

$$y_n = \sum_{k=0}^{N-1} X_k W_N^{nk} \quad \text{where} \quad W_N^{nk} = e^{j2\pi nk/N}$$

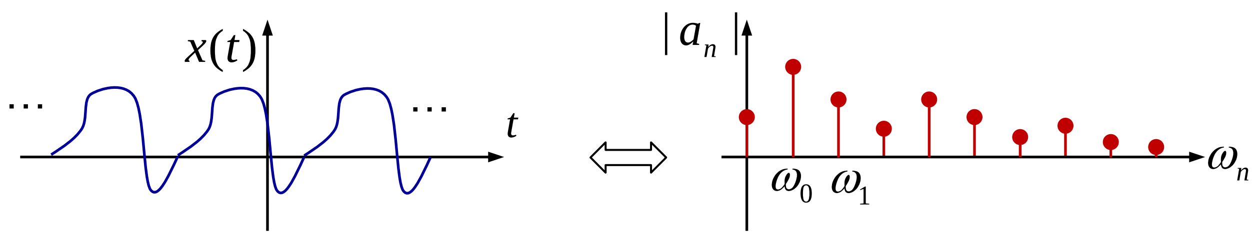
Recall : Fourier …

if $x(t)$ is continuous arbitrary integrable function : $X(\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt$ (Fourier Transform)



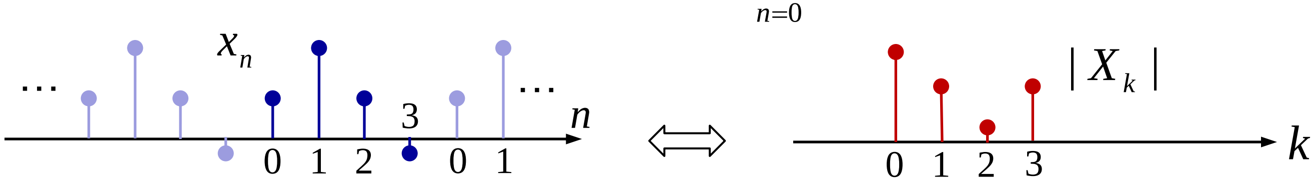
infinite sum of sinusoidals whose magnitude and phase are given by $X(f)$

if $x(t)$ is periodic : $a_n = \frac{1}{T} \int_{-T/2}^{T/2} x(t) e^{-jn\omega_o t} dt$ (Fourier Series Coefficients)



infinite sum of sinusoidals with frequencies of integer multiples of fundamental frequency

if x_n are samples of periodic $x(t)$: $X_k = \sum_{n=0}^{N-1} x_n e^{-j2\pi nk/N}$ (DFT Coefficients)



finite sum of sinusoidals with frequencies of integer multiples of fundamental frequency

Recall : Discrete Fourier Transform (DFT)

Forward Transform $$X_k = \sum_{n=0}^{N=1} x_n e^{-j2\pi nk/N} \qquad k = 0, 1, \cdots, N-1$$

Inverse Transform $$x_n = \tfrac{1}{N} \sum_{k=0}^{N=1} X_k e^{j2\pi nk/N} \qquad n = 0, 1, \cdots, N-1$$

In matrix form

$$\begin{bmatrix} X_0 \\ X_1 \\ \vdots \\ X_{N-1} \end{bmatrix} = \begin{bmatrix} W_N^{0\cdot 0} & W_N^{0\cdot 1} & \cdots & W_N^{0\cdot(N-1)} \\ W_N^{1\cdot 0} & W_N^{1\cdot 1} & \cdots & W_N^{1\cdot(N-1)} \\ \vdots & \vdots & \ddots & \vdots \\ W_N^{(N-1)\cdot 0} & W_N^{(N-1)\cdot 1} & \cdots & W_N^{(N-1)\cdot(N-1)} \end{bmatrix} \begin{bmatrix} x_0 \\ x_1 \\ \vdots \\ x_{N-1} \end{bmatrix} \qquad W_N^{nk} = e^{-j2\pi nk/N}$$

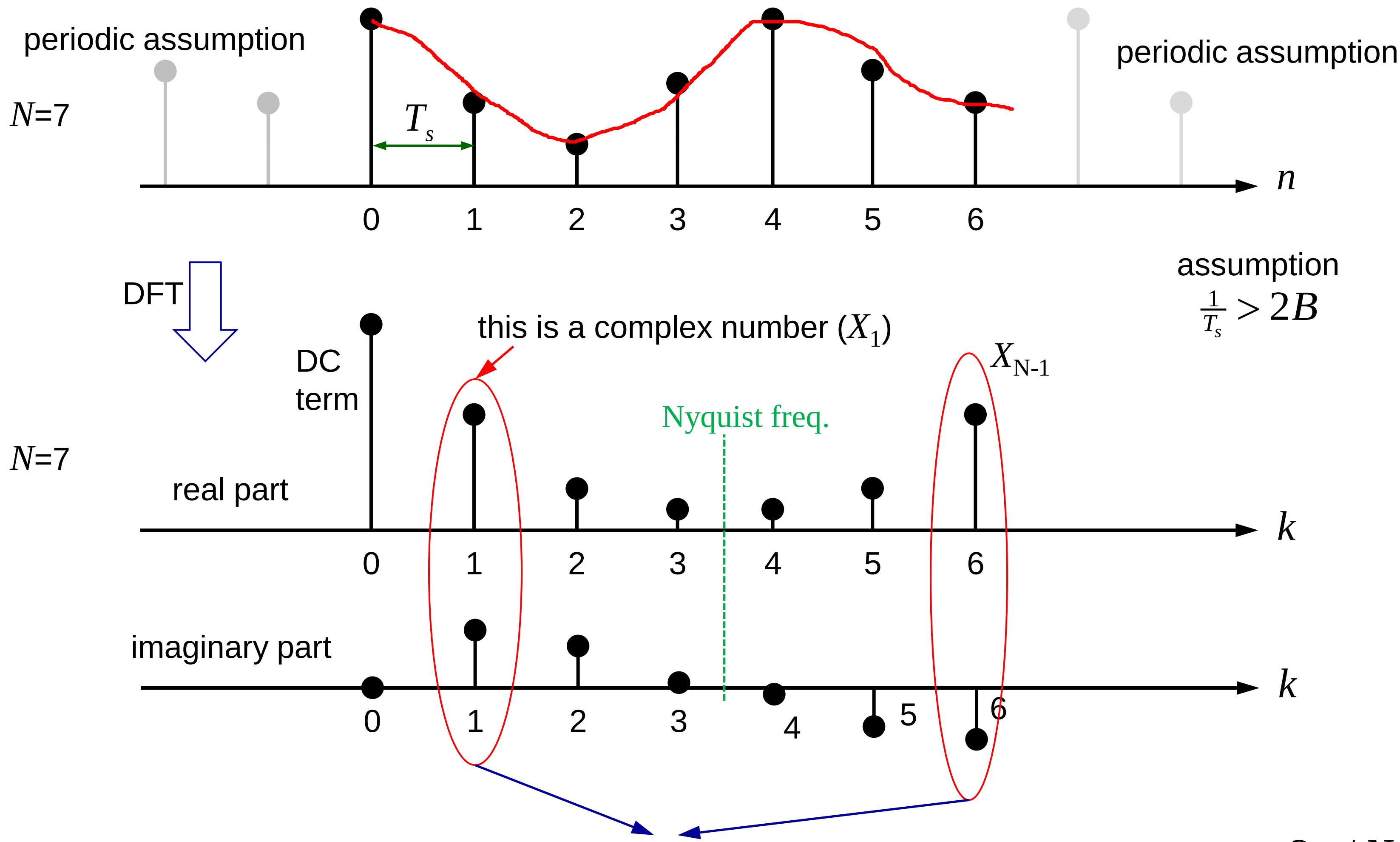
* Cross correlation of waveform samples and complex sinusoids at frequencies *k/N*

* Discrete version of Fourier Series

$$a_n = \frac{1}{T} \int_{-T/2}^{T/2} y(t) e^{-jn\omega_o t} dt \qquad y(t) = \sum_{n=-\infty}^{\infty} a_n e^{jn\omega_o t}$$

Recall : Discrete Fourier Transform (DFT)

DFT is a unitary transform from a set of N complex numbers to another set of complex numbers



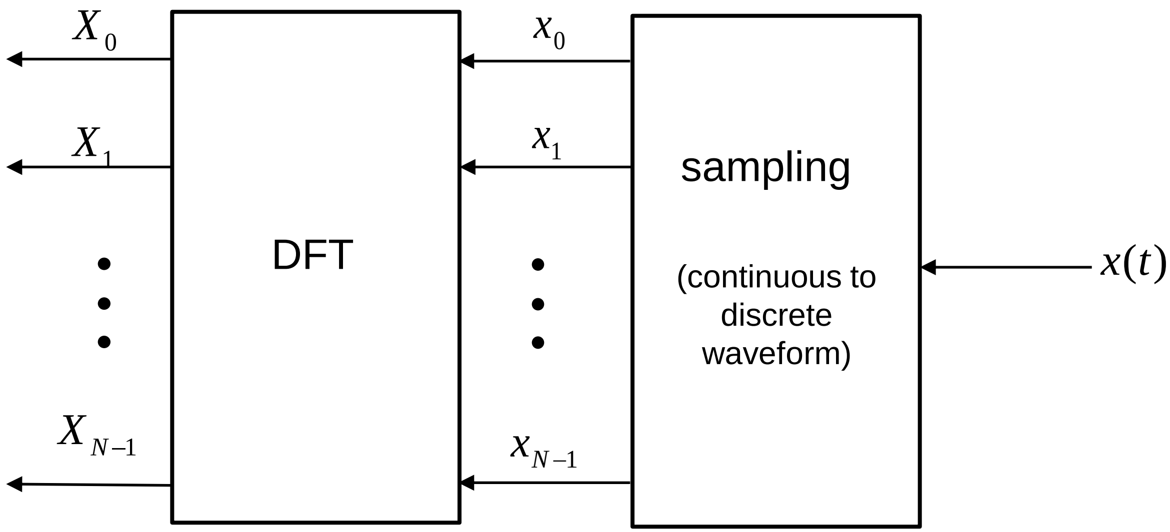
(Euler's identity) these two pairs make up a sinusoidal with some phase & amplitude at freq. $2\pi / N$

(if the input samples are real)

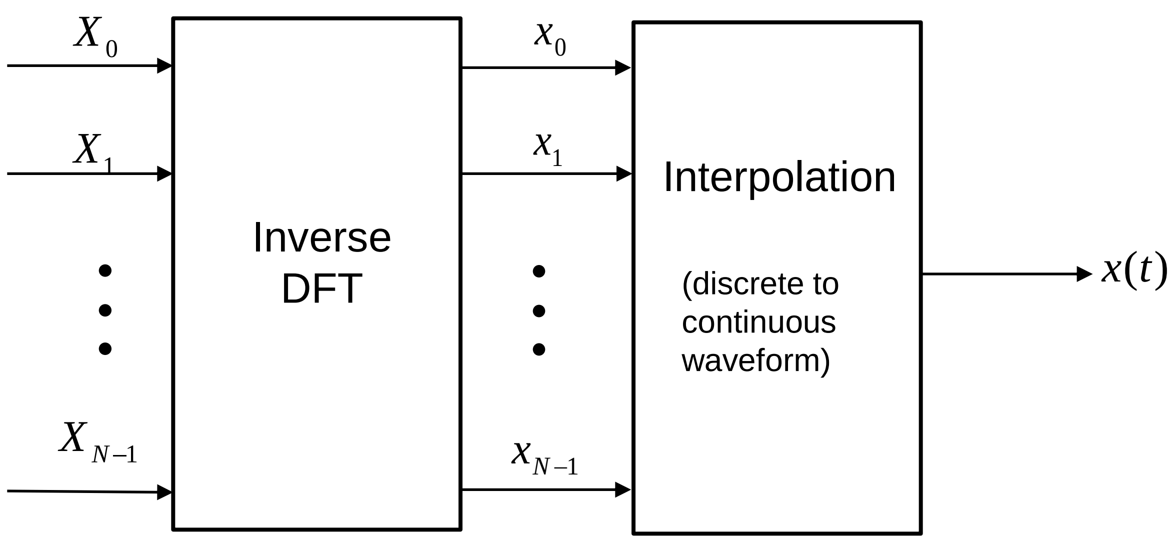
$f_0 = \frac{1}{NT_s}$

Therefore

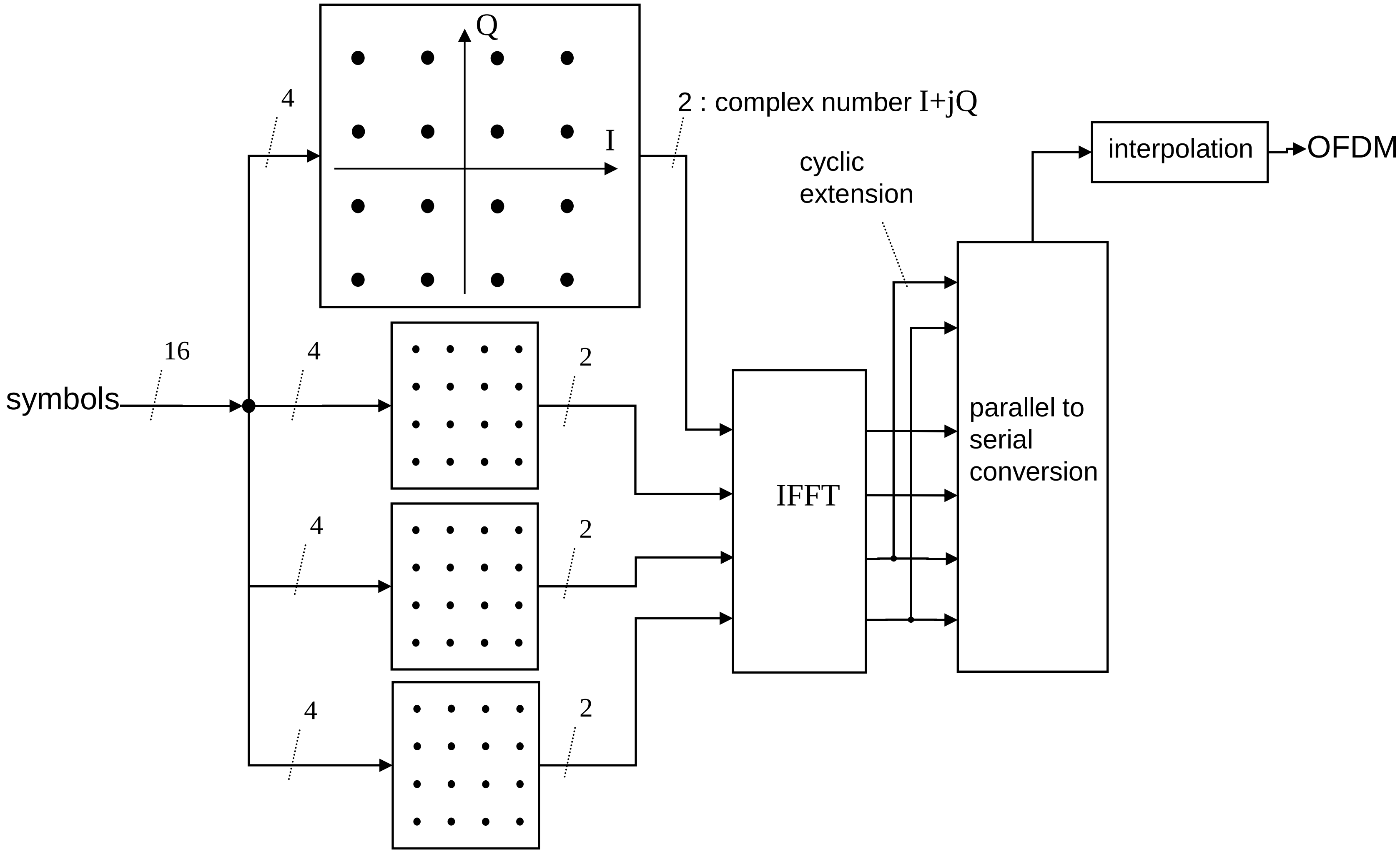
These complex numbers determine the phase and magnitude of the sinusoidal at freq. n/T_s



The sinusoidals represented by these complex numbers add up to $x(t)$



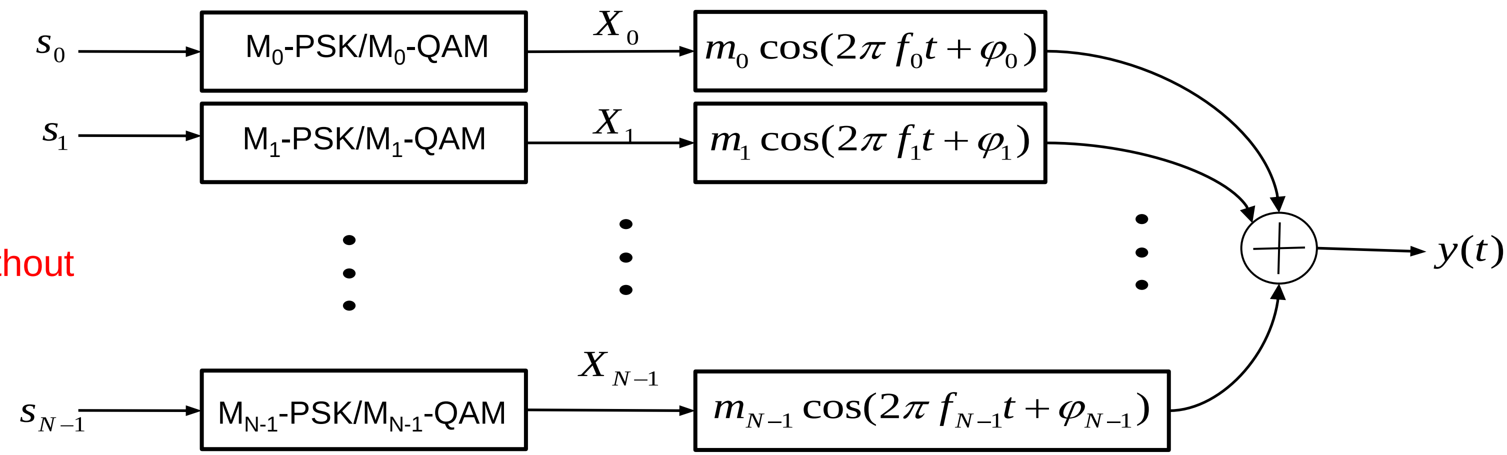
Example with 4 Carriers (4 sub-symbols), each having 16-QAM



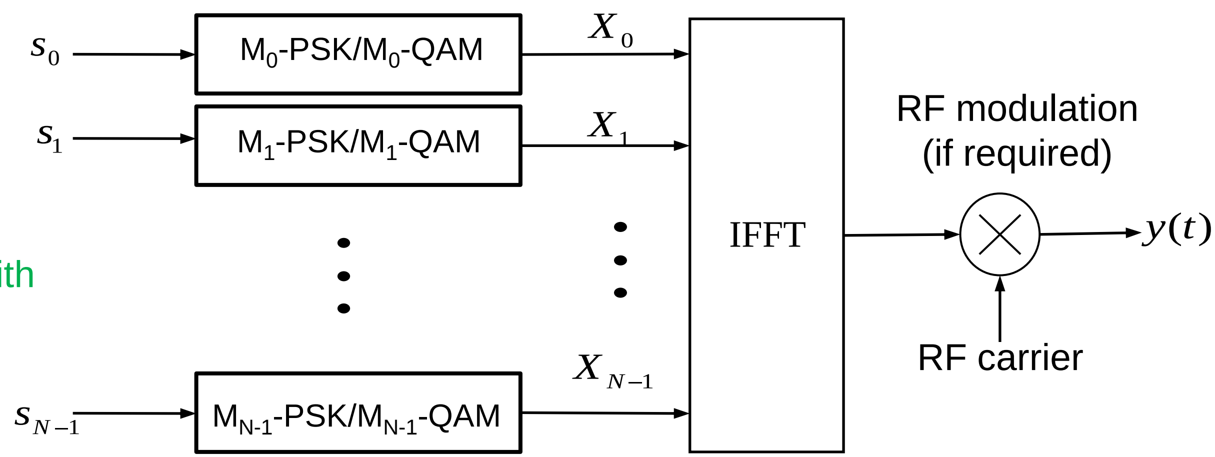
There is no OFDM Without FFT/IFFT

Of course, it is possible to have OFDM without FFT/IFFT.
But the actual benefits come from FFT/IFFT.
It would be so cumbersome/difficult otherwise, it would not be worth doing it.

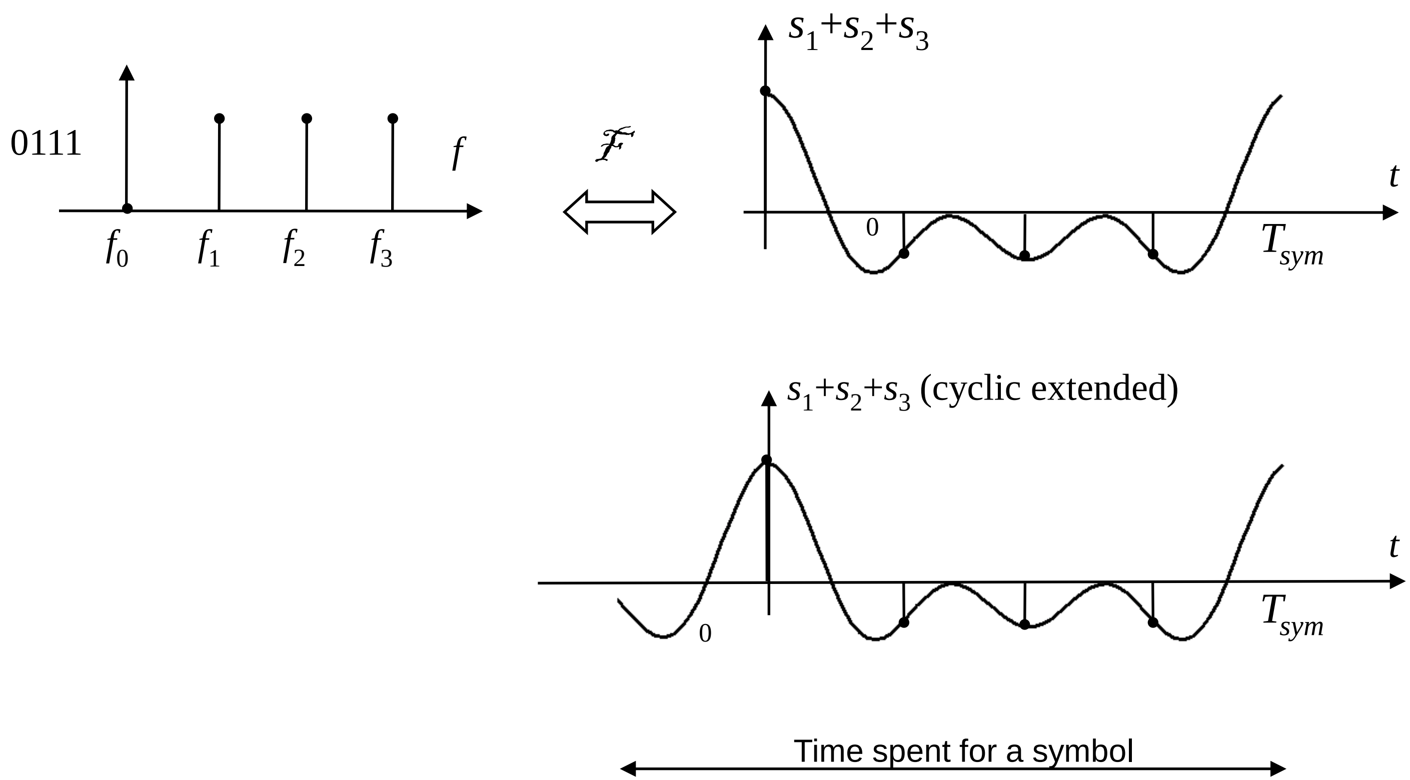
Without



With

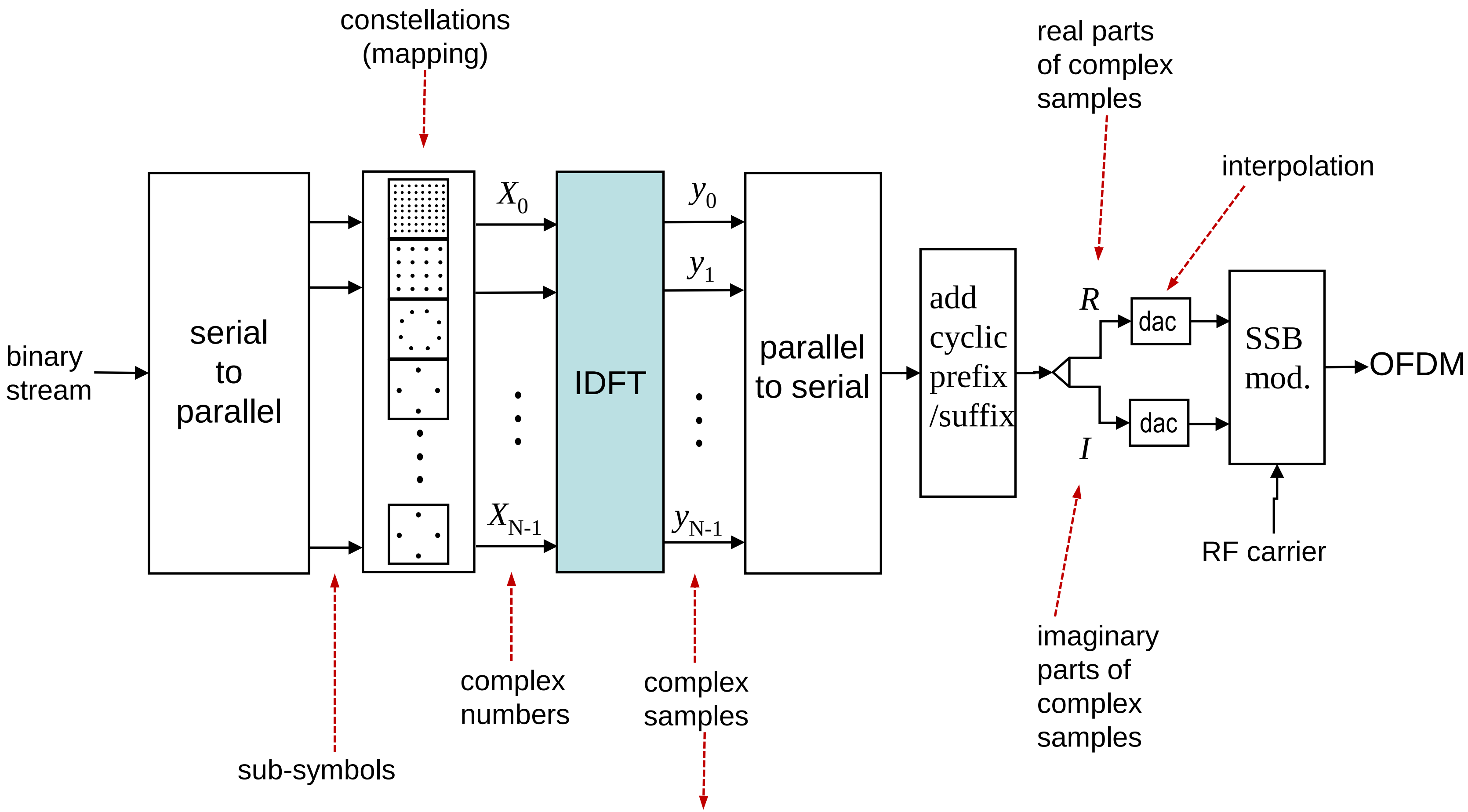


Cyclic Extension (cyclic prefix)



reason : homework

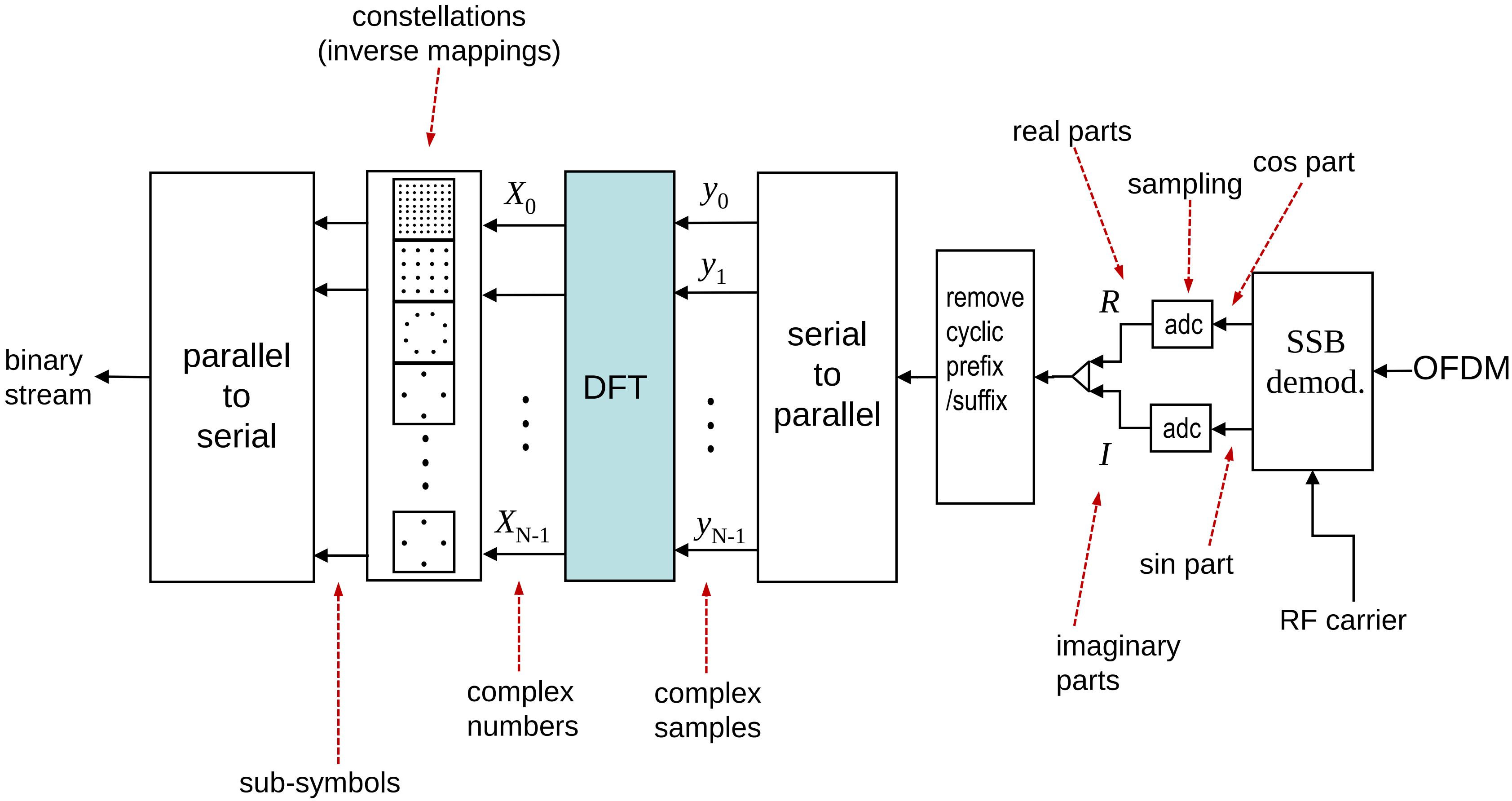
General Structure of OFDM Transmitter



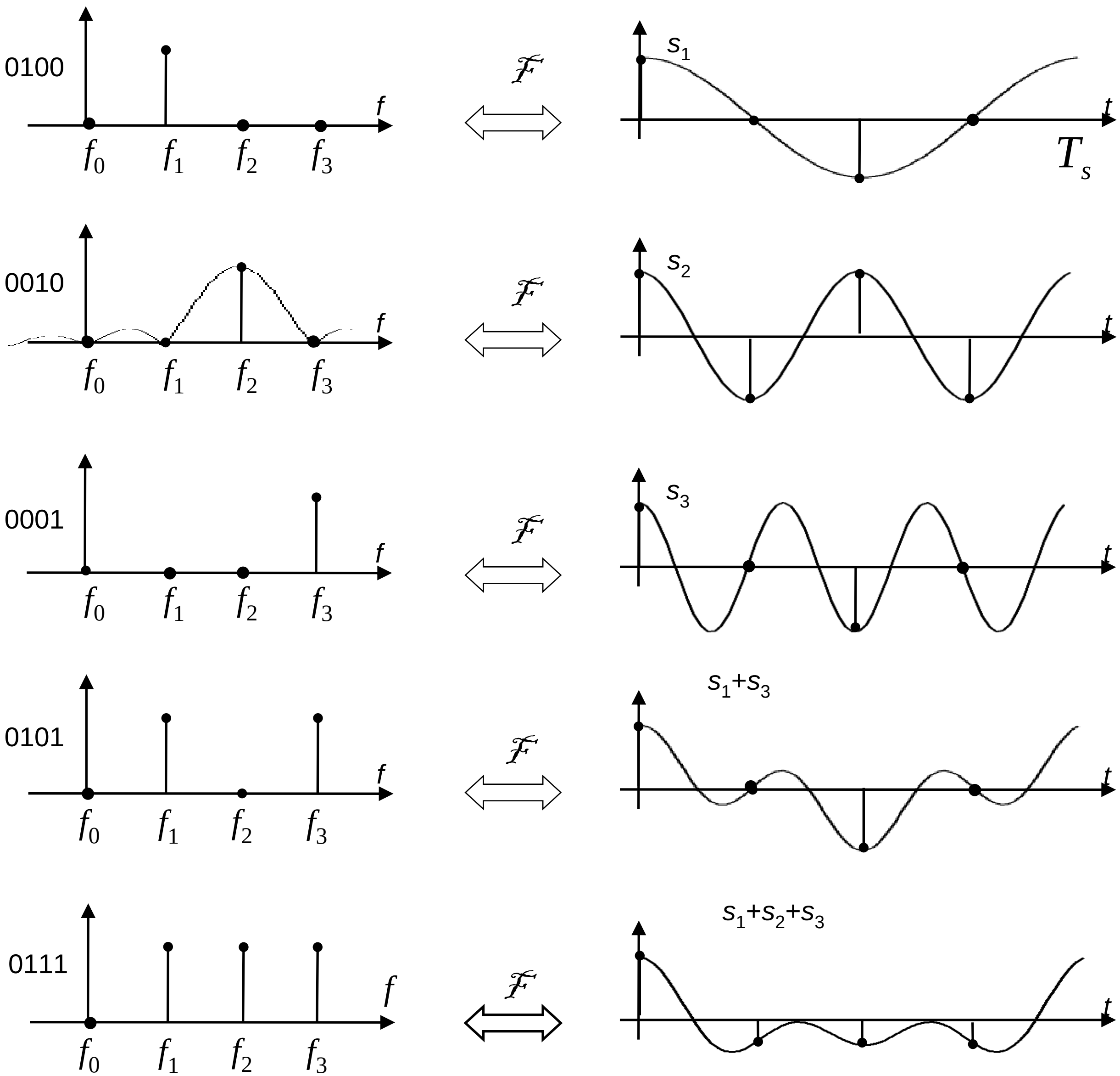
Real parts : samples of sums of I components of carriers

Imag parts : samples of sums of Q components of carriers

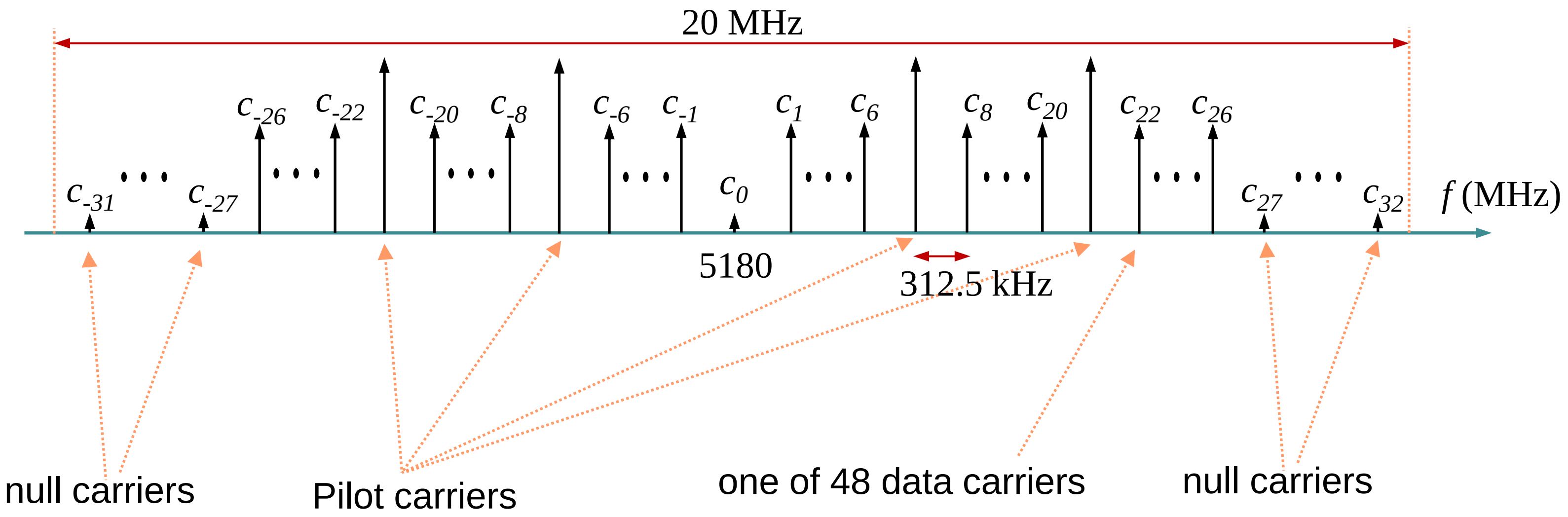
General Structure of OFDM Receiver



{0,A} ASK examples for 4 carriers



IEEE 802.11a-1999



$T_s = 3.2\mu s = 1/312.5k$

$T_{withCP} = 3.2\mu s + 0.8\mu s = 4\mu s$

1/4th of T_s (required by 802.11a)

Physical Bit rate = $6x48/4x10^{-6}$ (if all sub-carriers use 64-QAM)

= 72 Mbps

Actual Bit rate = ¾ x72 = 54 Mbps (channel coded)

END