

# Spread Spectrum

(Part 2)

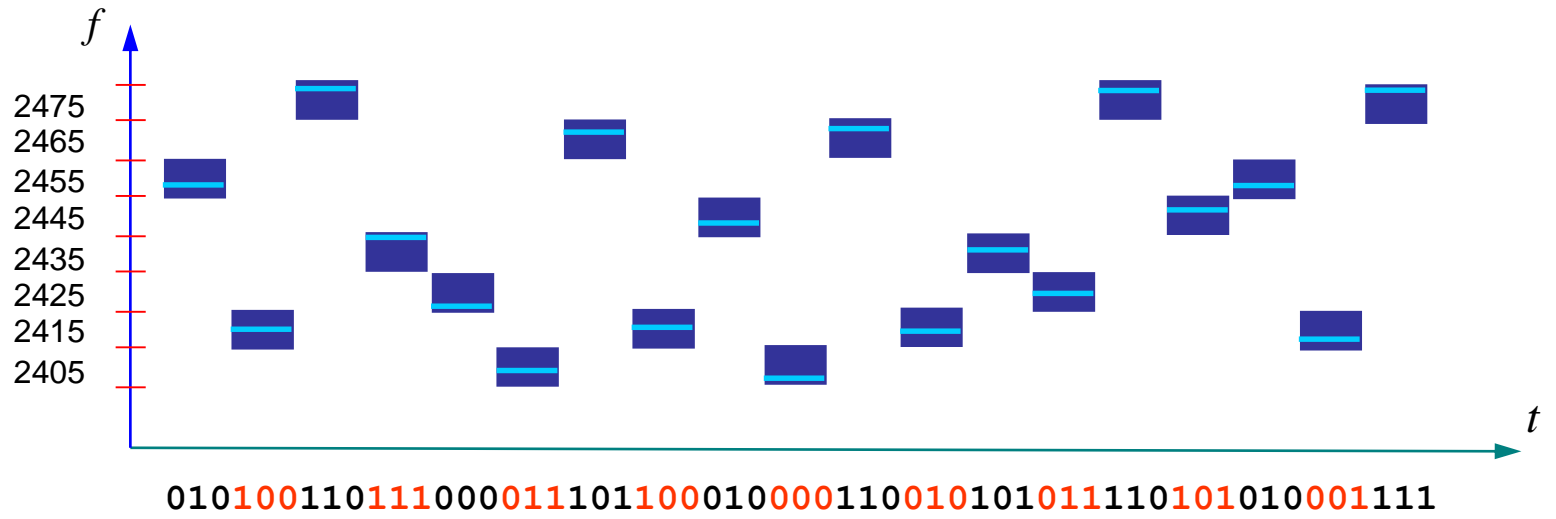
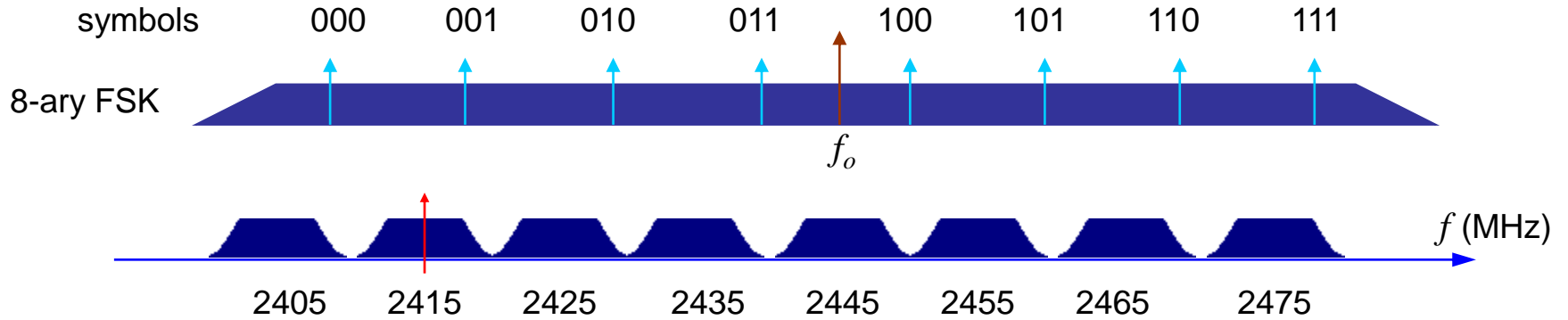
by Erol Seke

For the course “**Communications**”



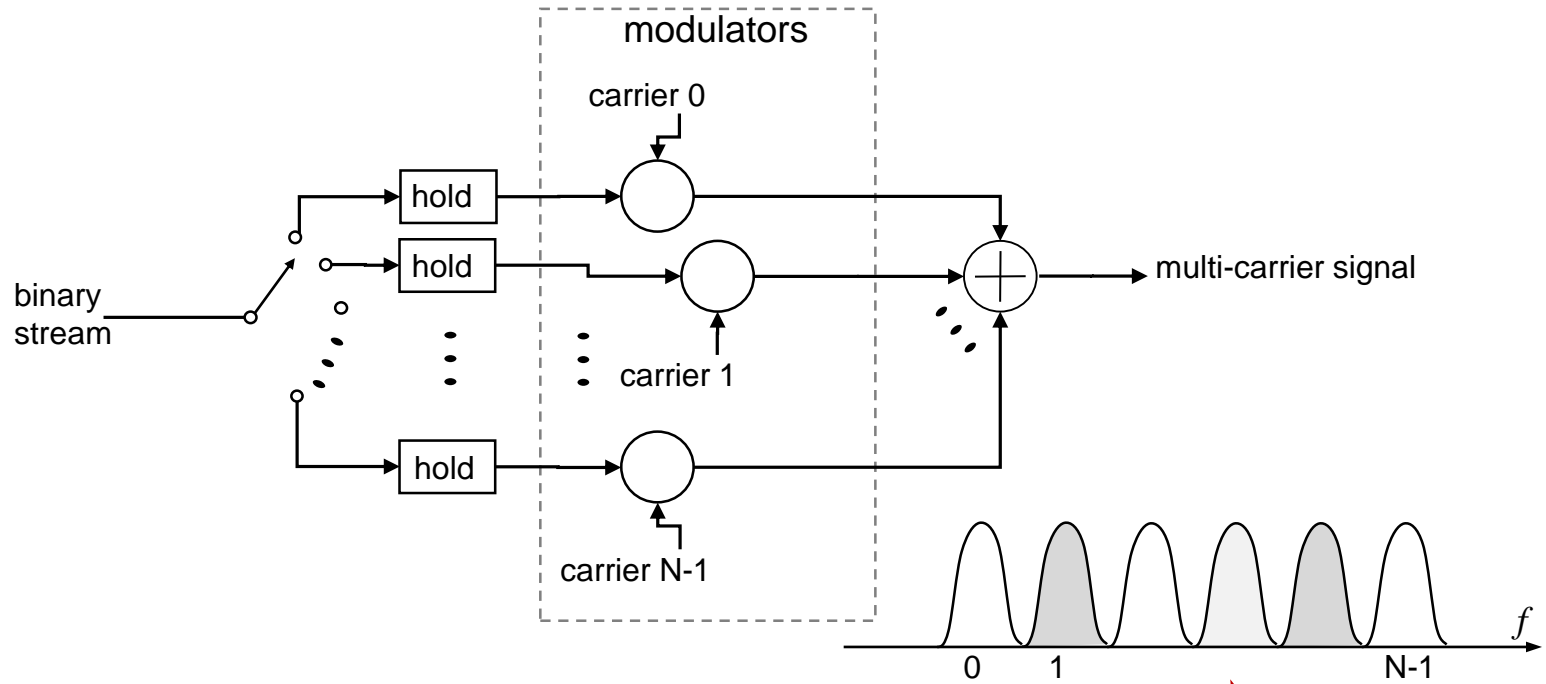
**ESKİŞEHİR OSMANGAZI UNIVERSITY**

Remember the sub-channels in frequency hopping example



What if they are all used in parallel by the same single transmitter?

## FDM for Single Binary Source



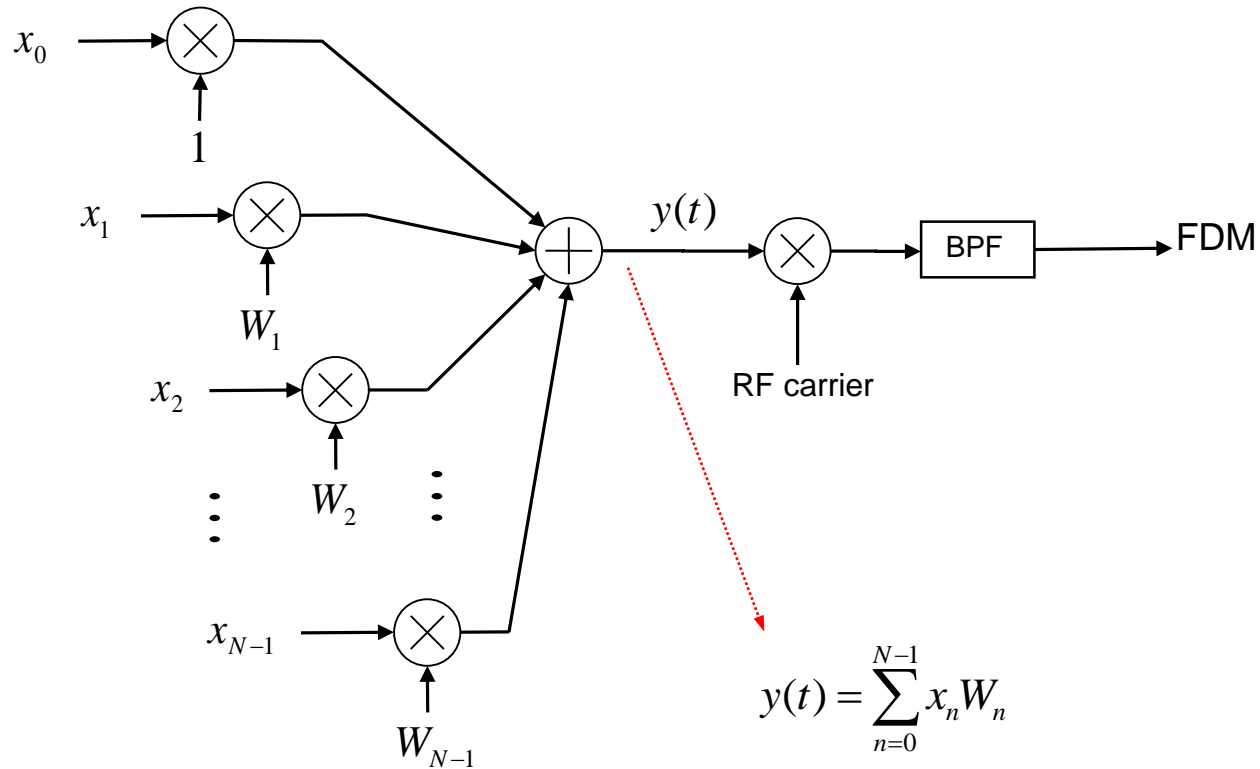
example:     ...010111001011 111001000111 010000110110

current-symbol

sub-symbols    11   1001   000   1   11

each sub-symbol is transmitted using  $T_{si}$  symbol rate to have a non-overlapping sub-spectrums  
(using M-PSK or M-QAM)

## Frequency Upconversion of All FDM Channels

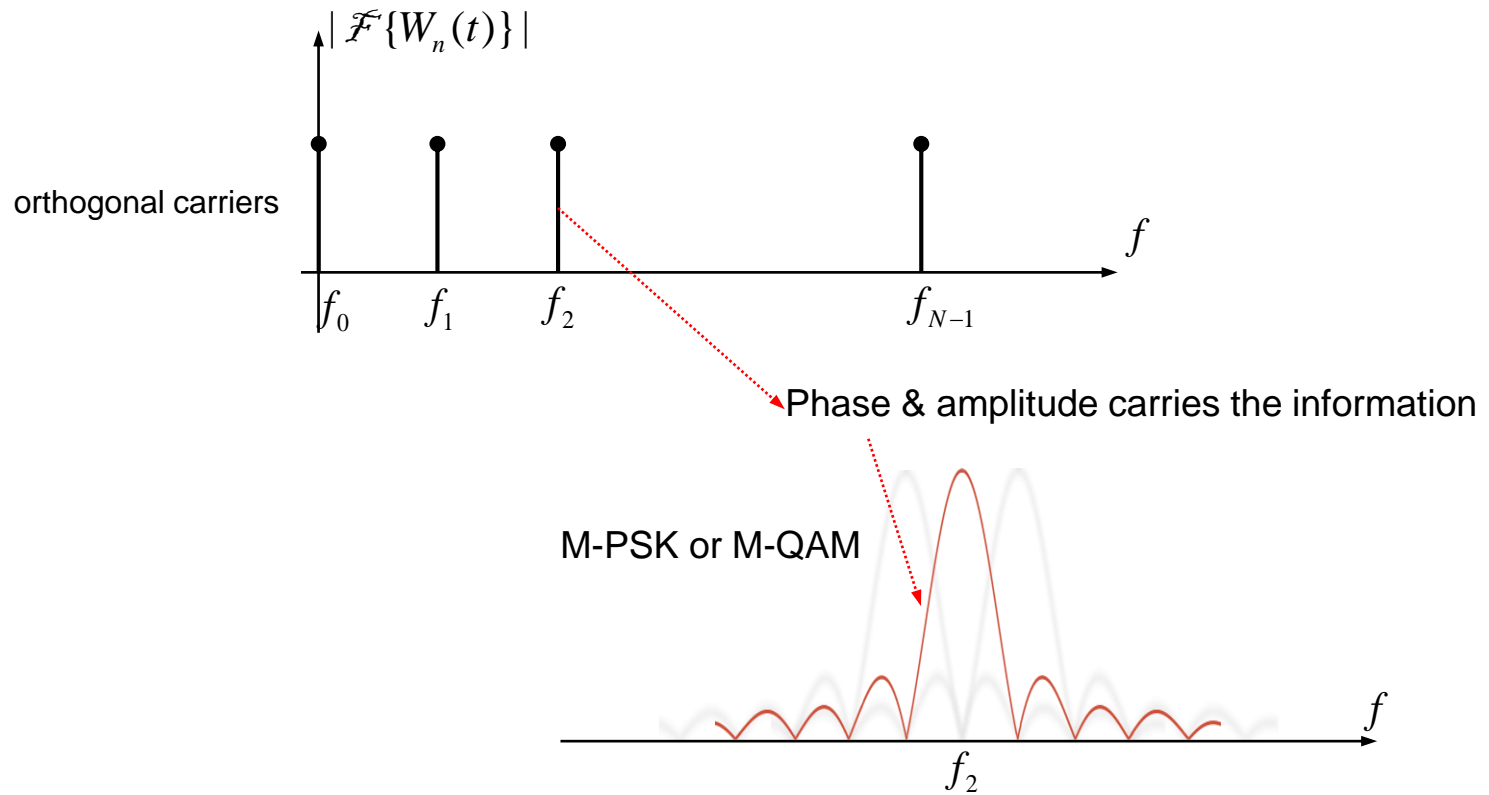


If  $W_n$  's are orthogonal and  $x_n$  's are constant for a symbol period  $T_s$  , then it is called OFDM

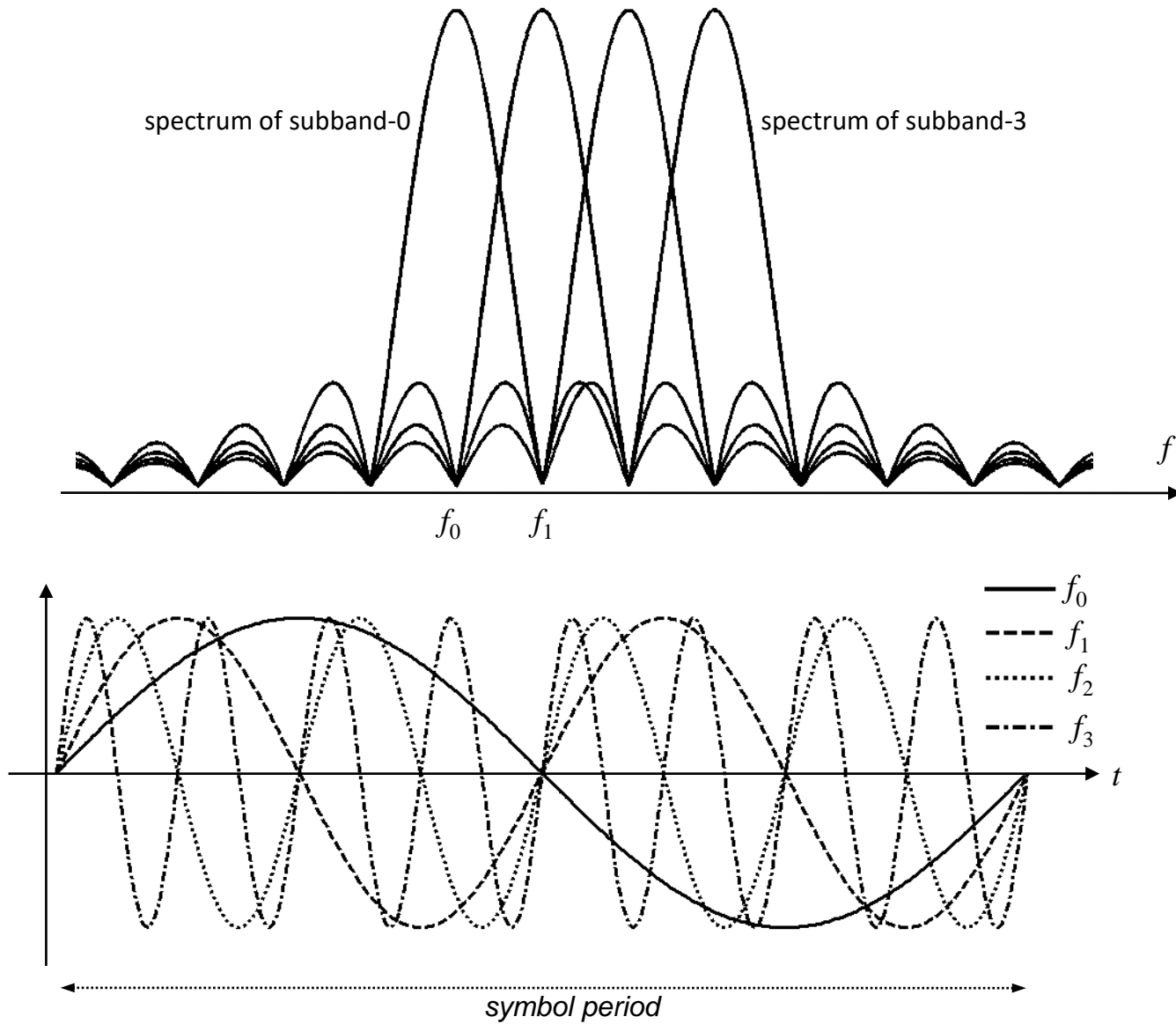
Orthogonality is maintained even when phase and/or amplitude changes between symbol periods (M-PSK, M-QAM)

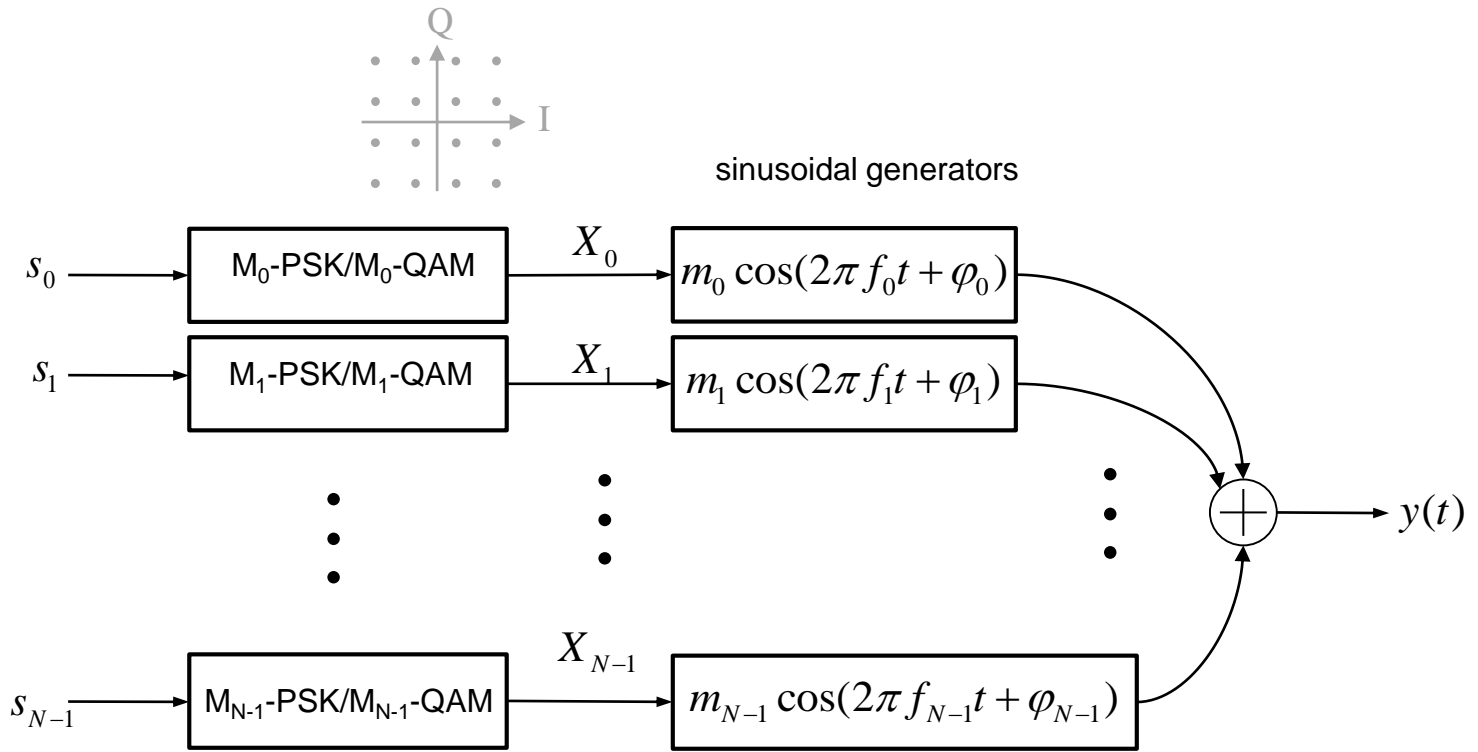
$W_n(t)$ 's are orthogonal when  $W_n(t) = \cos(2\pi f_n t)$  and  $f_n = n/T_s$

Information is carried by the carriers' phase and amplitude : M-PSK or M-QAM



# Orthogonal Carriers for a Symbol Period





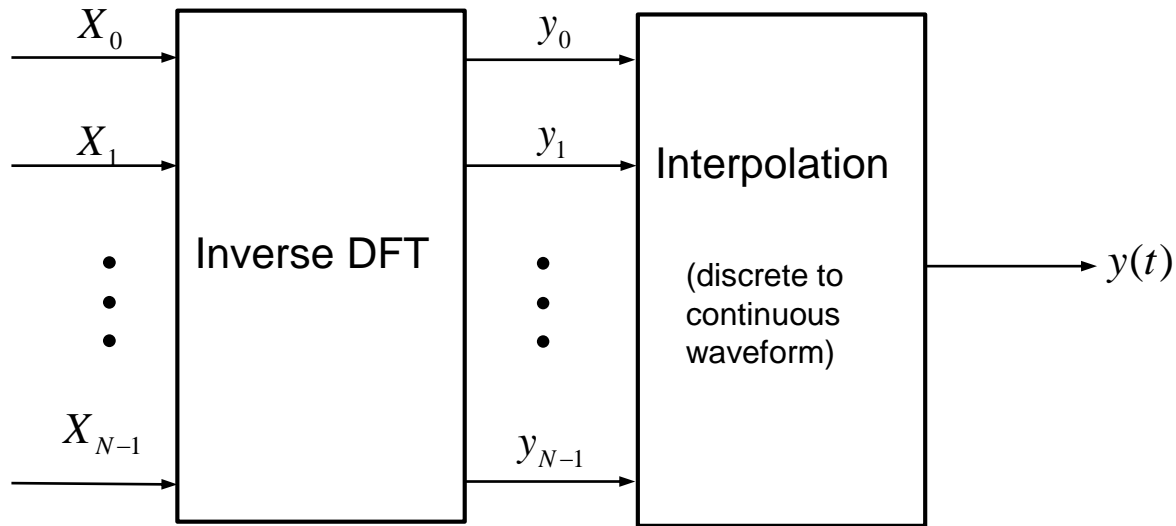
$s_n$  :  $M_{N-1}$ -ary symbol

$X_n$  : complex numbers for one symbol period

$m_n$  : magnitudes

$\varphi_n$  : phases

right block is equivalent to



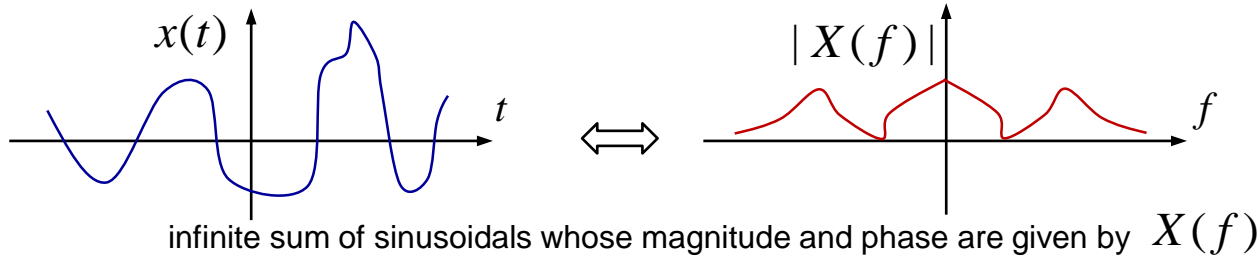
where  $y_n$ 's are samples of  $y(t)$

$$y_n = \sum_{k=0}^{N-1} X_k W_N^{nk} \quad \text{where} \quad W_N^{nk} = e^{j2\pi nk/N}$$

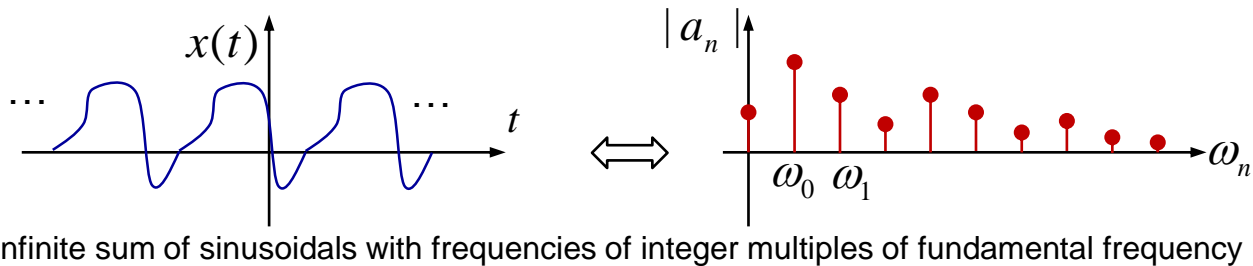


## Recall : Fourier ...

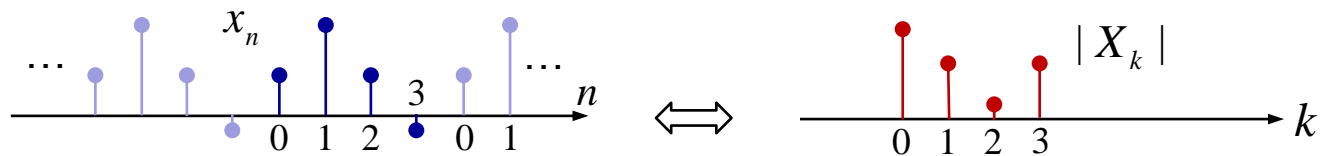
if  $x(t)$  is continuous arbitrary integrable function :  $X(\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t} dt$  (Fourier Transform)



if  $x(t)$  is periodic :  $a_n = \frac{1}{T} \int_{-T/2}^{T/2} x(t)e^{-jn\omega_0 t} dt$  (Fourier Series Coefficients)



if  $x_n$  are samples of periodic  $x(t)$  :  $X_k = \sum_{n=0}^{N-1} x_n e^{-j2\pi nk/N}$  (DFT Coefficients)



## Recall : Discrete Fourier Transform (DFT)

Forward Transform  $X_k = \sum_{n=0}^{N-1} x_n e^{-j2\pi nk/N} \quad k = 0, 1, \dots, N-1$

Inverse Transform  $x_n = \frac{1}{N} \sum_{k=0}^{N-1} X_k e^{j2\pi nk/N} \quad n = 0, 1, \dots, N-1$

In matrix form

$$\begin{bmatrix} X_0 \\ X_1 \\ \vdots \\ X_{N-1} \end{bmatrix} = \begin{bmatrix} W_N^{0 \cdot 0} & W_N^{0 \cdot 1} & \dots & W_N^{0 \cdot (N-1)} \\ W_N^{1 \cdot 0} & W_N^{1 \cdot 1} & \dots & W_N^{1 \cdot (N-1)} \\ \vdots & \vdots & \ddots & \vdots \\ W_N^{(N-1) \cdot 0} & W_N^{(N-1) \cdot 1} & \dots & W_N^{(N-1) \cdot (N-1)} \end{bmatrix} \begin{bmatrix} x_0 \\ x_1 \\ \vdots \\ x_{N-1} \end{bmatrix} \quad W_N^{nk} = e^{-j2\pi nk/N}$$

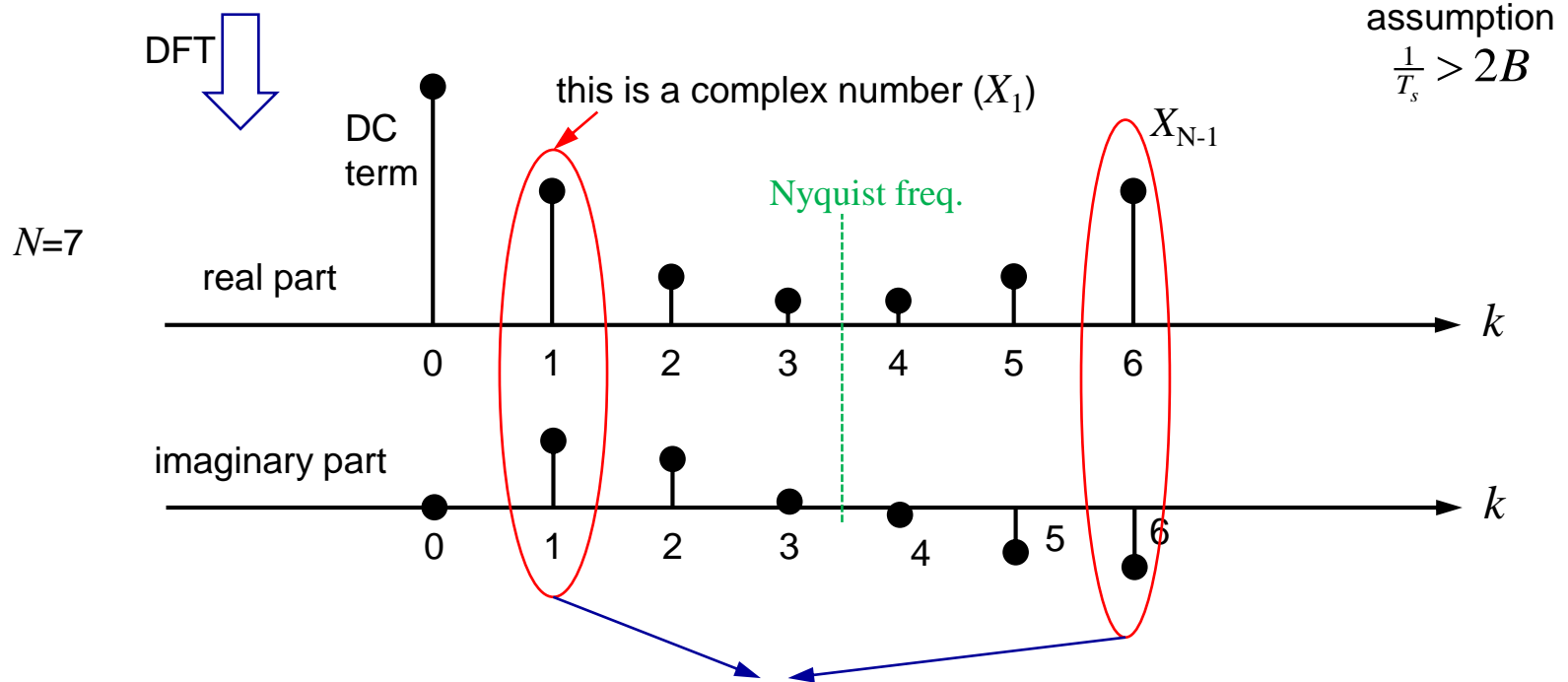
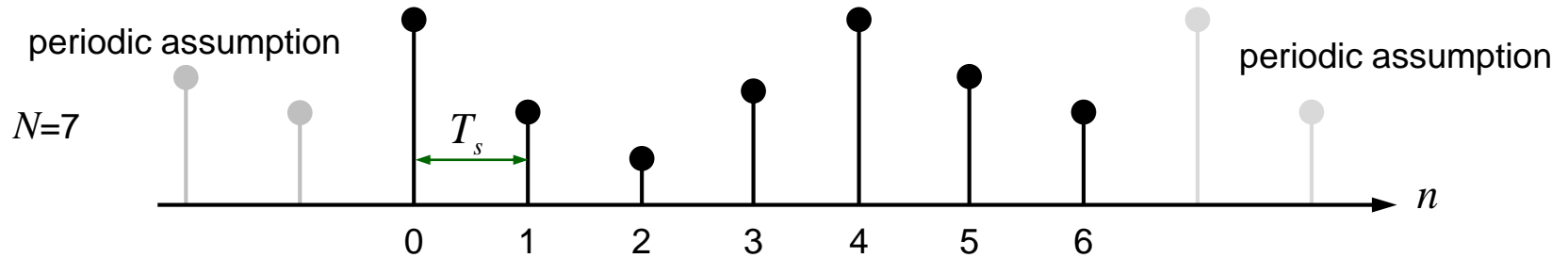
\* Cross correlation of waveform samples and complex sinusoids at frequencies  $k/N$

\* Discrete version of Fourier Series

$$a_n = \frac{1}{T} \int_{-T/2}^{T/2} y(t) e^{-jn\omega_0 t} dt \quad y(t) = \sum_{n=-\infty}^{\infty} a_n e^{jn\omega_0 t}$$

## Recall : Discrete Fourier Transform (DFT)

DFT is a unitary transform from a set of  $N$  complex numbers to another set of complex numbers

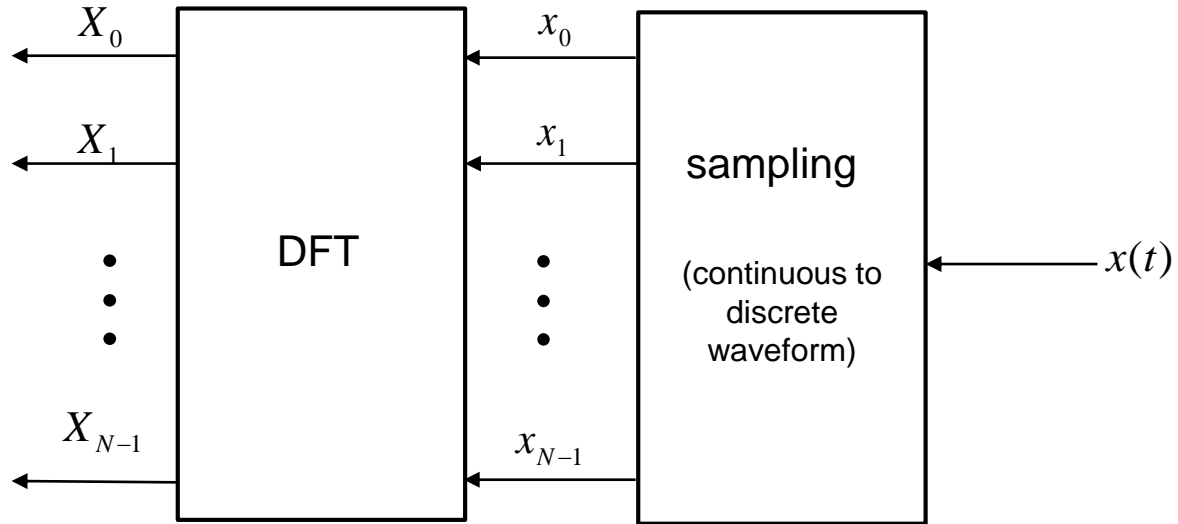


(Euler's identity) these two pairs make up a sinusoidal with some phase & amplitude at freq.  $\frac{2\pi}{N}$   
 (if the input samples are real)

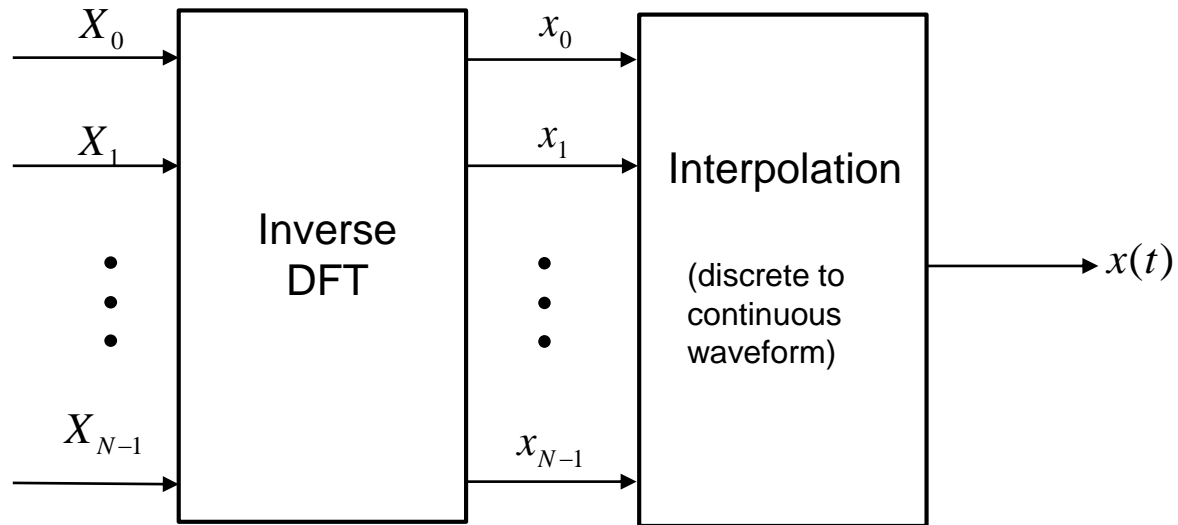
$f_0 = \frac{1}{NT_s}$

Therefore

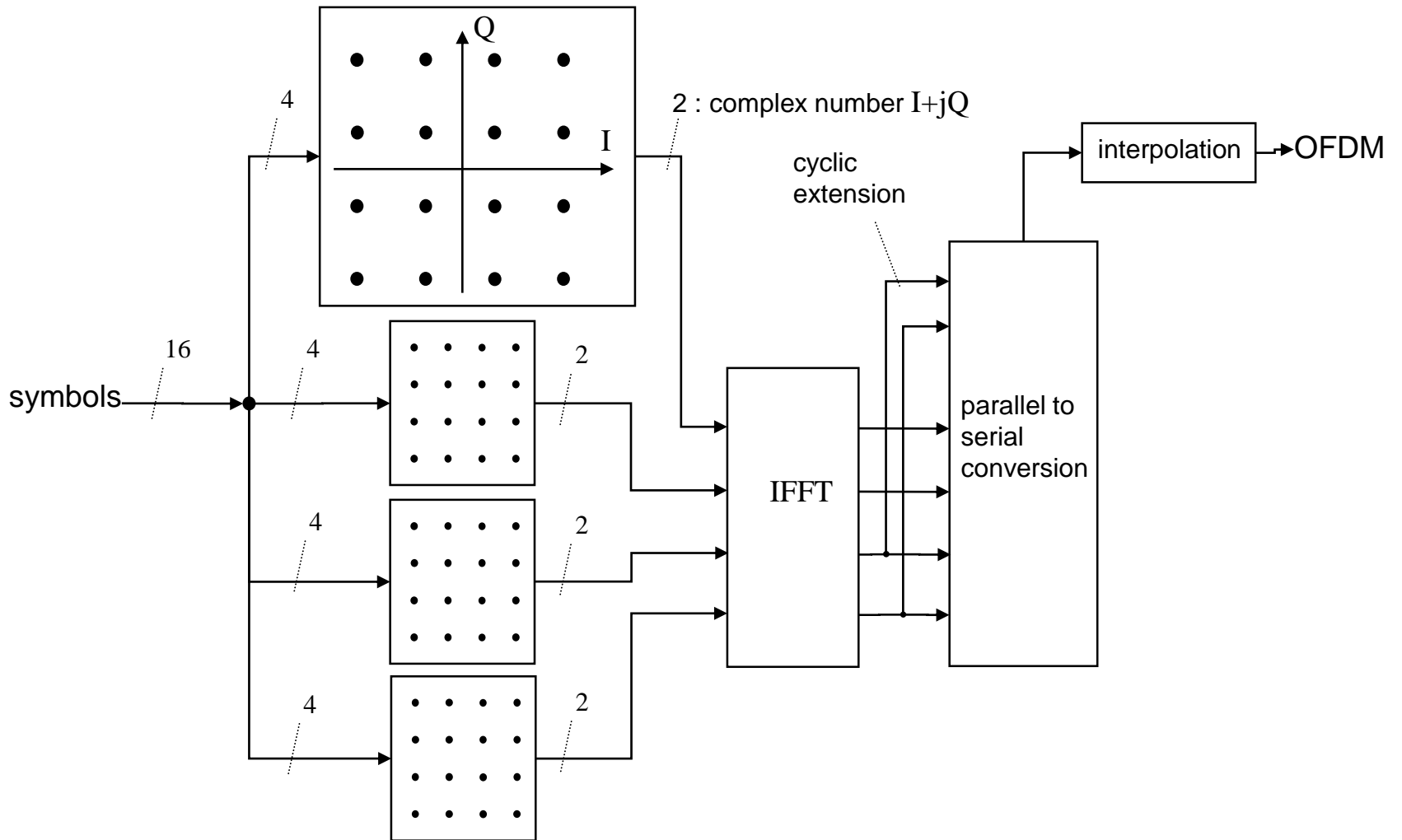
These complex numbers determine the phase and magnitude of the sinusoidal at freq.  $n/T_s$



The sinusoids represented by these complex numbers add up to  $x(t)$



Example with 4 Carriers (4 sub-symbols), each having 16-QAM

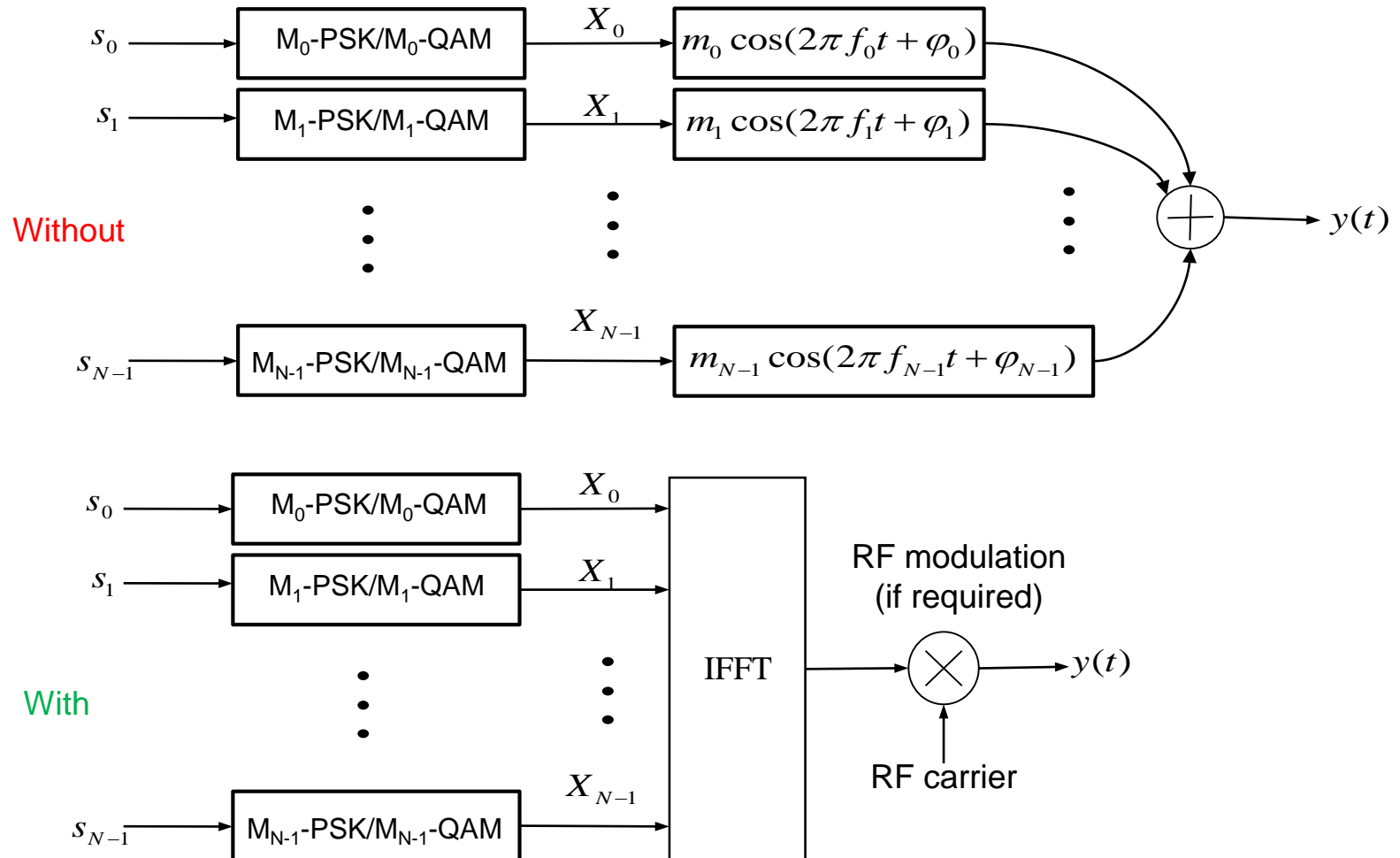


## There is no OFDM Without FFT/IFFT

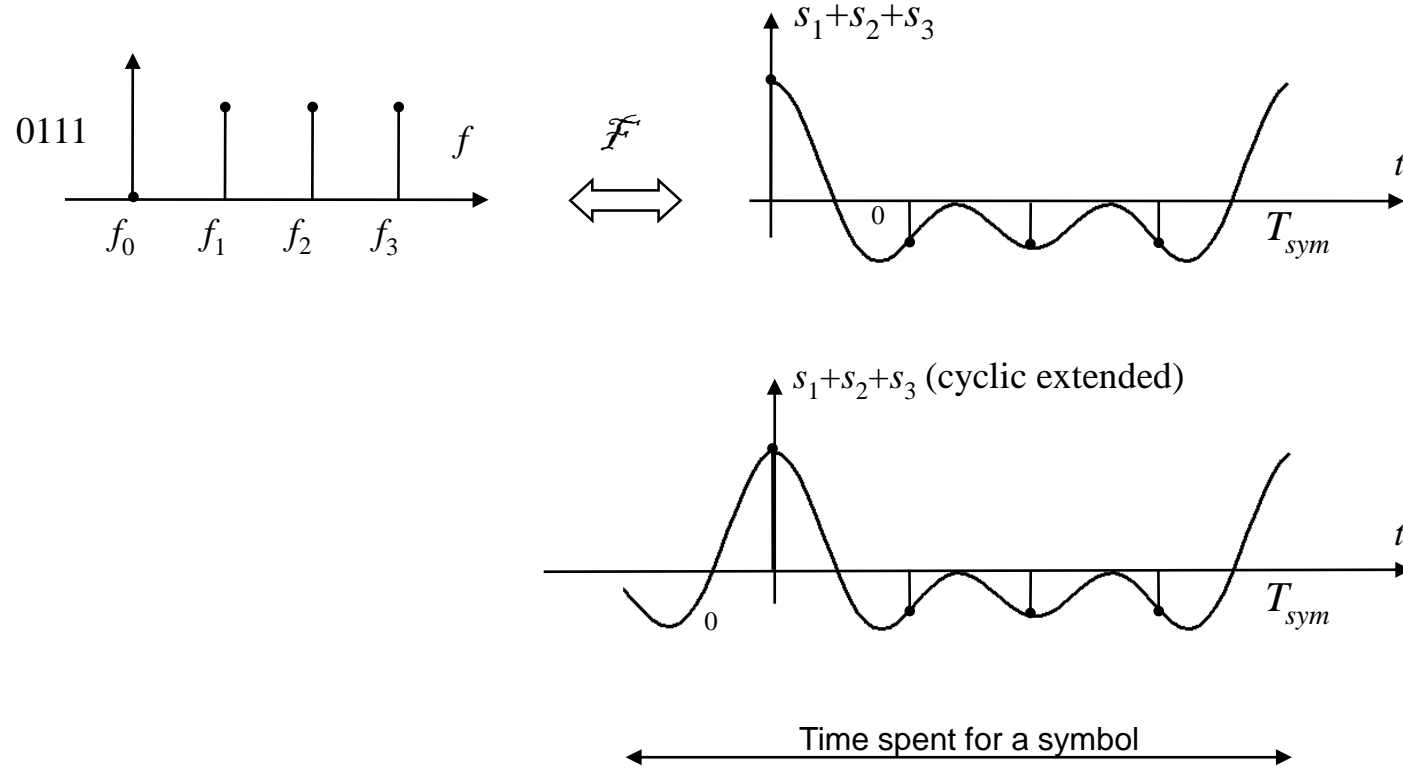
Of course, it is possible to have OFDM without FFT/IFFT.

But the actual benefits come from FFT/IFFT.

It would be so cumbersome/difficult otherwise, it would not be worth doing it.

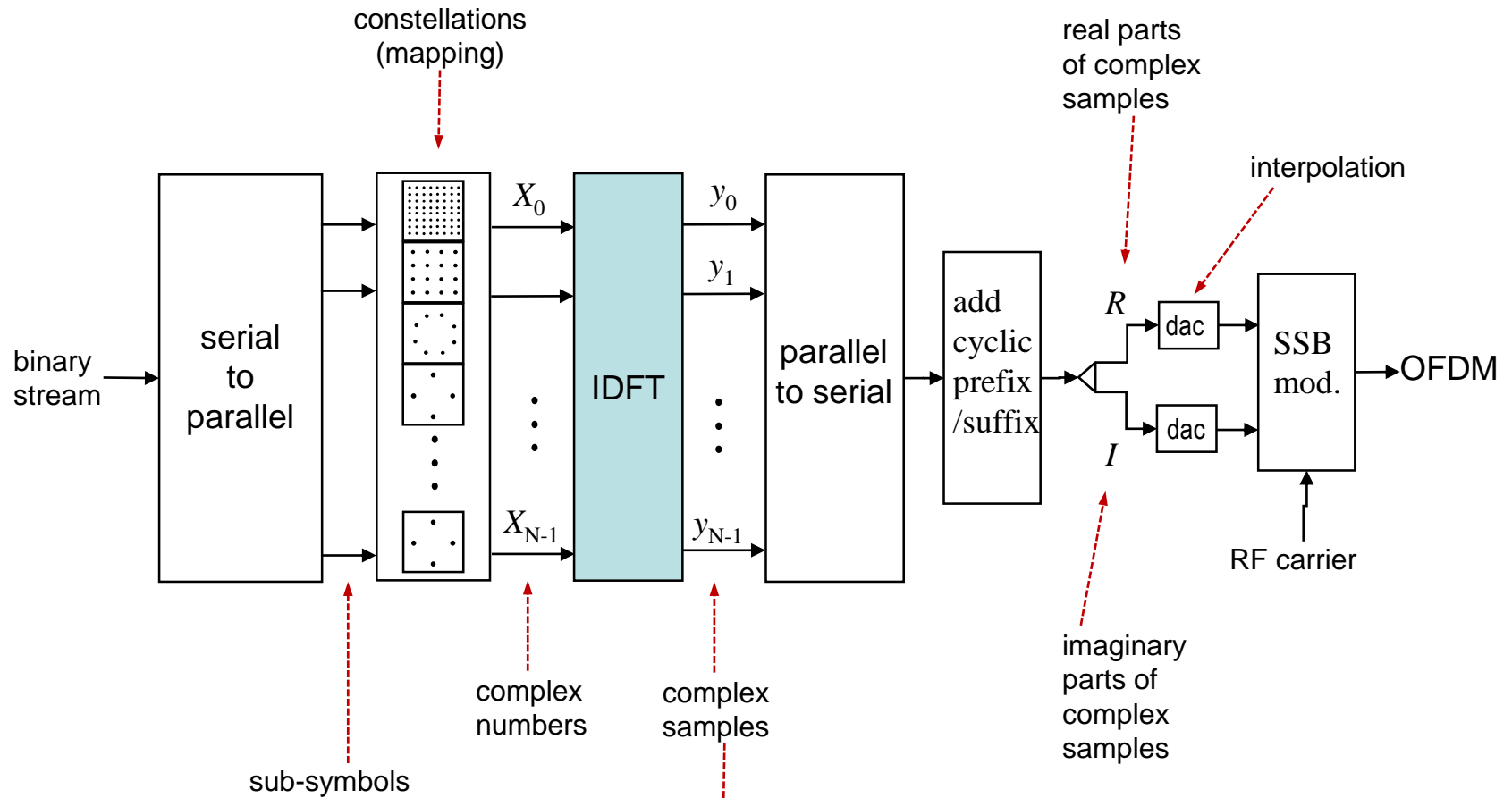


## Cyclic Extension (cyclic prefix)



reason : homework

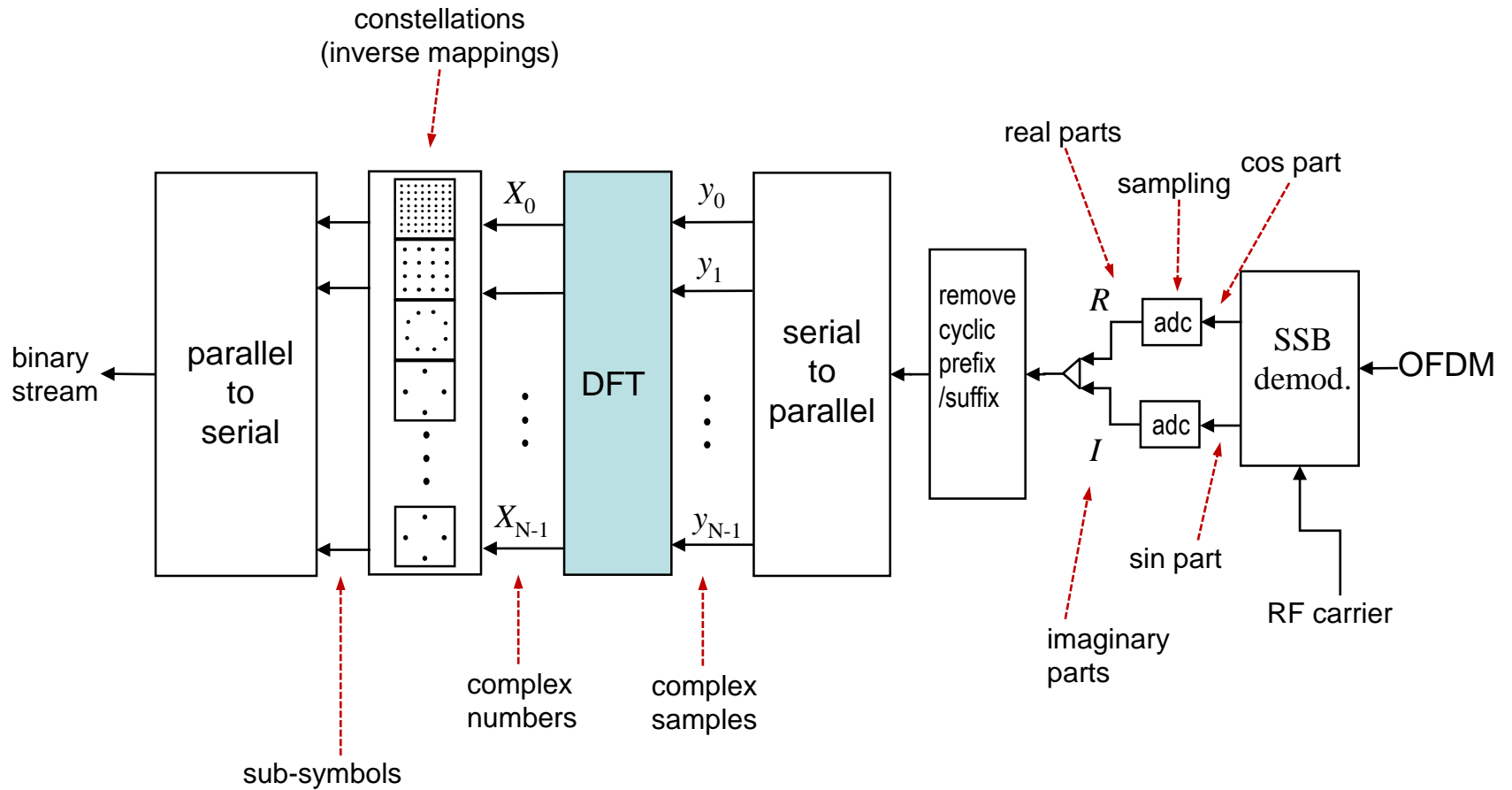
## General Structure of OFDM Transmitter



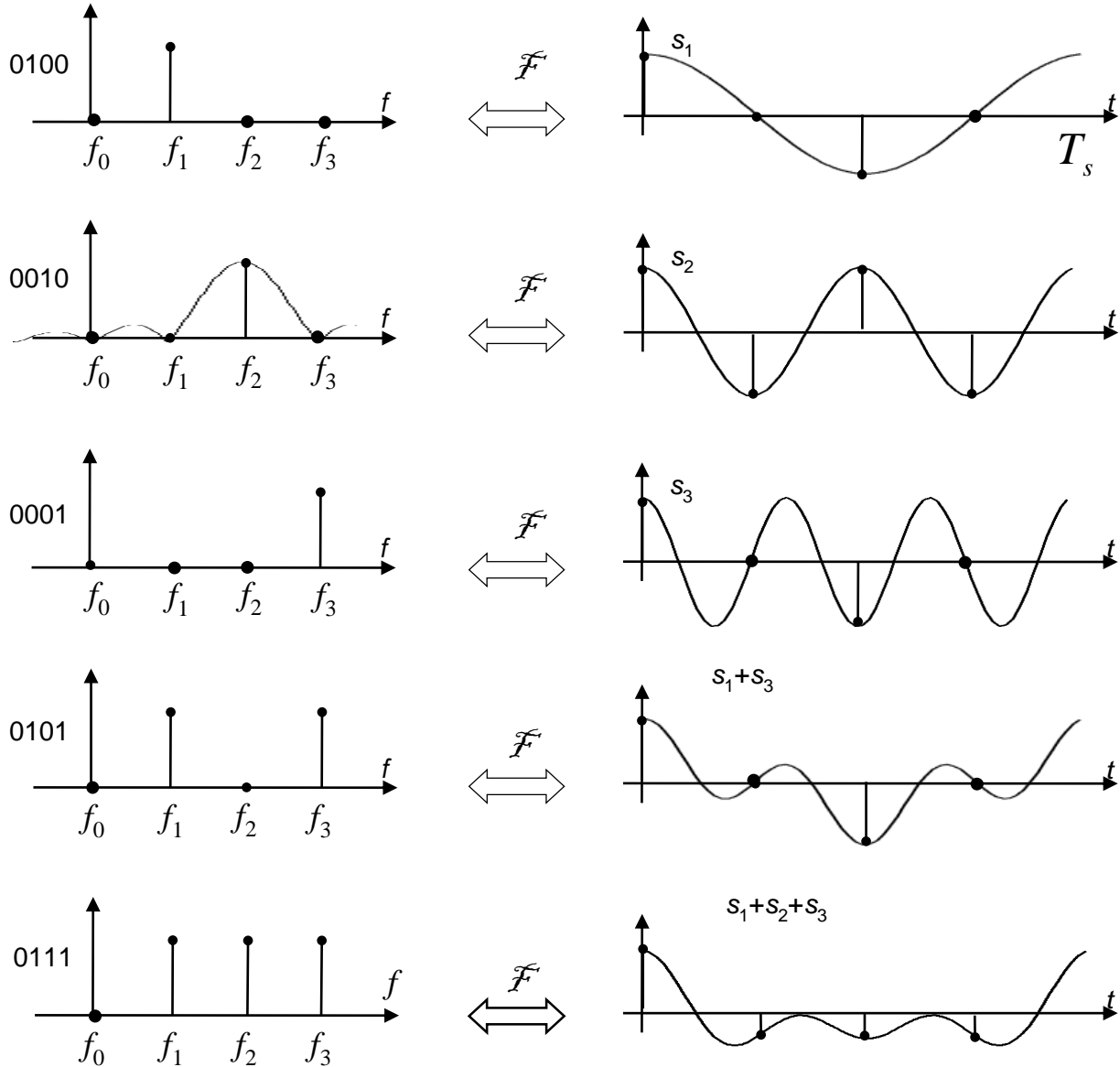
Real parts : samples of sums of I components of carriers  
Imag parts : samples of sums of Q components of carriers



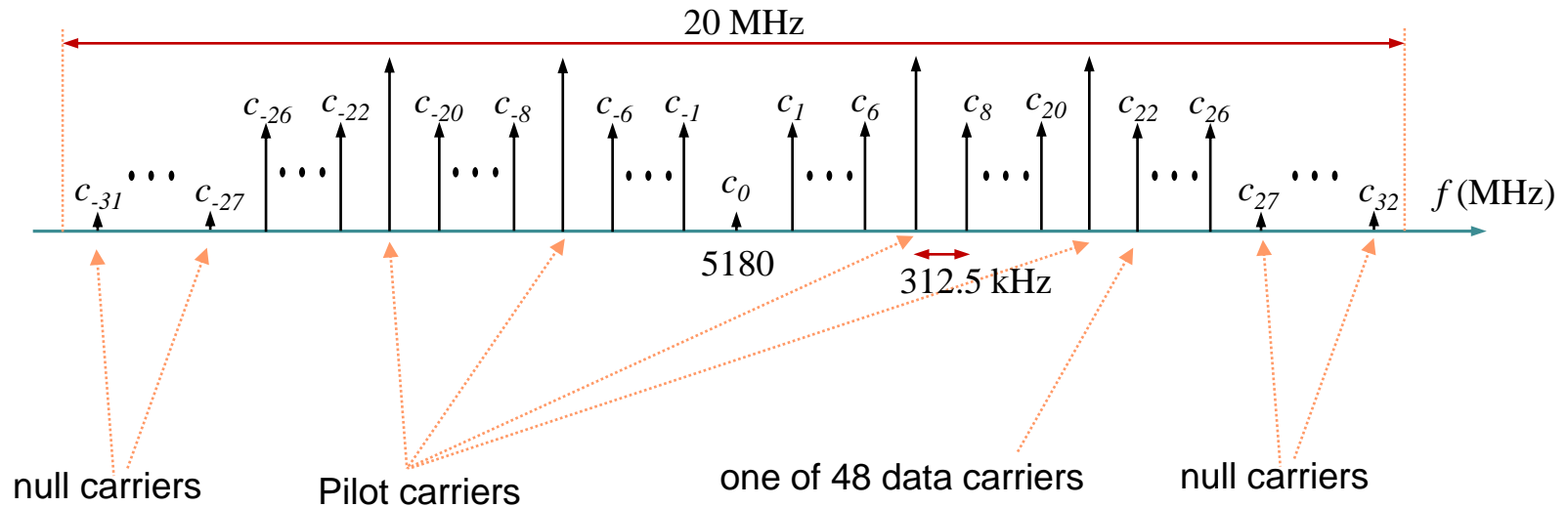
# General Structure of OFDM Receiver



## {0,A} ASK examples for 4 carriers



# IEEE 802.11a-1999



$$T_s = 3.2\mu\text{s} = 1/312.5\text{k}$$

$$T_{withCP} = 3.2\mu\text{s} + 0.8\mu\text{s} = 4\mu\text{s}$$

1/4<sup>th</sup> of  $T_s$  (required by 802.11a)

Physical Bit rate =  $6 \times 48 / 4 \times 10^{-6}$  (if all sub-carriers use 64-QAM)  
 = 72 Mbps

Actual Bit rate =  $\frac{3}{4} \times 72 = 54$  Mbps (channel coded)

**END**