Introduction

Definitions

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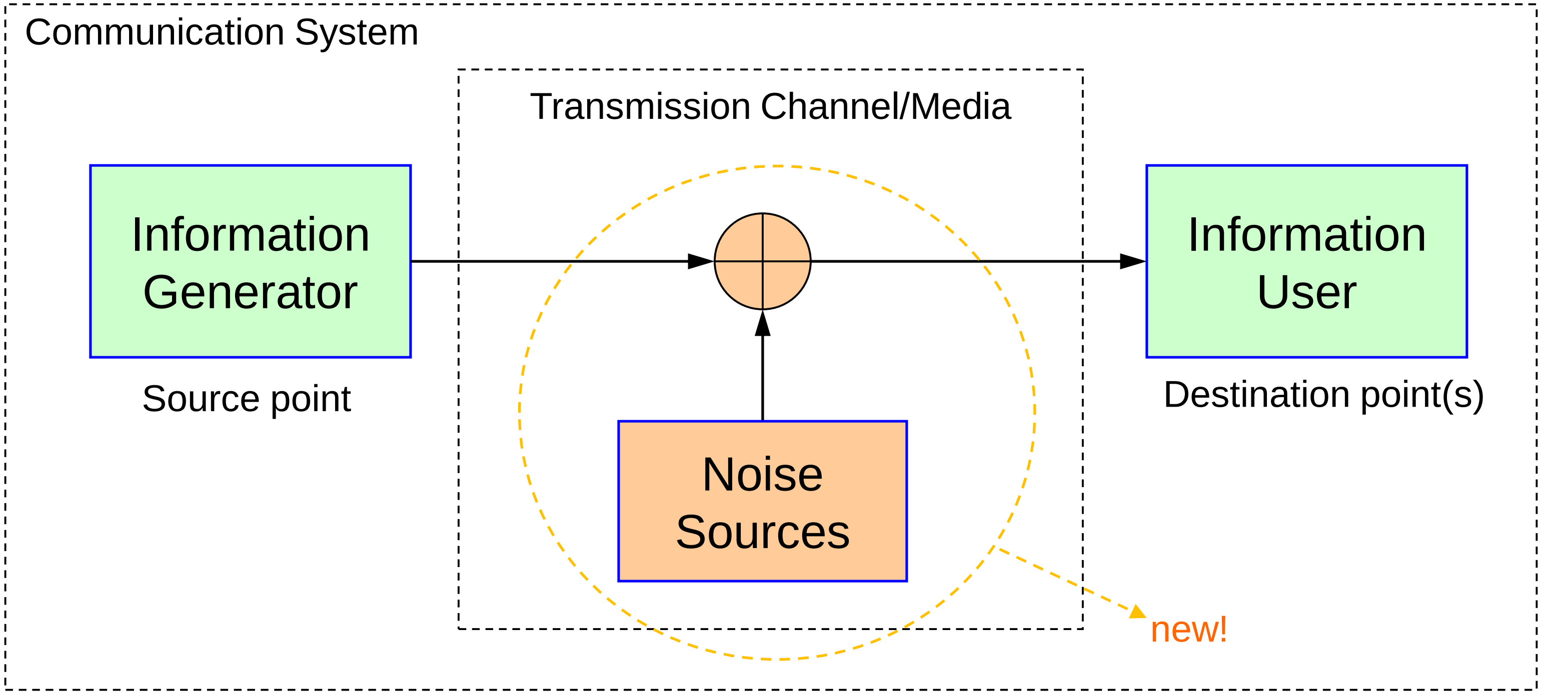
For the course "**Communications**"

ESKİŞEHİR OSMANGAZİ UNIVERSITY

The Goal

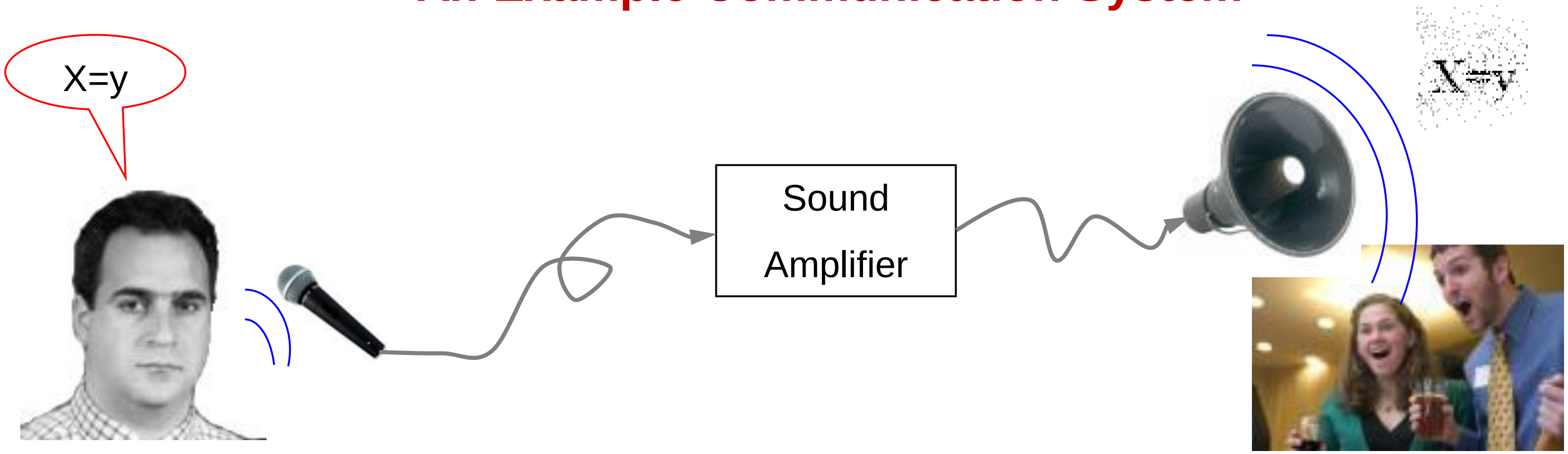
Transfer information from source point to one or more destinations correctly

(using least amount of resources, in most cases)



Resources = material, voltage, spectrum, health, circuit, ... = time, money

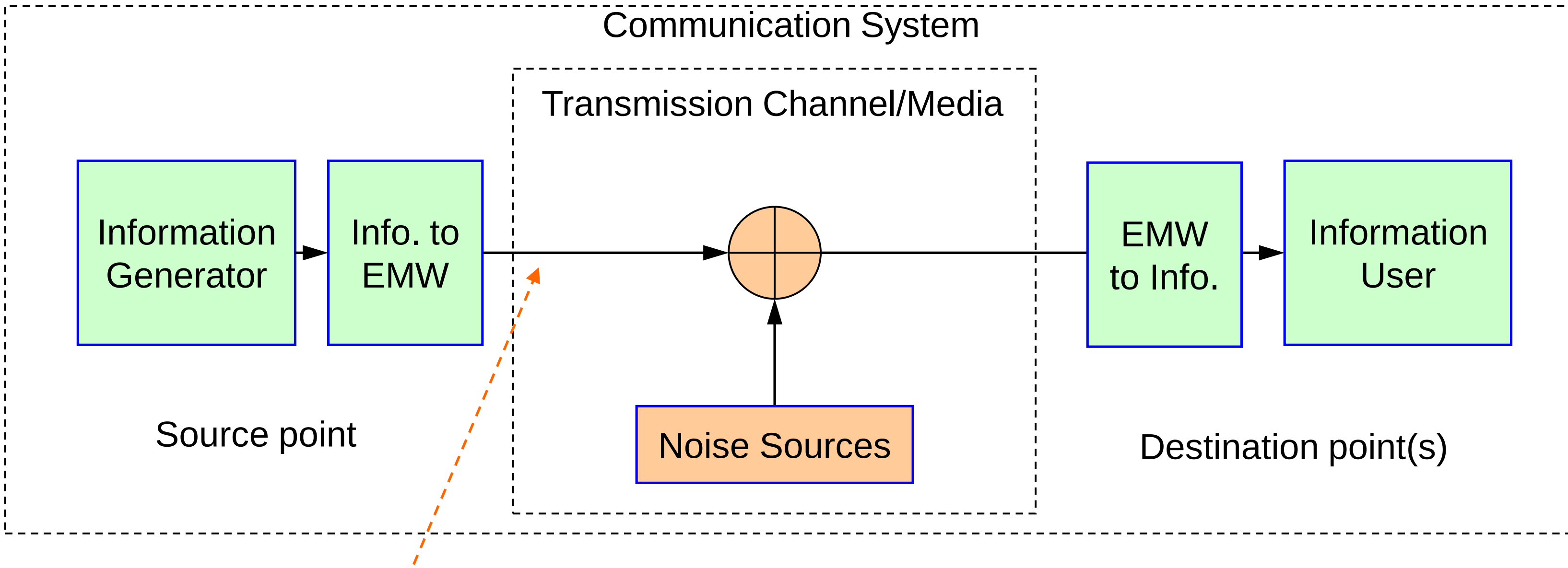
An Example Communication System



Some other examples of (electronic) source, channel and destination

Microphone – Twisted pair of wire - Amplifier
Modem - Twisted pair of telephone line - Modem
Fax scanner – Telephone system – Fax printer
Computer – Ethernet cable - Computer
Computer’s data storage medium – Fiber optic network – Another computer’s storage
Digital data generator – Magnetic disk – Digital data user
Radio transmitter – Air/Space – Radio receiver
Digital TV data from satellite – Atmosphere – Digital TV receiver
TV Remote controller – Air – IR sensor/receiver on TV

Another Block Diagram



$\psi(r, t) = A cos(kr - \omega t + \varphi)$ (basic travelling wave)

t : time
r : distance from source
A : amplitude
k : spatial frequency
ω: temporal frequency

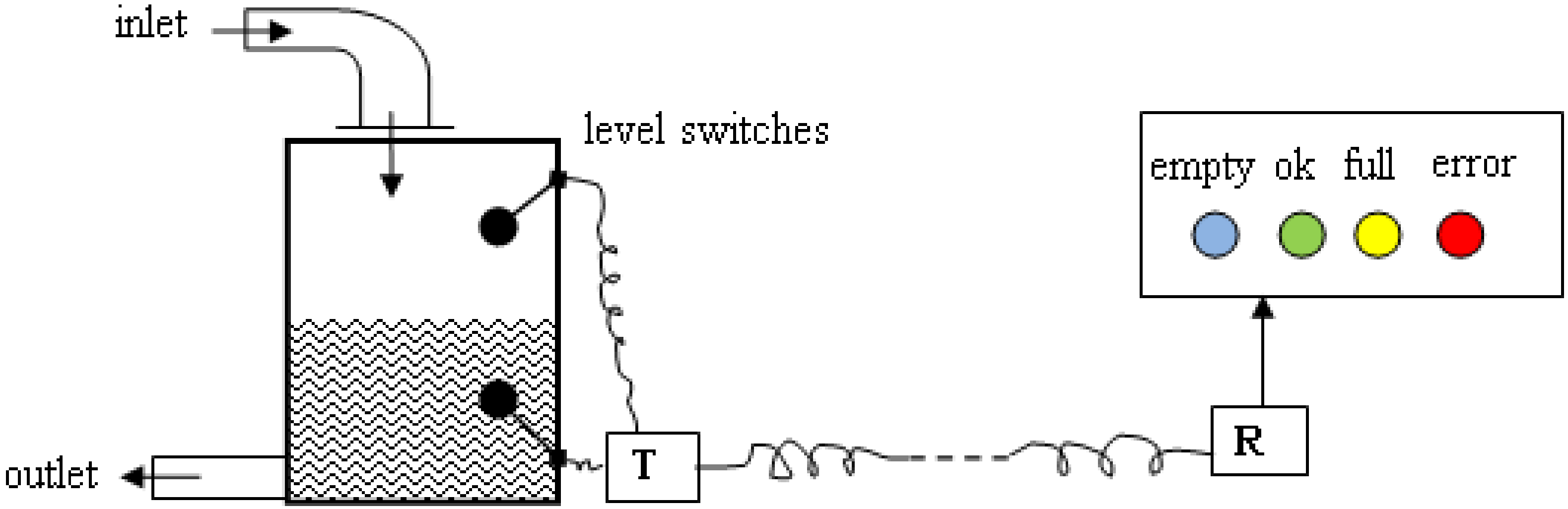
Magnetic wave is always perpendicular to electric field and travel direction

It turns out that we need to control value(s) of one ore more of these in order to carry information

We can control some properties of electric field, magnetic field occurs automatically

We can also control polarity of the wave using superposition of multiple waves

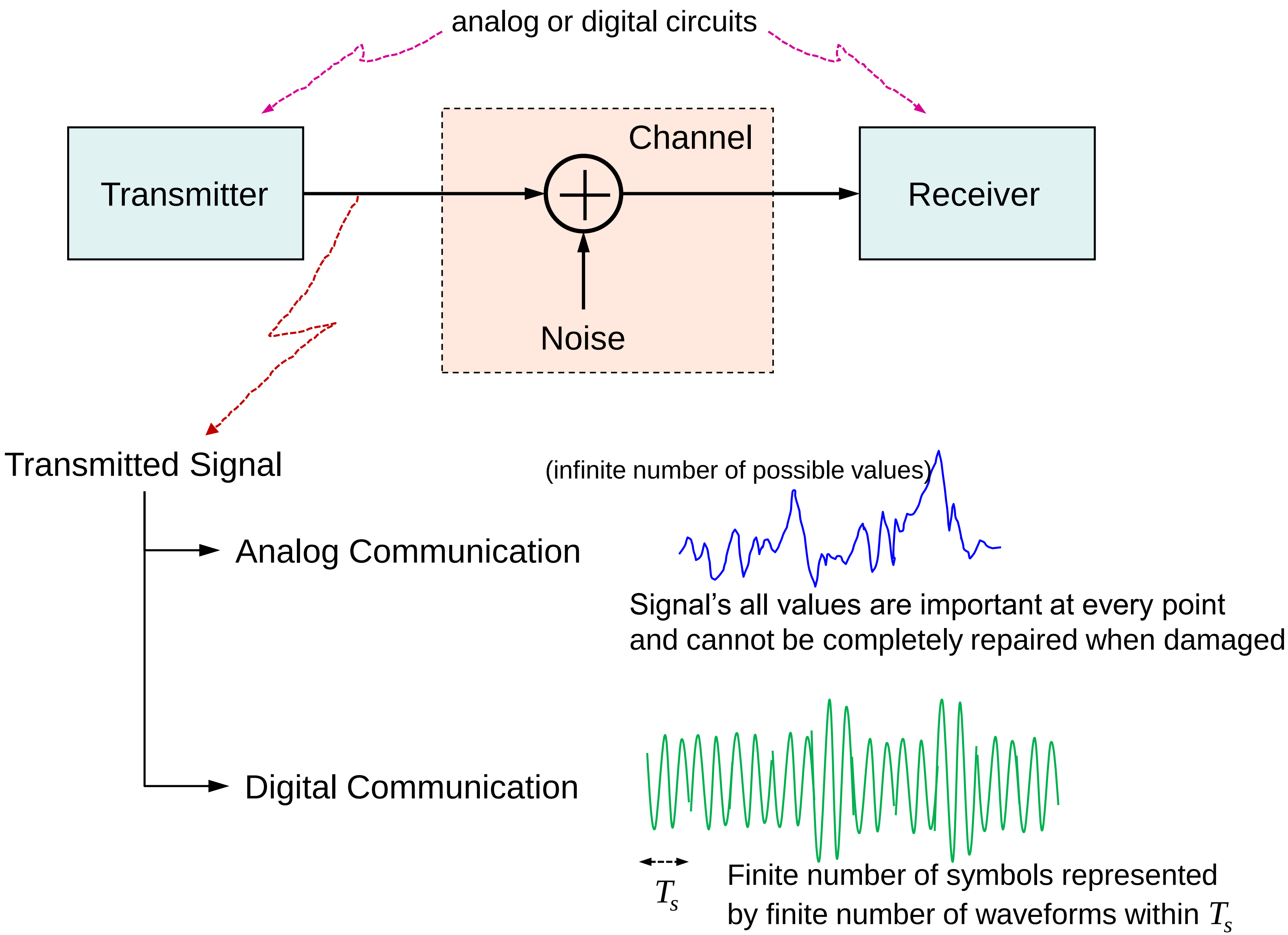
An Example Digital Communication System



There are only 4 messages/values to sent

Consider: An analog system doing a similar job

Analog / Digital Electronic Communication



Digital Communication

Advantages :

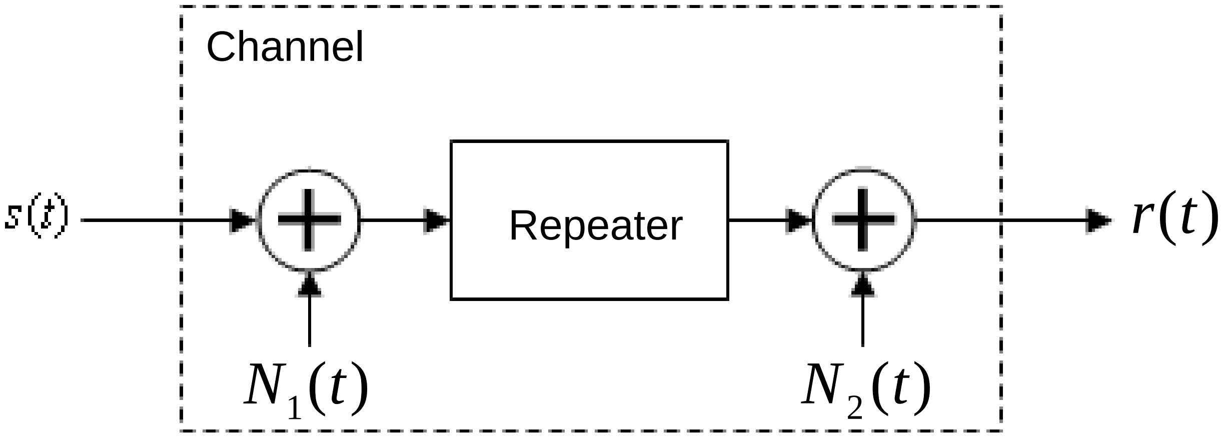
- Mathematical/Logical Processing on the data is possible
- Therefore : higher protection against noise
- More flexible when performed using reconfigurable / reprogrammable elements
- ?

Disadvantages : (against analog communication)

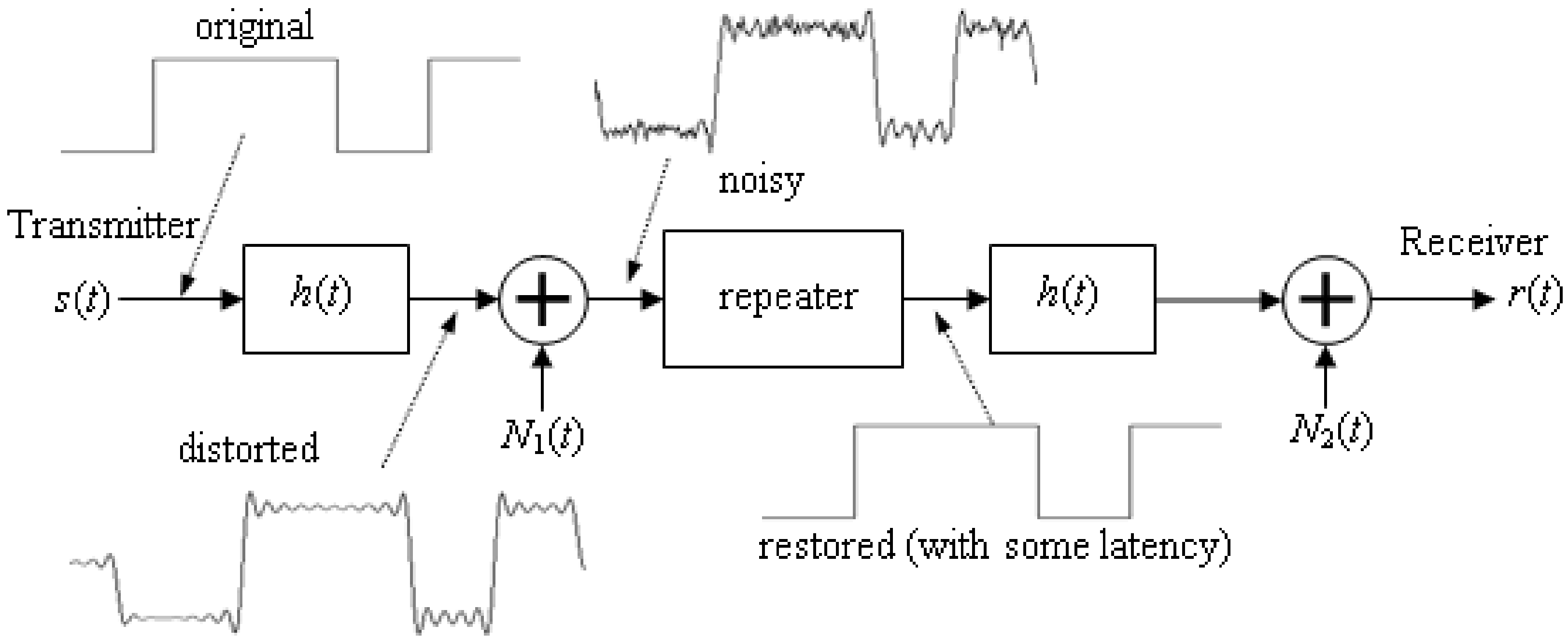
- Complexity is higher
- Higher speed devices are required
- Sometimes analog signals need to be converted/deconverted using ADC/DAC
- ?

An Advantage of Digital Communication

Signal can be restored and resent halfway between transmitter and receiver

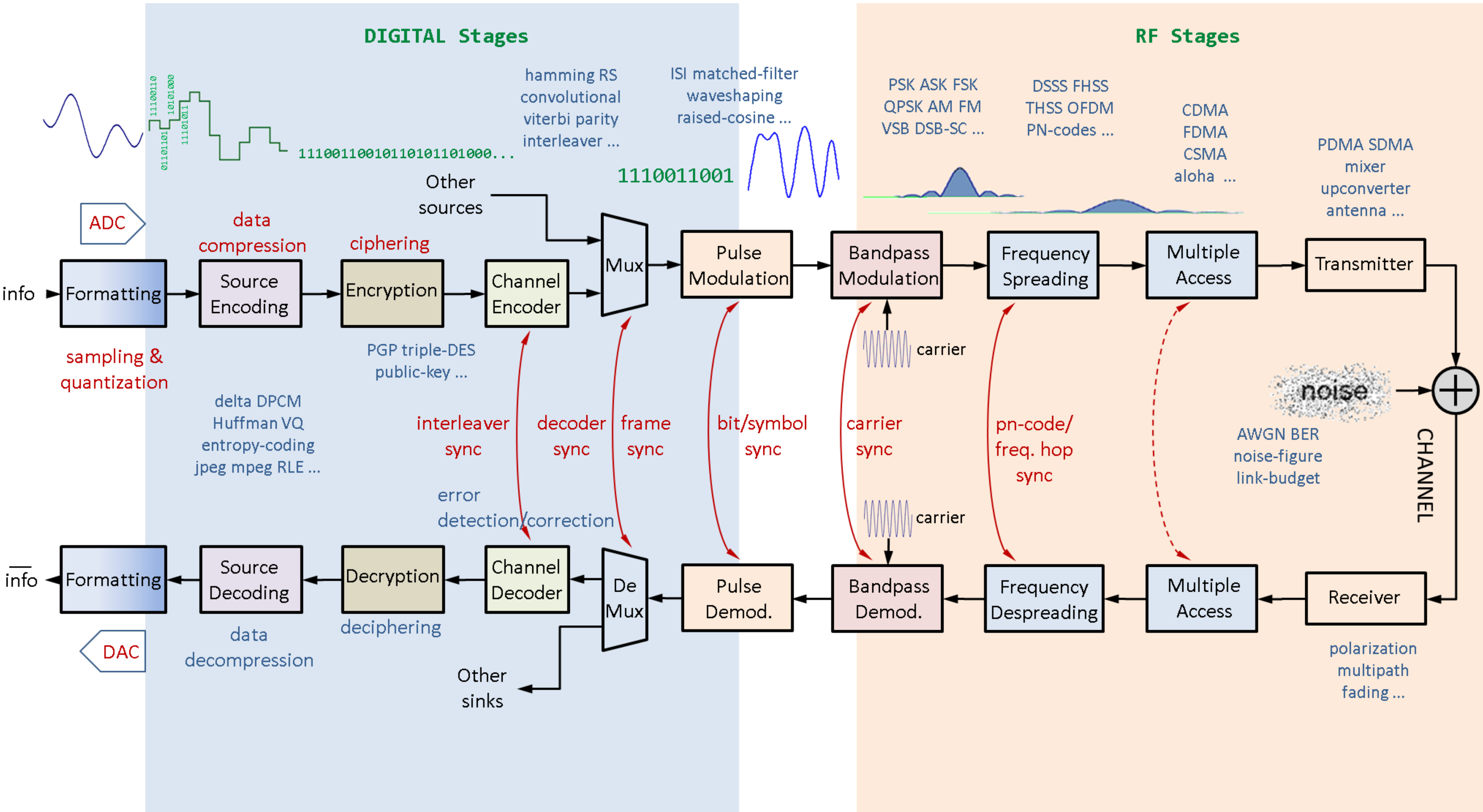


An Advantage of Digital Communication



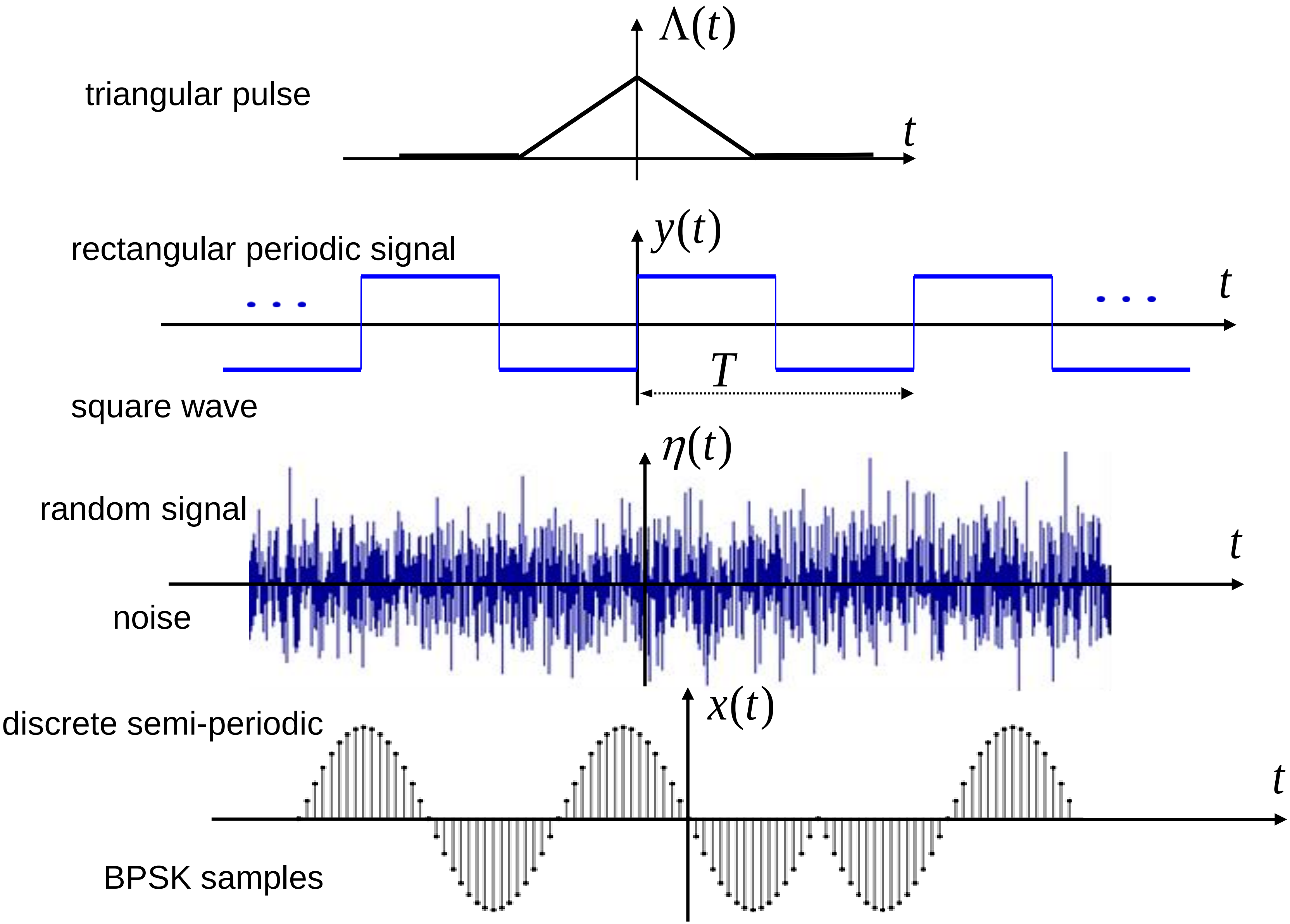
So that the signal is received with minimum (or no) error (but with additional delay)

General Communication System

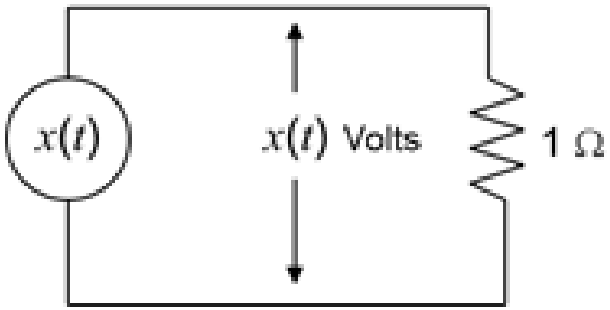


we will get back to this schema time-to-time.

Various Signals



Energy of a Signal



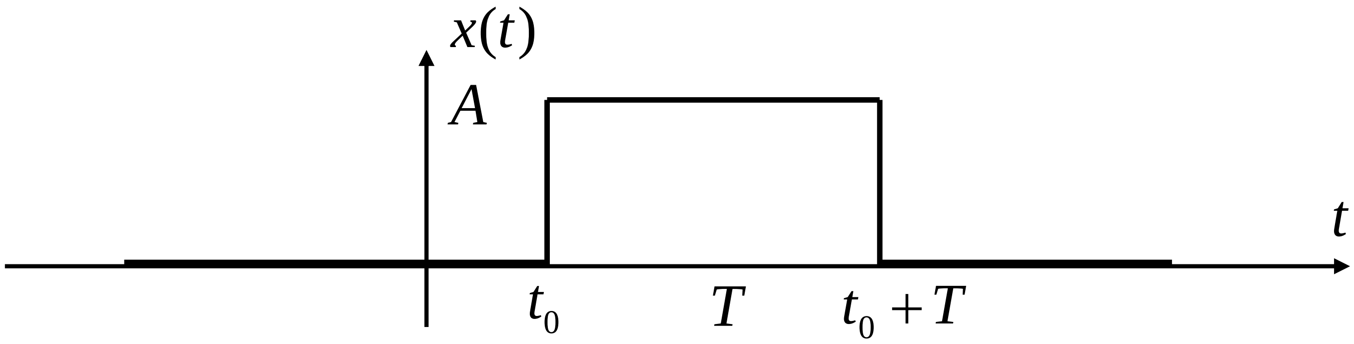
Energy of a signal $x(t)$ is defined as the energy spent on a 1 Ohm load

$$E_x = \int_{-\infty}^{\infty} |x(t)|^2 \, dt$$

Its unit is Volts^2/Ohm = Joules

Example

Energy of a rectangular pulse

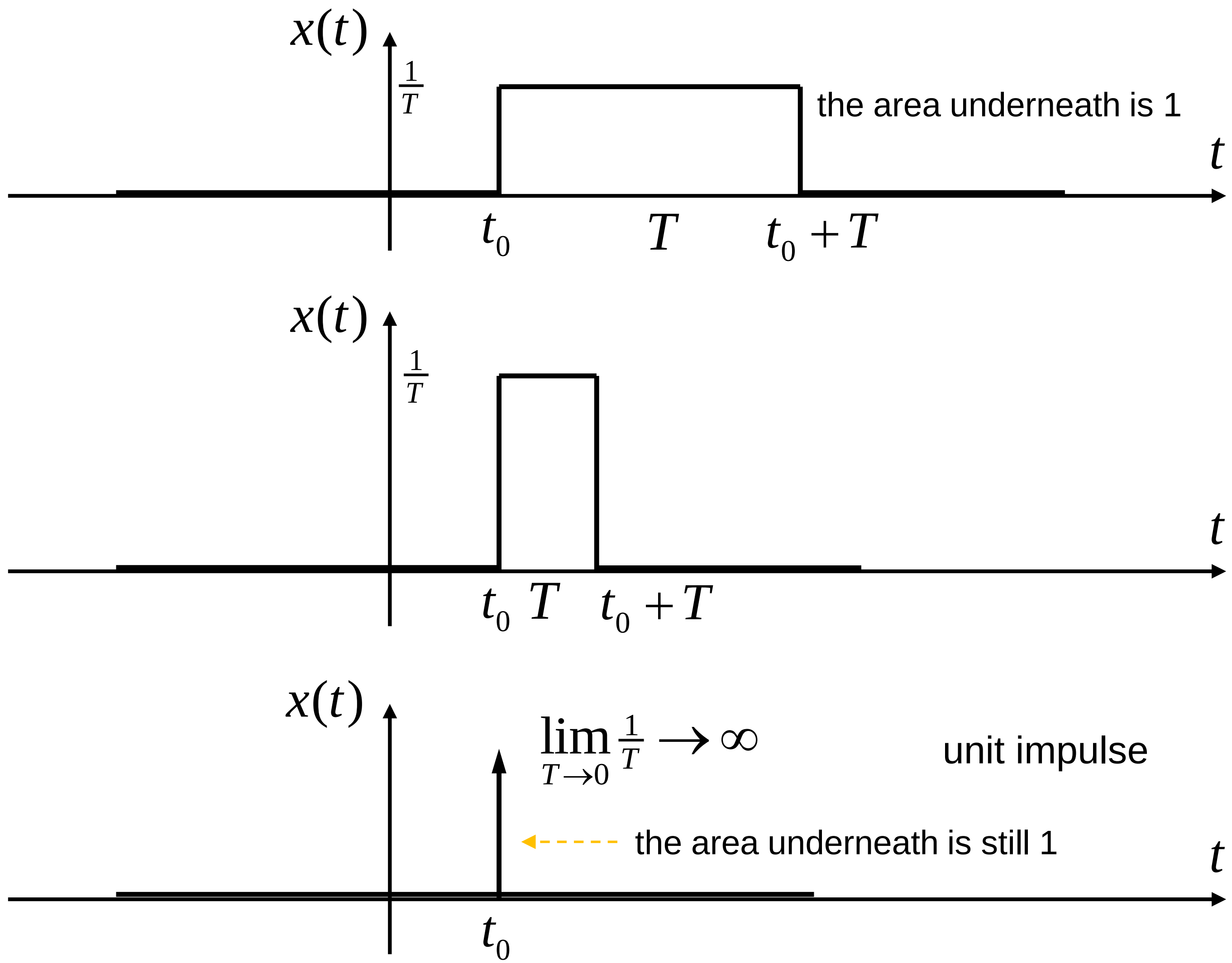


$$E_x = \int_{-\infty}^{\infty} \left| x(t) \right|^2 dt = \int_{t_0}^{t_0+T} A^2 dt = A^2 \, t \Big|_{t_0}^{t_0+T} = A^2 T$$

independent of the position on time axis (valid for all signals)

Example

Let $AT = 1 \quad \Rightarrow \quad A = \frac{1}{T}$



Power of a Signal

If the energy is infinite, then we talk about the energy spent in unit time.

$$P_x = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{+T/2} |x(t)|^2 \, dt$$

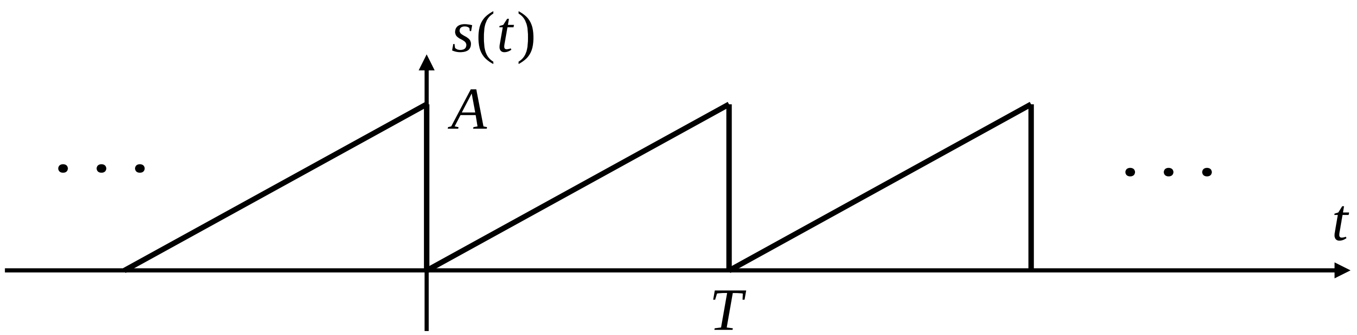
For periodic signals

$$P_x = \frac{1}{T} \int_{\alpha}^{\alpha + T_0} |x(t)|^2 \, dt$$

Its unit is Watts

Example

Find the power of the saw-tooth signal



$$P_s = \frac{1}{T}\int_0^T |s(t)|^2\, dt = \frac{1}{T}\int_0^T \left|\frac{At}{T}\right|^2 dt = \frac{A^2}{3T^3} t^3 \Big|_0^T$$

$$P_s = \frac{A^2}{3}$$ power is independent of the period / frequency / time-shift

Example

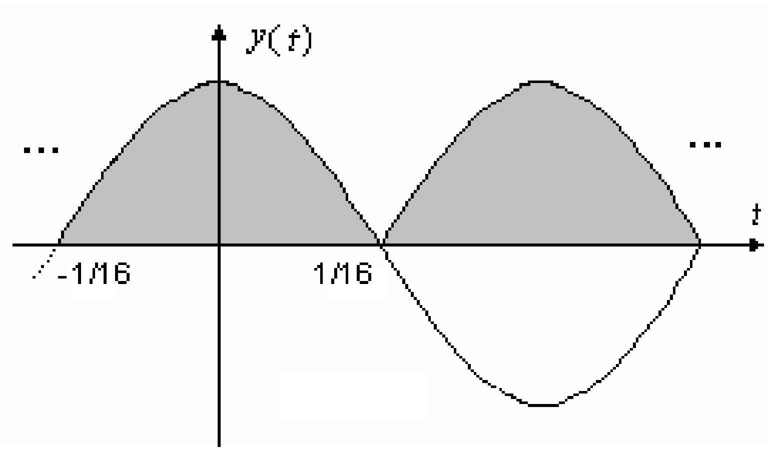
Find the power and energy of the waveform $y(t) = |\cos(8\pi t)|$

$$P = \frac{1}{T}\int_a^{a+T} |x(t)|^2\, dt$$

$$P = \frac{1}{1/8}\int_{-1/16}^{1/16} \cos^2(8\pi t)\,dt$$

$$P = 8\left[\frac{t}{2} + \frac{\sin(16\pi t)}{32\pi}\right]_{-1/16}^{1/16} = \frac{1}{2}$$

(verify!)



Since $\frac{1}{2} < \infty$ it is a power signal. Therefore it is not an energy signal. So, $E = \infty$

Since power is independent of the phase & frequency, it is ok to shift & squeze the waveform & take advantage of symmetry in order to have easier integration

Average / Expected Values

Average value of a continuous signal $m_x = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{+T/2} x(t)dt$

Average value of discrete samples $x_{avg} = \frac{1}{N} \sum_{i=1}^{N} x_i$

X is a random process, x is its generated values.

(we will define *random process* later)

Expected value of a continuous random process $E\{X\} = \int_{-\infty}^{\infty} x f(x) dx$

where $f(x)$ the pdf of x.

Average and expected values are equal when observation duration (or number of samples) is infinite

Example Let us have two sinusoidal signals

$$x(t) = \sin(2\pi t / T) \quad \text{and} \quad y(t) = \cos(2\pi t / T)$$

$$E_x = \int_{-\infty}^{\infty} |x(t)|^2 \, dt = \infty \qquad E_y = \int_{-\infty}^{\infty} |y(t)|^2 \, dt = \infty$$

energies are both infinite (periodic signals)

$$P_x = \tfrac{1}{T} \int_{\alpha}^{\alpha+T_0} |x(t)|^2 \, dt = \tfrac{1}{T} \int_0^T |\sin(2\pi t / T)|^2 \, dt = \frac{1}{2}$$

$$P_y = \tfrac{1}{T} \int_{\alpha}^{\alpha+T_0} |y(t)|^2 \, dt = \tfrac{1}{T} \int_0^T |\cos(2\pi t / T)|^2 \, dt = \frac{1}{2}$$

powers are the same, too

$$m_y = m_x = \lim_{T \to \infty} \tfrac{1}{T} \int_{-T/2}^{+T/2} \sin(2\pi t / T) dt = 0$$ averages are the same

They are both sinusoidal

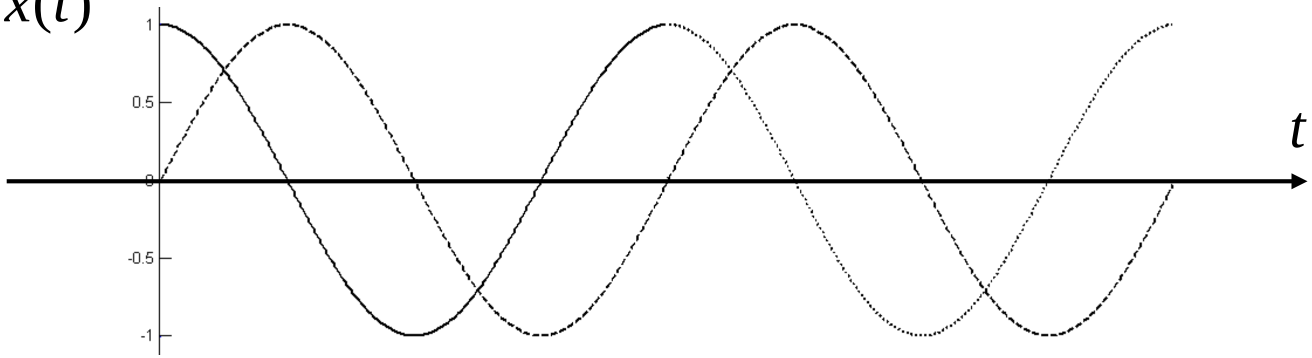
question : what is different?

Similarity / Dissimilarity Measure

Similarity of signals is measured using an inner product

$$\langle y(t), x(t) \rangle = \int_{-\infty}^{\infty} y(t)x(t)dt$$

$y(t), x(t)$



It is obvious that the integral, except for finite duration signals, will be infinite. Therefore, we need to have some kind of normalization.

$$< x(t), y(t) >= \int_0^T \sin(2\pi t/T)\cos(2\pi t/T)dt = 0!$$

Does this mean these two are dissimilar?

We need to check similarity for shifted versions of signals too.

Cross-Correlation

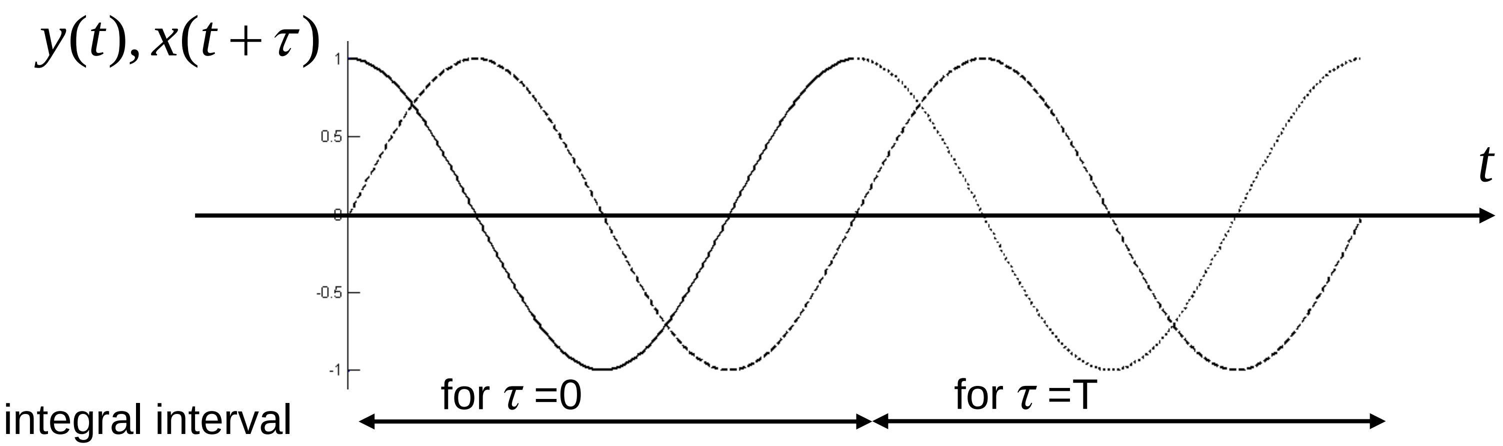
Similarity of shifted versions of signals

$$R''_{xy}(\tau) = \int_{-\infty}^{\infty} x(t)\,y(t+\tau)\,dt$$

called Cross-Correlation Function, where τ represents the time/spatial shift

$$R'_{xy}(\tau) = \int_{0}^{T} x(t)\,y(t+\tau)\,dt$$ for periodic signals

$$R_{xy}(\tau) = R_{xy}(\tau) / R_{\max}$$ (normalized)

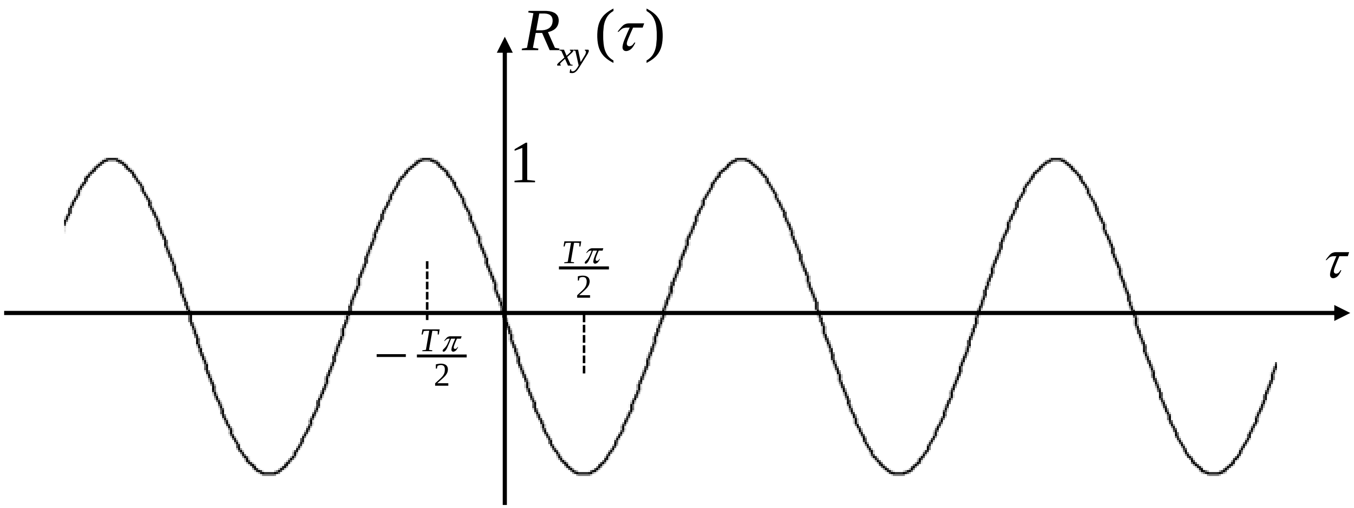


Example

Since our signals are periodic, $T_x = T_y = T$

we can select an integral interval of T

$$R'_{xy} = \int_0^T \sin(2\pi t / T)\cos(2\pi(t+\tau)/T)dt \;\; = -\tfrac{T}{2}\sin(2\pi\tau / T)$$



The signals are similar to each other on periodic intervals

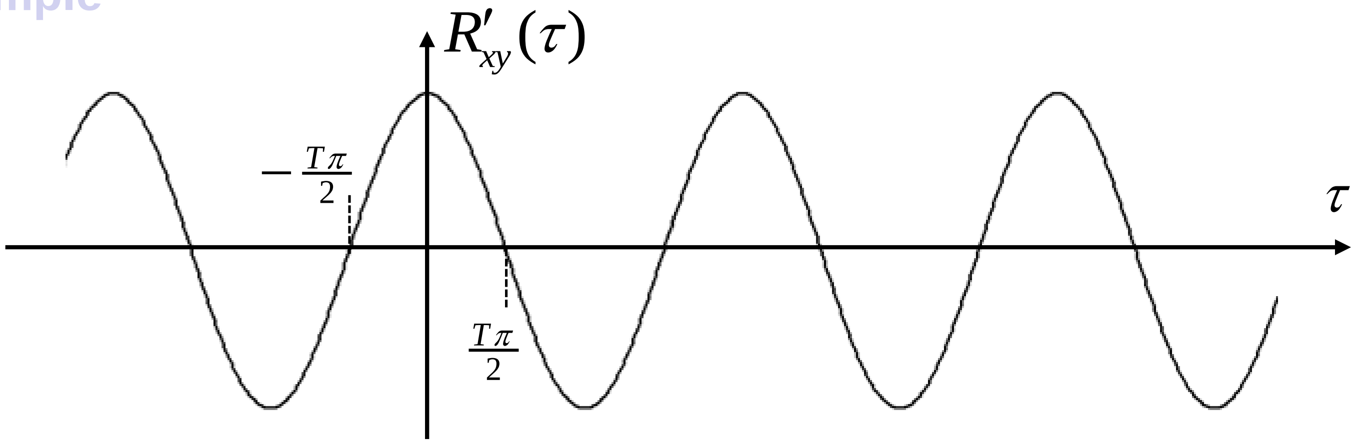
Autocorrelation

if both signals are same; $y(t) = x(t)$

the similarity is named as autocorrelation (function)

$$R''_{xy}(\tau) = \int_{-\infty}^{\infty} x(t)x^*(t+\tau)dt$$

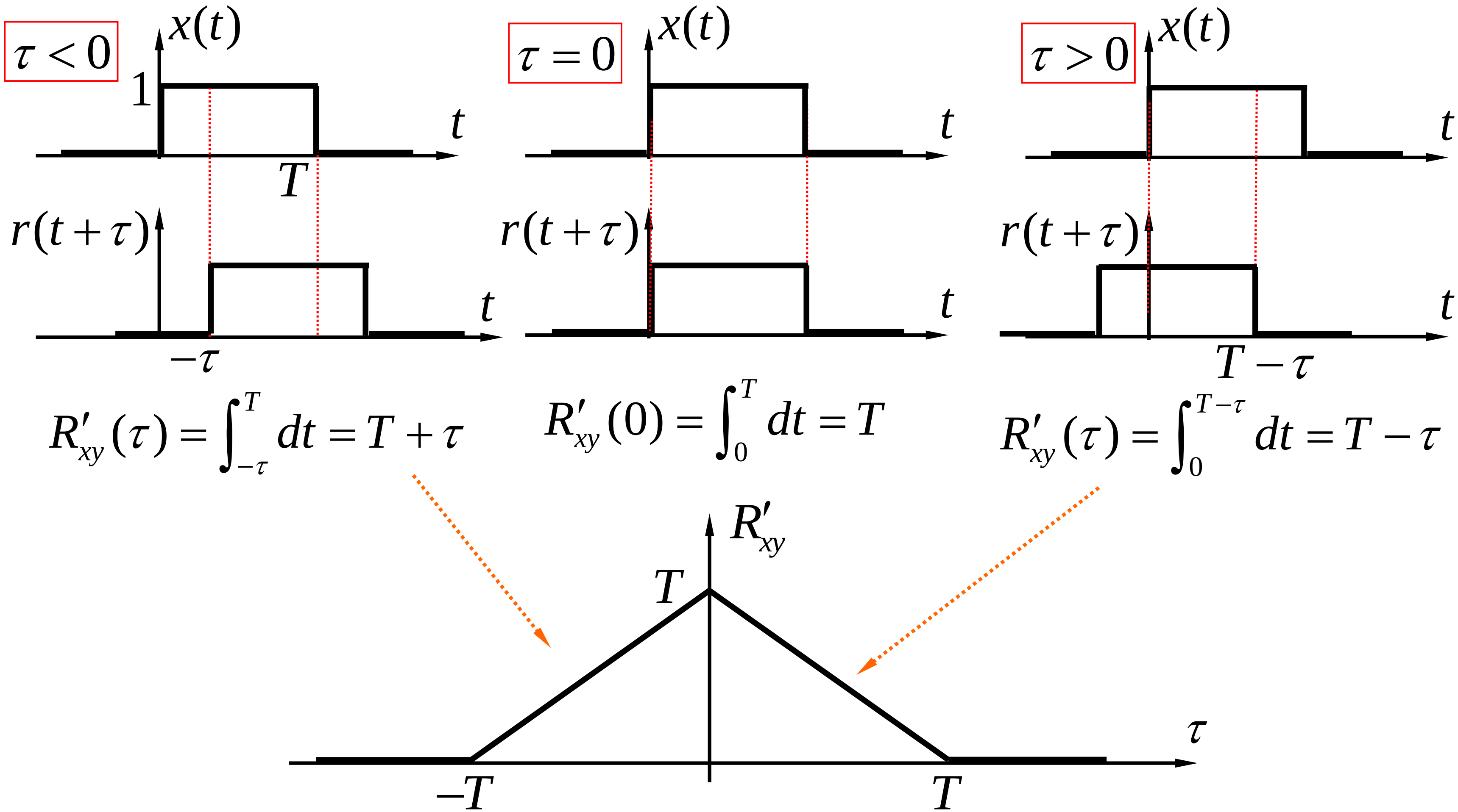
Example



signal x is similar to itself on periodic intervals

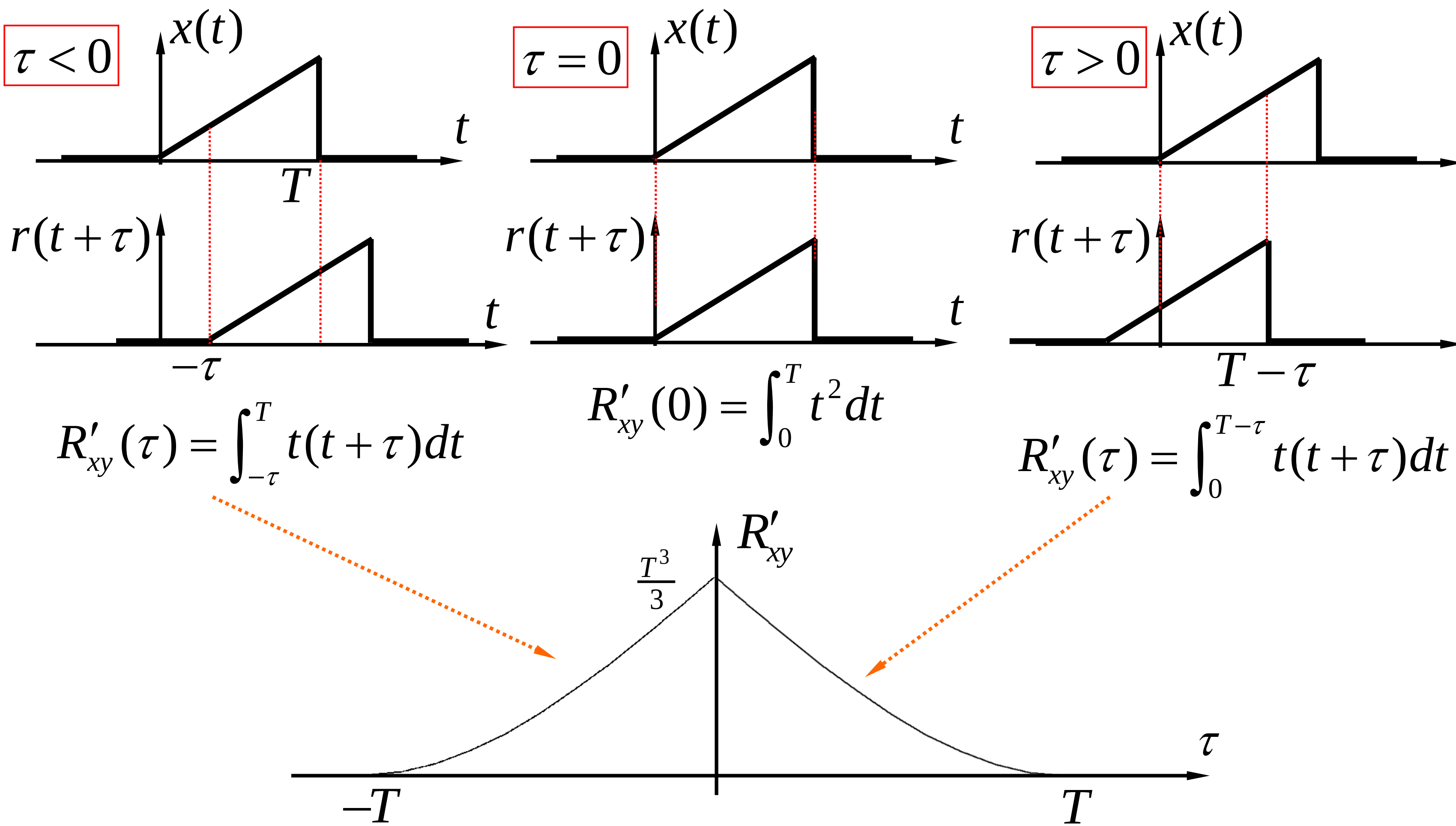
Example

$$r(t) = \begin{cases} 1 & , 0 < t < T \\ 0 & , otherwise \end{cases} \quad \text{and} \quad x(t) = r(t)$$



Example

$$r(t) = \begin{cases} t & , 0 < t < T \\ 0 & , otherwise \end{cases} \quad \text{and} \quad x(t) = r(t)$$



$$R'_{xy}(\tau) = \int_{-\tau}^{T} t(t+\tau)dt$$

$$R'_{xy}(0) = \int_{0}^{T} t^2 dt$$

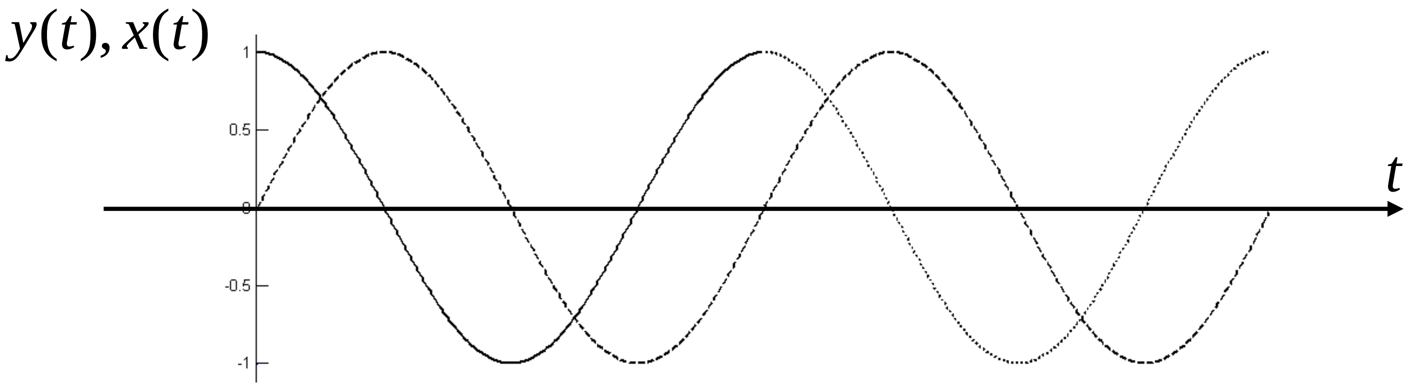
$$R'_{xy}(\tau) = \int_{0}^{T-\tau} t(t+\tau)dt$$

Orthogonal Signals

Inner product also tells us if the signals are orthogonal

If $\langle y(t), x(t) \rangle = \int_{-\infty}^{\infty} y(t)x(t)dt = 0$

then $y(t), x(t)$ are said to be orthogonal

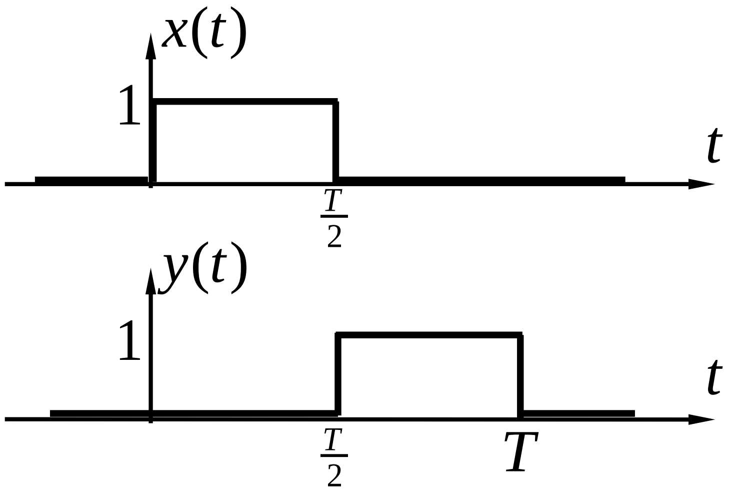


$$< x(t), y(t) >= \int_0^T \sin(2\pi t / T)\cos(2\pi t / T)dt = 0$$

meaning that $y(t)$ does not have any component of $x(t)$ within

(shifted versions of signals may not be orthogonal)

Example



$$< x(t), y(t) >= \int_0^T (u(t) - u(t - \tfrac{T}{2}))(u(t - \tfrac{T}{2}) - u(t - T))dt = 0$$

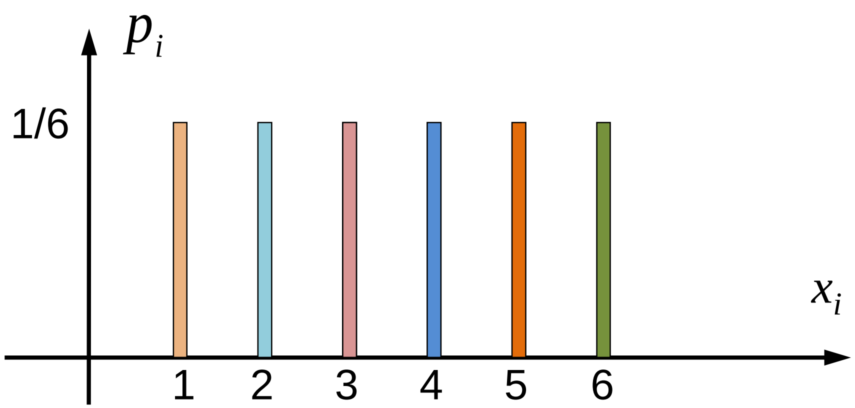
Given a set of waveforms $x_i(t)$, we can find an orthogonal waveform set $\psi_k(t)$ so that $x_i(t)$ can be written as a weighted linear sum of $\psi_k(t)$

$$x_i(t) = \sum_{k=0}^{M-1} c_{k,i} \psi_k(t)$$

Hmw : study this subject (*orthogonalization*) from the referenced sources

Probability

Die throwing experiment



$$\sum_i p(x_i) = 1$$

Random variable : An event or value which is measured

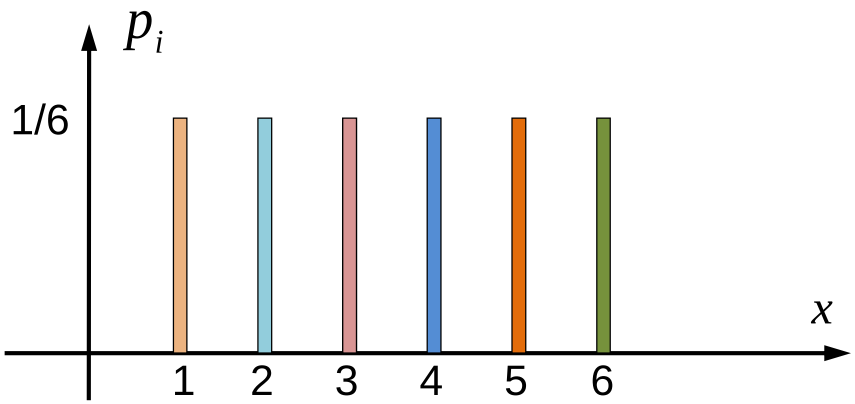
x : Value read on die after throwing event (random variable)
p_i : probability of x ($p(x_i)$)

Expected Value of Discrete Experiment

$$E(X) = \sum_i x_i p(x_i)$$

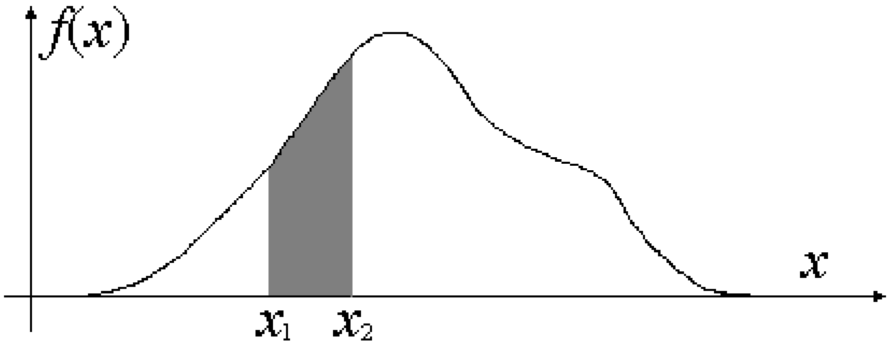
The name "expected value" does not imply that it is expected to happen

Expected value of die throwing experiment = 3.5 which will never happen



This graph is called Probability Mass Function (pmf)

Probability Density Function



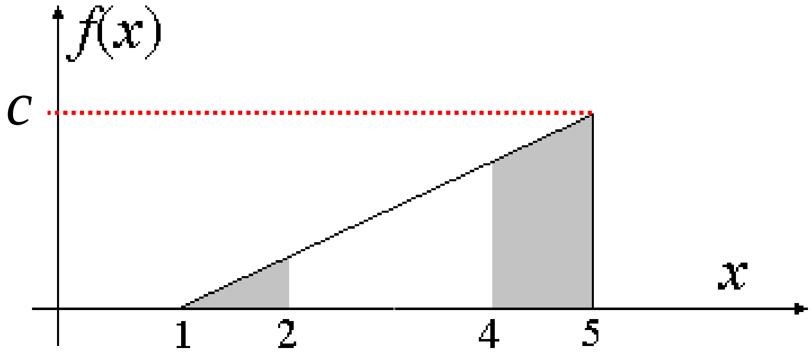
$$\int_{-\infty}^{+\infty} f(x)dx = 1.0$$

$$P(x_1 \le x < x_2) = \int_{x_1}^{x_2} f(x)dx$$

if this is a "total of something" then this is a "density"

shaded area gives the probability of $P(x_1 \le x < x_2)$

Example What is c?



$$\int_{-\infty}^{+\infty} f(x)dx = 1.0 \Rightarrow \int_1^5 \tfrac{c}{4}(x-1)dx = \tfrac{c}{8}(x-1)^2\Big|_1^5 = 2c \Rightarrow c = \tfrac{1}{2}$$

$$E(X) = \int_1^5 xf(x)dx = \tfrac{1}{8}\int_1^5 (x^2 - x)dx = \tfrac{1}{8}\left[\tfrac{1}{3}x^3 - \tfrac{1}{2}x^2\right]_1^5 \simeq 3.67$$

$$p(x \le 2) = \int_1^2 \tfrac{1}{8}(x-1)dx = \tfrac{1}{16} \qquad p(x > 4) = \int_4^5 \tfrac{1}{8}(x-1)dx = \tfrac{7}{16}$$

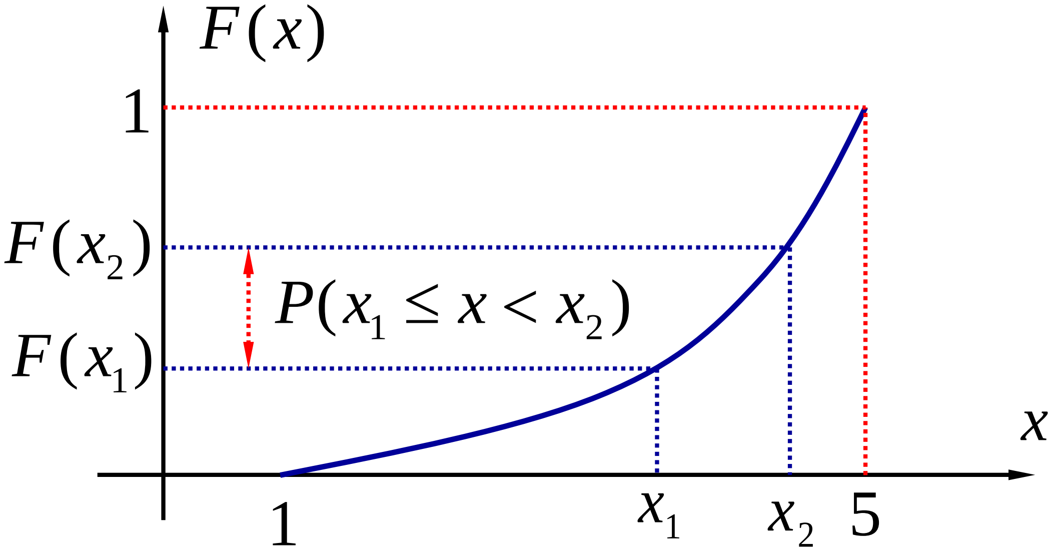
Q : if x is a periodic function, what are the possibilities of $x(t)$?

Cumulative Distribution Function

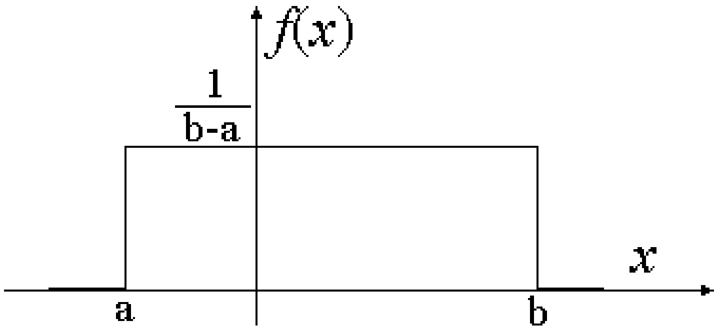
$$F(x) = \int_{-\infty}^{x} f(u)du$$

so that

$$P(x_1 \leq x < x_2) = F(x_2) - F(x_1)$$

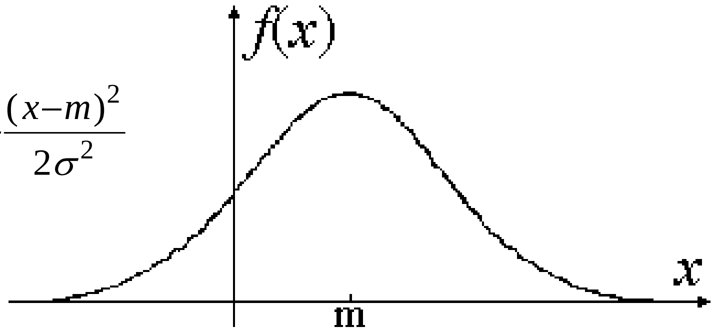


Well Known Distributions



uniform pdf

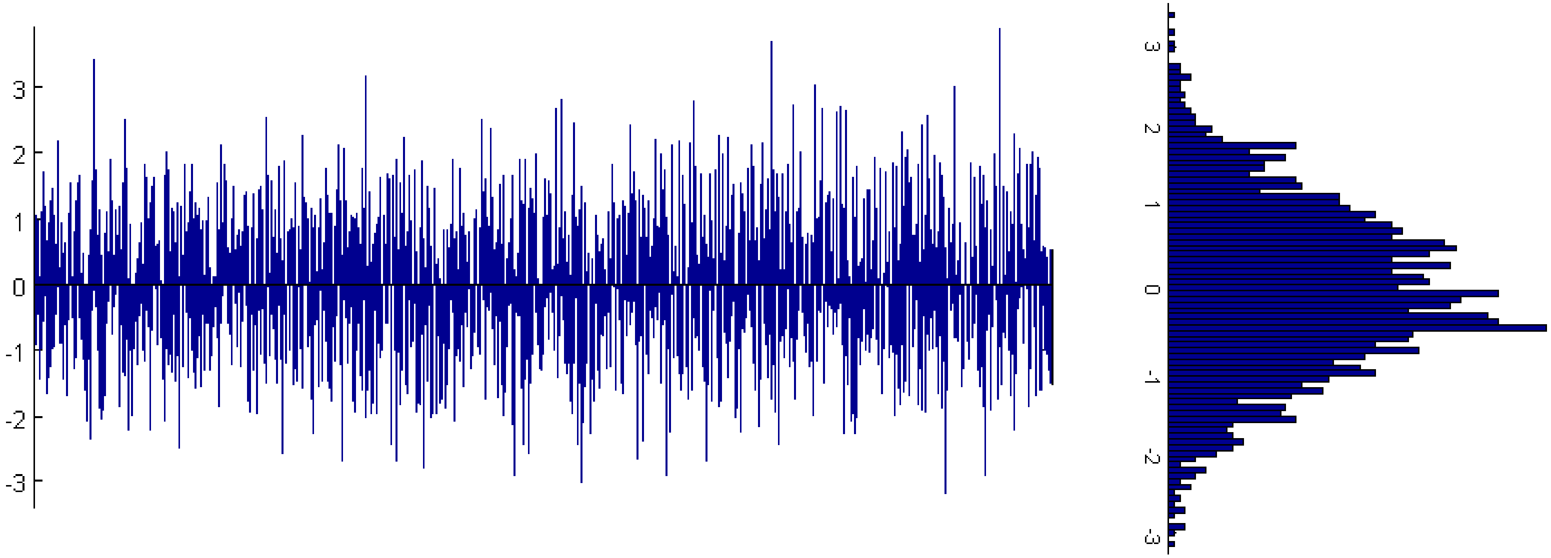
$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-m)^2}{2\sigma^2}}$$



Gaussian (normal) pdf

$$\sigma = \sqrt{E((x-m)^2)}$$ Standard deviation

Histogram



Histogram : A graph showing the #occurrences of r.v. within each bin.
for a random experiment repeated N times.

For the given signal : See the resemblance to Gaussian r.v.

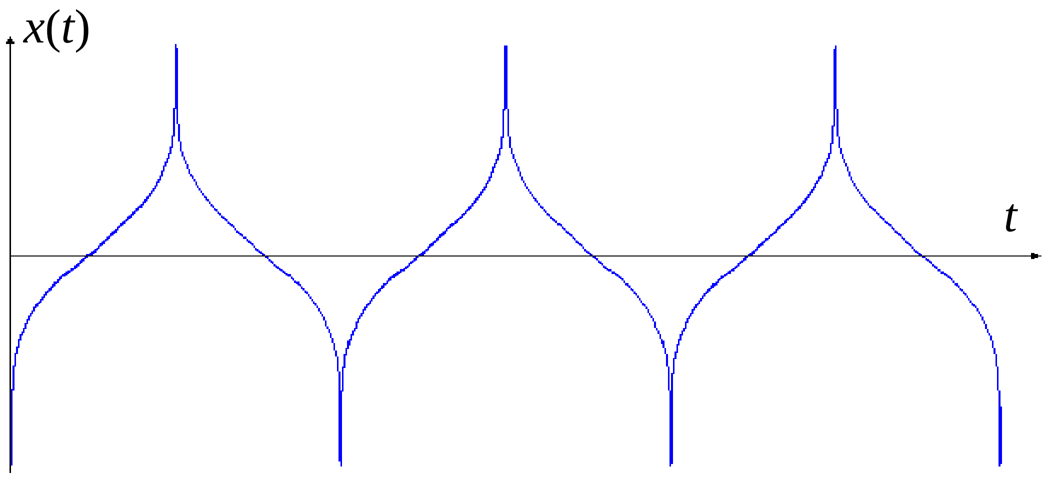
Histogram shows the distribution for the measured results.

pdf is the expected distribution (may not have done yet)

For large number of experiments, histogram becomes a representation of pdf

pdf shows only the probabilistic distribution, not the time function itself

For example, pdf of the following periodic function looks like a Gaussian



There may be infinite number of functions that have the same pdf

(we might have an example on how to determine pdf for a given periodic waveform)

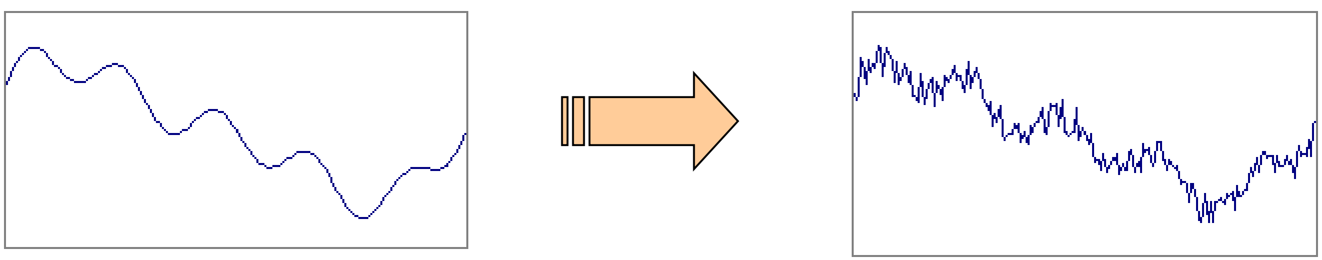
Noise

Any signal other than our structured signal, but intentionally or unintentionally added onto our signal, is categorized as noise.

Noise Sources

- Electronic / Thermal noise
- Electrical discharges in the atmosphere / nearby devices
- Interference / Crosstalk between channels and multipath effects
- Solar / Cosmic effects
- Distortion from nonlinearities of the electronics / media

(quantization noise and granular noise are evaluated in different contexts)



Channel input Channel output

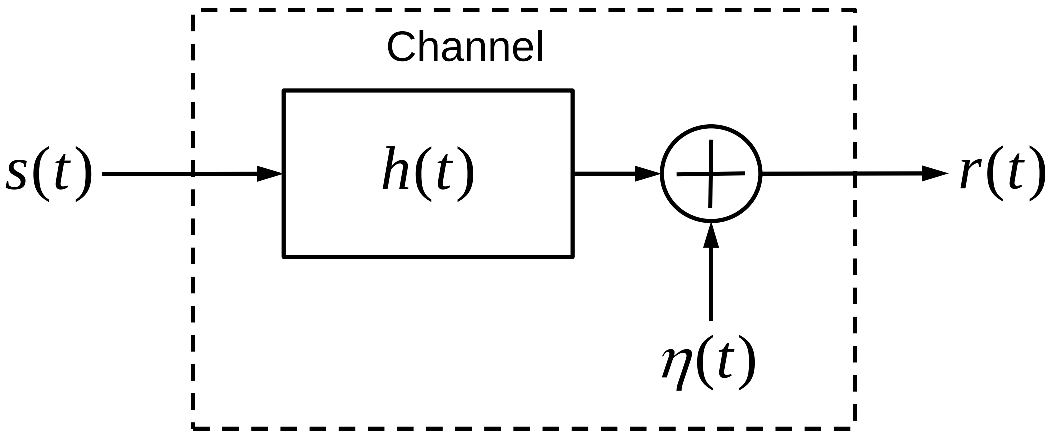
AWGN

Noise is usually assumed to be (this assumption is not baseless)

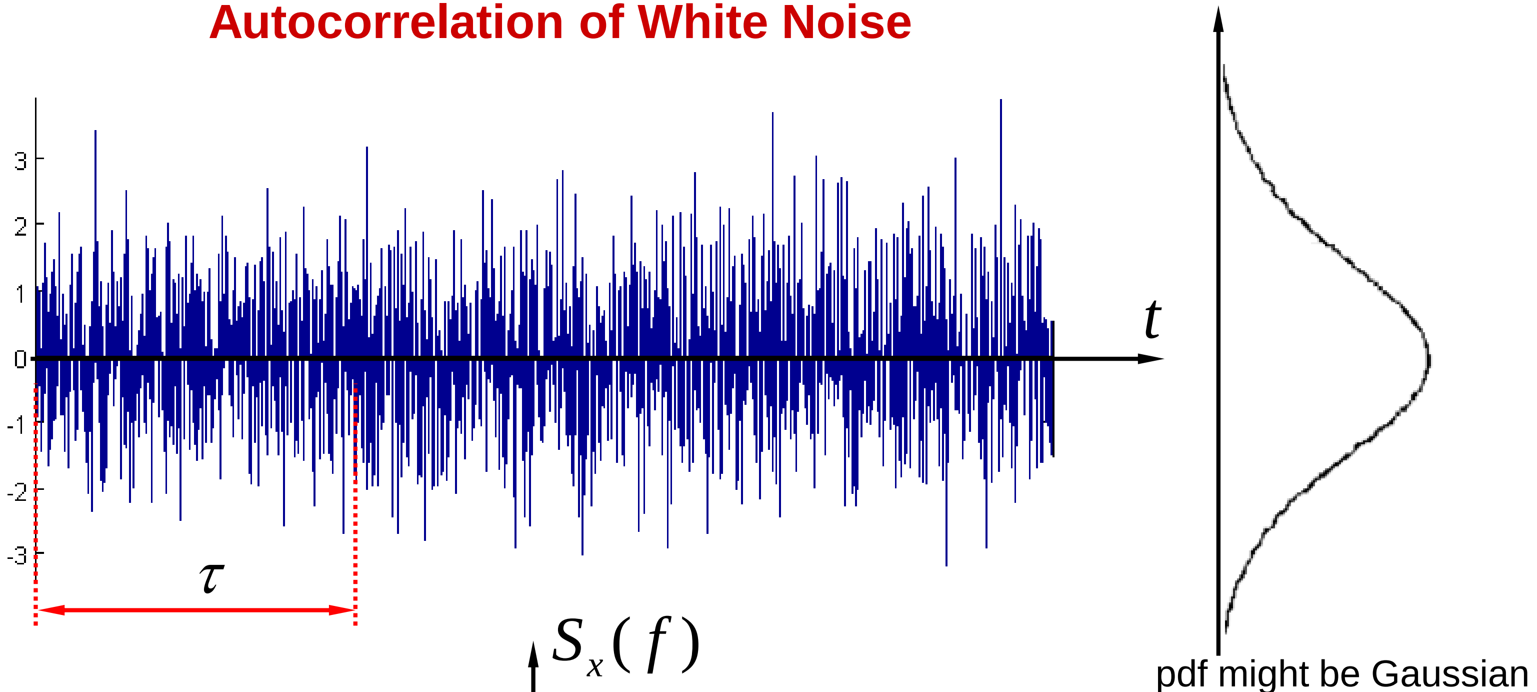
Additive : $S_o = kS_i + N$

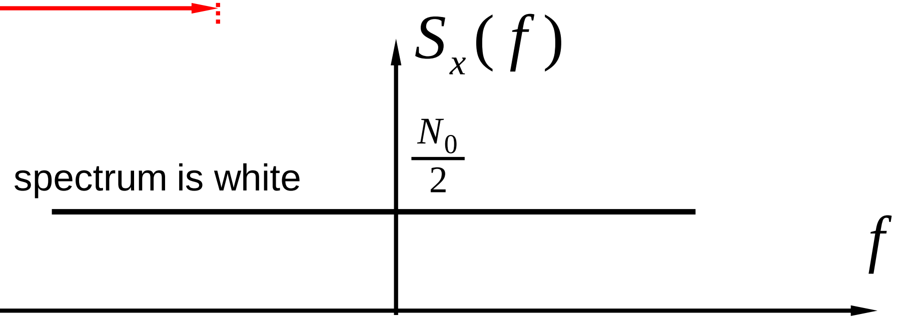
White : $|\mathcal{F}(N)| = c$ (has the same power at all frequencies)

Gaussian : Probability distribution function is Gaussian

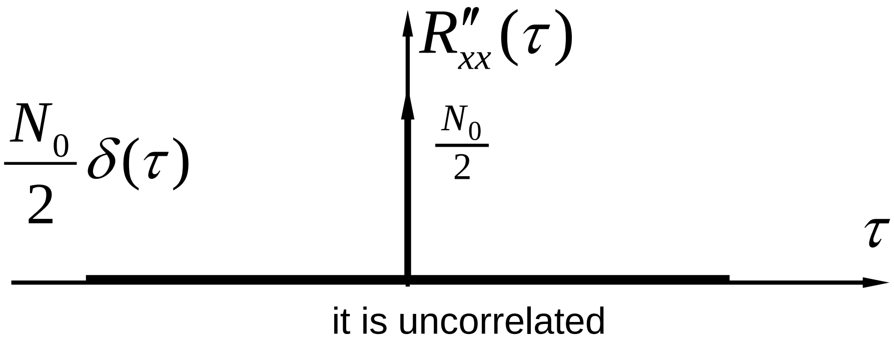


Autocorrelation of White Noise



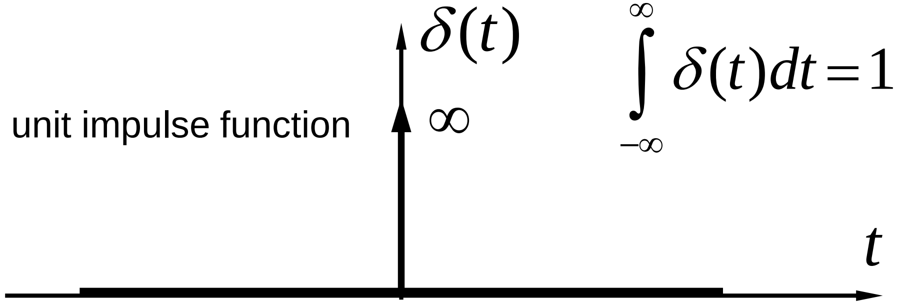


$$R''_{xx}(\tau) = \int_{-\infty}^{\infty} x(t)x^*(t+\tau)dt = \frac{N_0}{2}\delta(\tau)$$



Impulse

unit impulse function



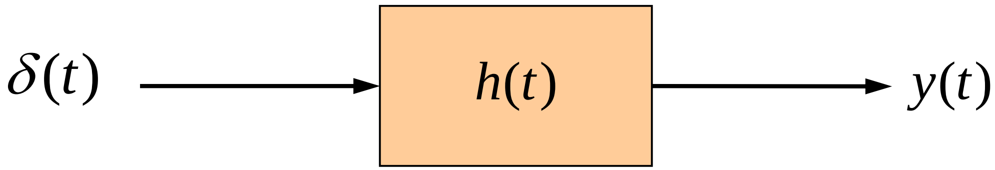
$$\int_{-\infty}^{\infty} \delta(t)dt = 1$$

$$E_x = \int_{-\infty}^{\infty} |\delta(t)|^2 dt = ?$$ (undefined)

$$\int_{-\infty}^{\infty} \delta(t)x(t)dt = x(0)$$

as a result $\int_{-\infty}^{\infty} \delta(t-\tau)x(t)dt = x(\tau)$ where τ is the position of impulse

Impulse Response of a System



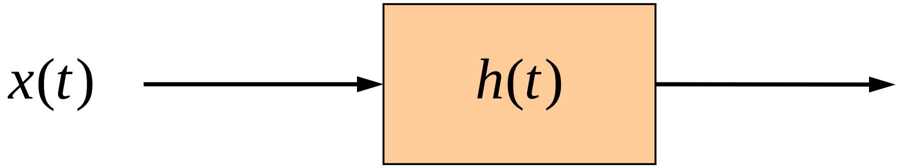
$$y(t) = \int_{-\infty}^{\infty} \delta(t-\tau)h(\tau)dt = h(t)$$

$y(t) = h(t)$ when input is $\delta(t)$

Convolution

Think of $x(t)$ as an infinite sum of $\int_{-\infty}^{\infty} \delta(\tau - t) x(\tau) dt = x(t)$

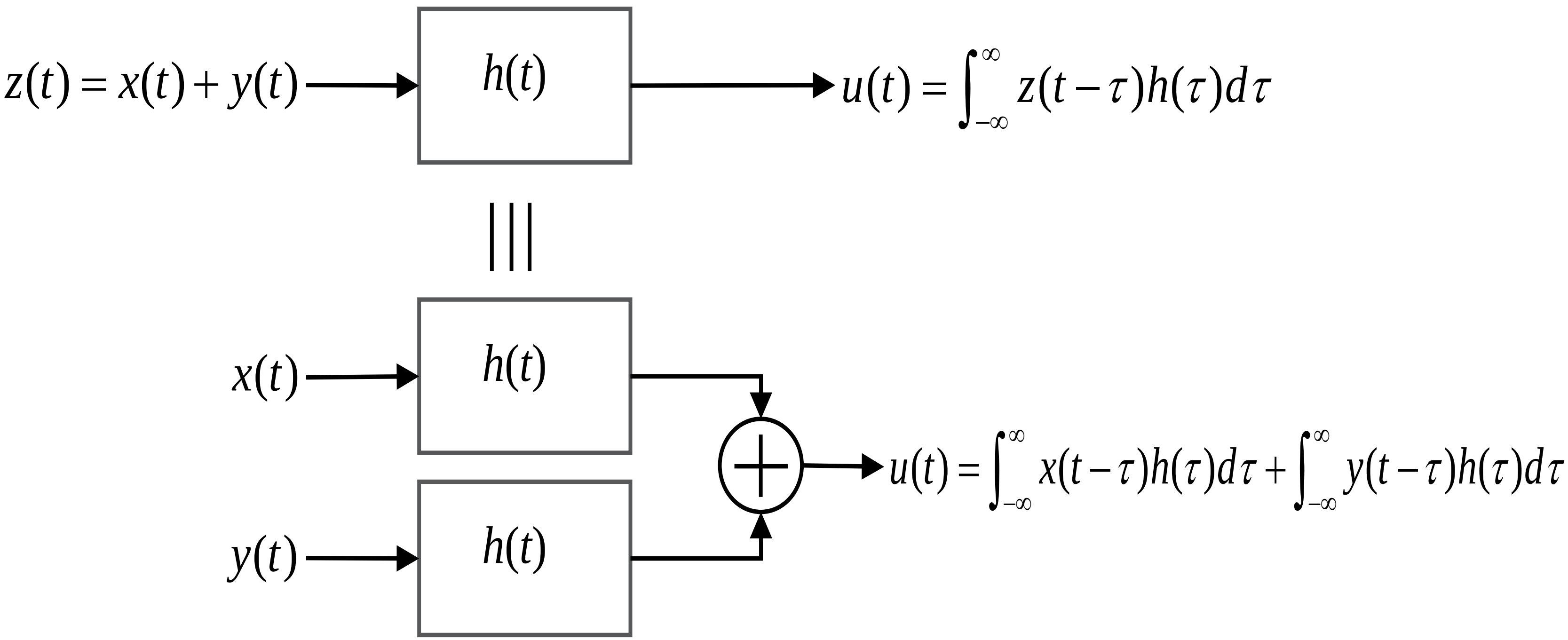
The output of the system will be an infinite sum of responses to each weighted impulse



$$y(t) = \int_{-\infty}^{\infty} x(\tau) h(t-\tau) d\tau$$

For this infinite summation to hold, the system must be a Linear System

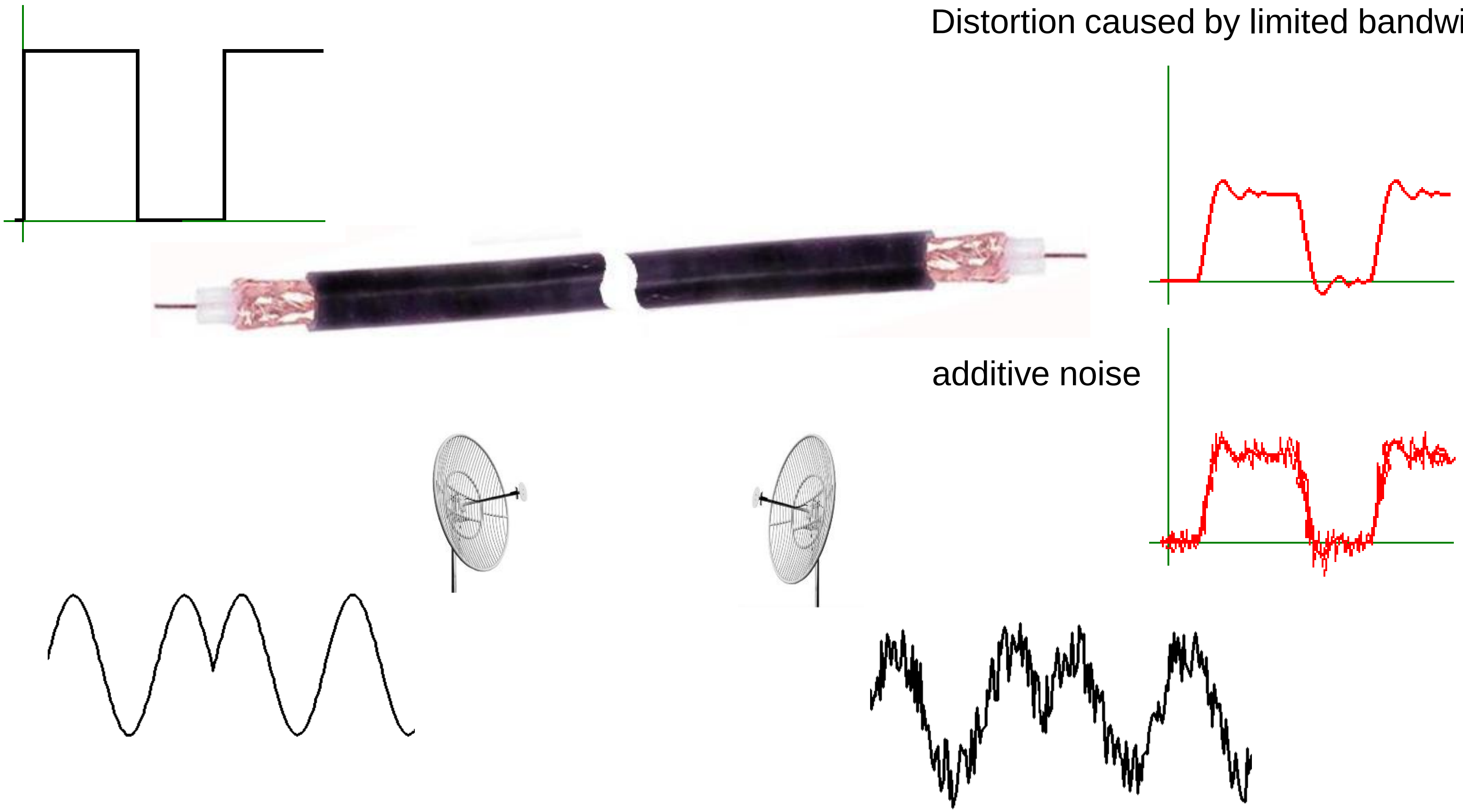
Linearity

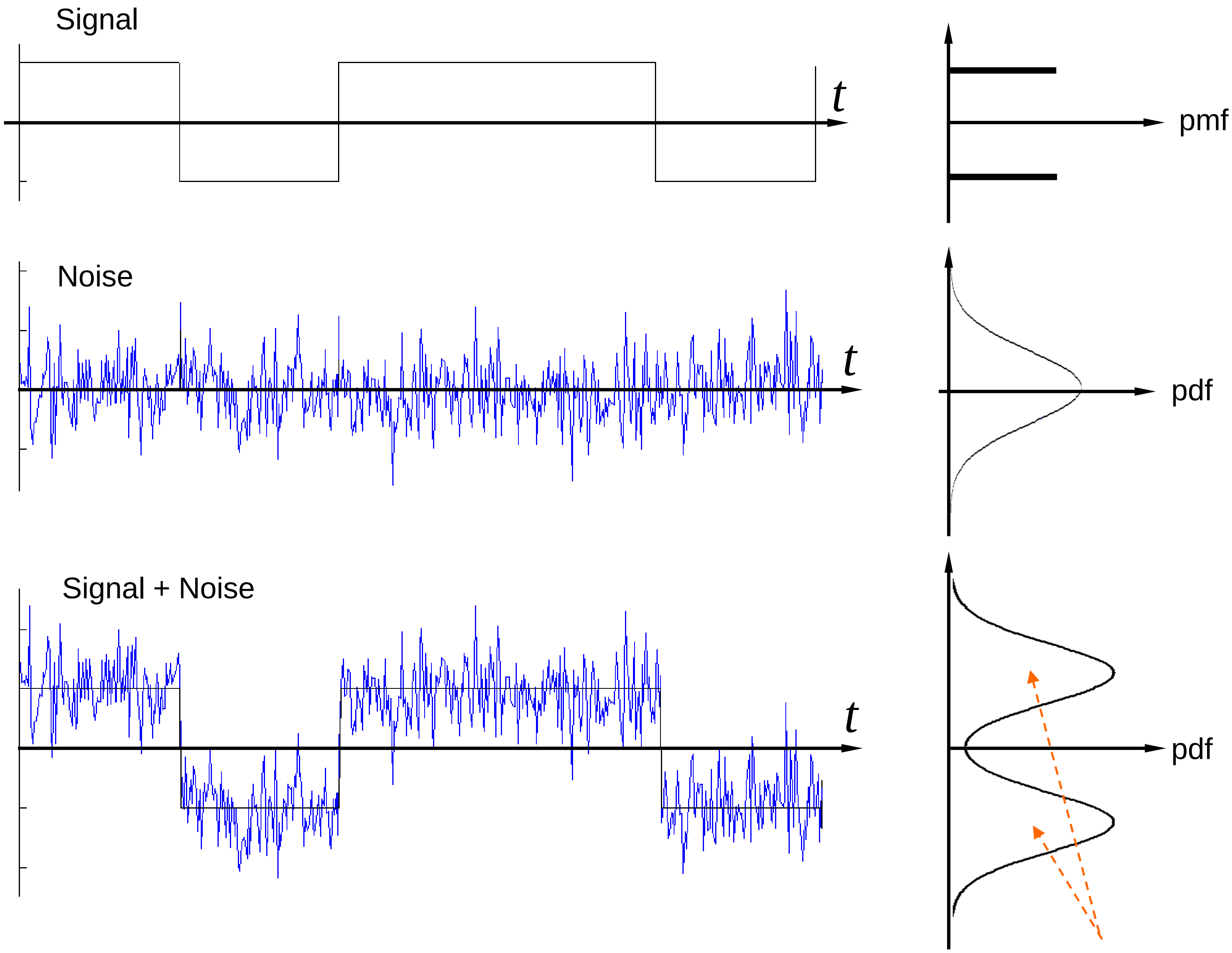


$h(t)$ is a Linear Time Invariant (LTI) system

Time Invariant : the system $h(t)$ does not change by time

Signal + Noise





Note that : If Signal/Noise decreases, it becomes more difficult to distinguish these hills

END