Amplitude Modulation (AM)

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For the course “`Communications`”

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Initial Problem : Carry voice signal over distances without using cable

The Solution : Radiate its electromagnetic wave through air
and pick this up at the receiver locations

Resulting Problems : 1. There can only be one EM-wave in the air since the receiver picks up all.
2. Transmitter antenna must be very long.

Hints : 1. Human ear can only hear 20Hz-20kHz†.
2. EM-spectrum is very large compared to this.

Second Approach : Load voice onto HF EM-wave.

So that : HF EM needs shorter antenna (~inverse of wavelength).
Each voice channel use different EM-band.

The Question : How?

The Answer : Modulate EM-wave with voice signal(s).

That is : Change one or more properties of EM-wave by the voice/message signal

$$\underset{\overset{\nearrow}{\text{Polarity}}}{\psi(r,t)} = \underset{\overset{\uparrow}{\text{Amplitude}}}{A}\cos(kr - \underset{\overset{\uparrow}{\text{Frequency}}}{\omega} t + \underset{\overset{\nwarrow}{\text{Phase}}}{\varphi})$$

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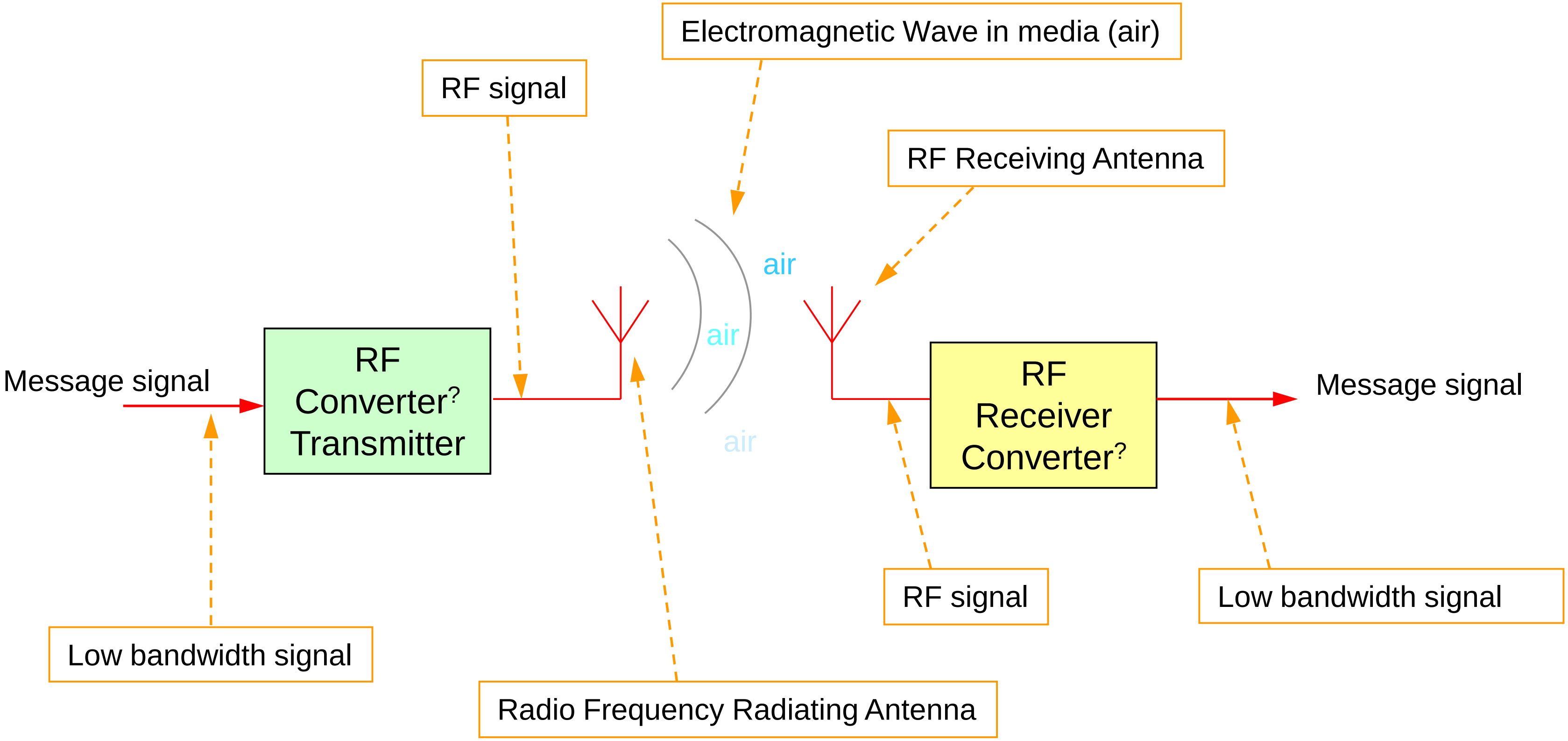
† : Various sources give different ranges depending on some parameters (age, health etc) but we are not interested in an exact number here anyway.

Summary of Radio Transmission



Electromagnetic Wave in media (air)

RF signal

RF Receiving Antenna

air

air

Message signal

RF Converter? Transmitter

RF Receiver Converter?

Message signal

air

Low bandwidth signal

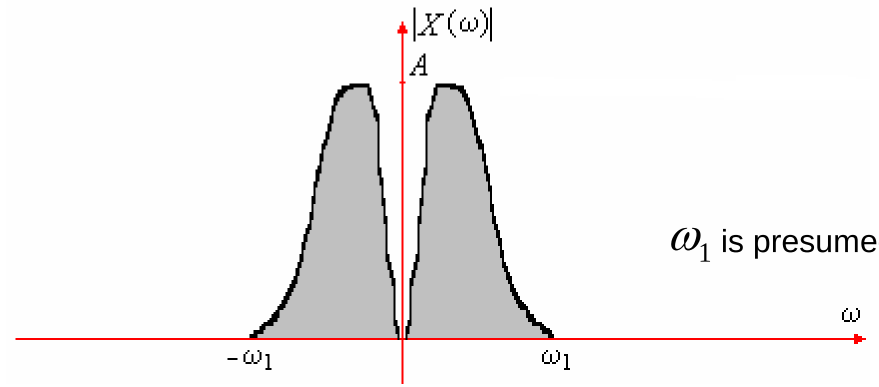
RF signal

Low bandwidth signal

Radio Frequency Radiating Antenna

Modulation property of Fourier Transform

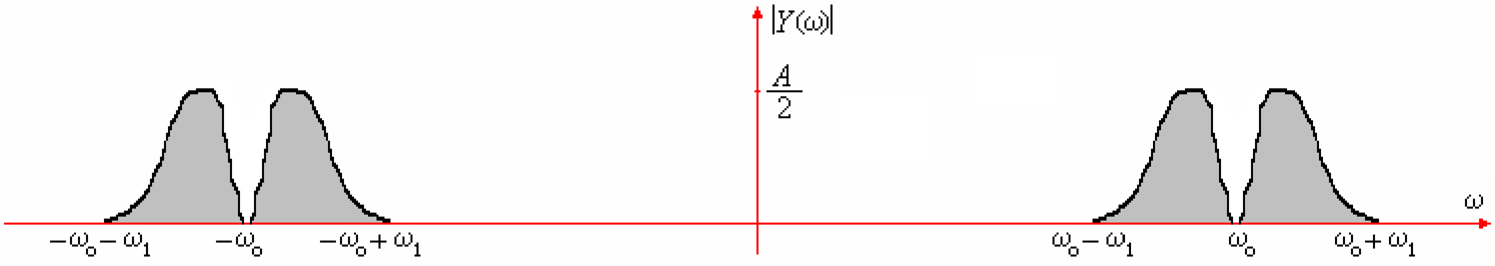
$$x(t) \Leftrightarrow X(\omega)$$



ω_1 is presumed to be cutoff freq. of $x(t)$

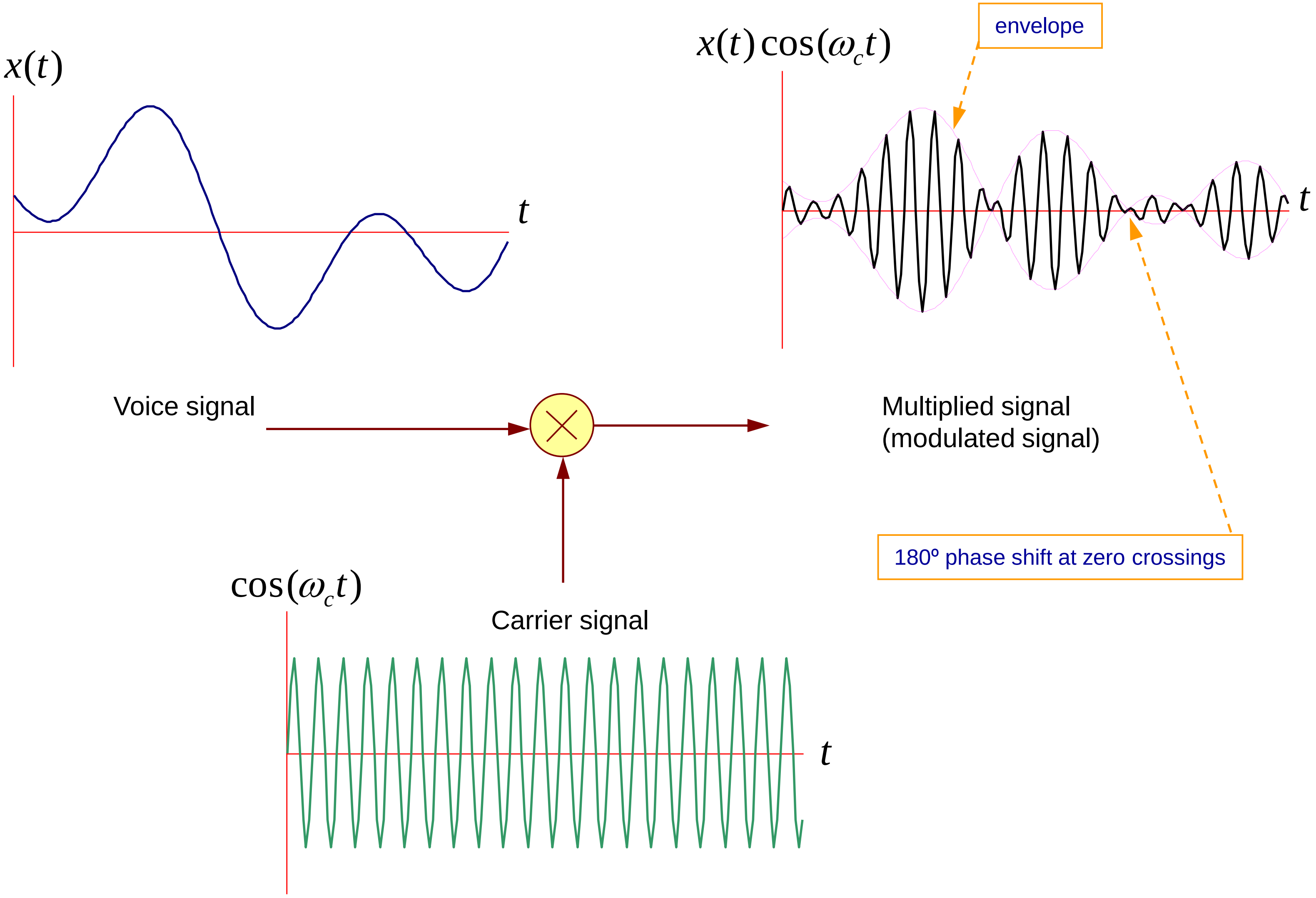
$$x(t)\cos(\omega_o t) \Leftrightarrow \tfrac{1}{2}X(\omega-\omega_o)+\tfrac{1}{2}X(\omega+\omega_o)$$

$\omega_o > \omega_1$



$\cos(\omega_o t)$ is called the carrier signal or carrier. ω_o is called carrier frequency

In time domain



$x(t)$

$x(t)\cos(\omega_c t)$

envelope

Voice signal

Multiplied signal
(modulated signal)

180º phase shift at zero crossings

$\cos(\omega_c t)$

Carrier signal

Example : Find/Draw $\mathcal{F}\{x(t)\cos(\omega_c t)\}$ for $x(t)=\sin(\omega_m t)$ where $\omega_c >> \omega_m$

Solution $$X(\omega)=\mathcal{F}\{\sin(\omega_m t)\}= j\pi(\delta(\omega+\omega_m)-\delta(\omega-\omega_m))$$

$$Y(\omega)=\mathcal{F}\{x(t)\cos(\omega_c t)\}=\frac{1}{2}X(\omega-\omega_c)+\frac{1}{2}X(\omega+\omega_c)$$

$$=\underbrace{\frac{j\pi}{2}\delta(\omega-\omega_c+\omega_m)}_{\mathrm{I}}-\underbrace{\frac{j\pi}{2}\delta(\omega-\omega_c-\omega_m)}_{\mathrm{II}}+\underbrace{\frac{j\pi}{2}\delta(\omega+\omega_c+\omega_m)}_{\mathrm{III}}-\underbrace{\frac{j\pi}{2}\delta(\omega+\omega_c-\omega_m)}_{\mathrm{IV}}$$

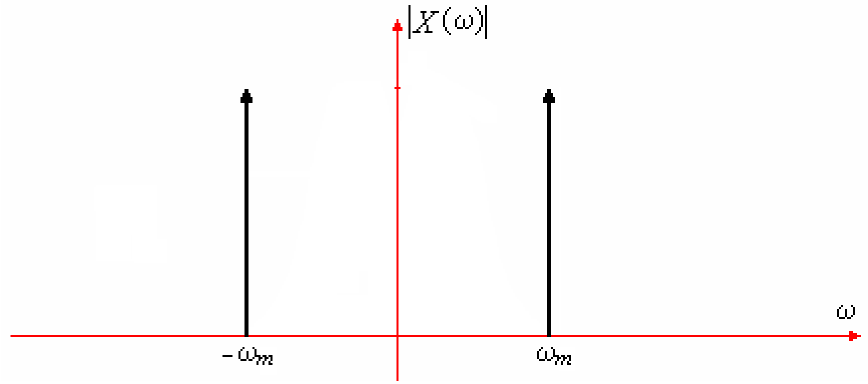
$$\mathrm{I}+\mathrm{IV}=\frac{j\pi}{2}\big(\delta(\omega-(\omega_c-\omega_m))-\delta(\omega+(\omega_c-\omega_m))\big)\Leftrightarrow -\frac{1}{2}\sin((\omega_c-\omega_m)t)$$

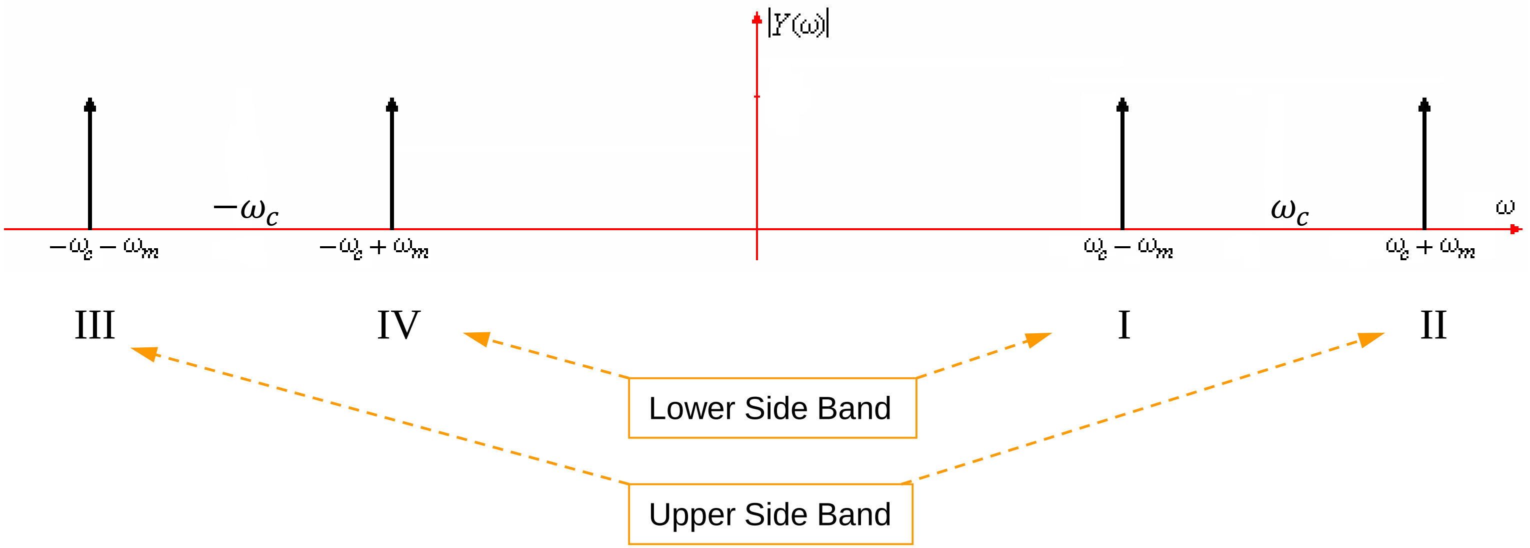
lower frequency components

$$\mathrm{II}+\mathrm{III}=\frac{j\pi}{2}\big(\delta(\omega+(\omega_c+\omega_m))-\delta(\omega-(\omega_c+\omega_m))\big)\Leftrightarrow \frac{1}{2}\sin((\omega_c+\omega_m)t)$$

upper frequency components

Entire signal $y(t) = \frac{1}{2}\sin((\omega_c + \omega_m)t) - \frac{1}{2}\sin((\omega_c - \omega_m)t)$



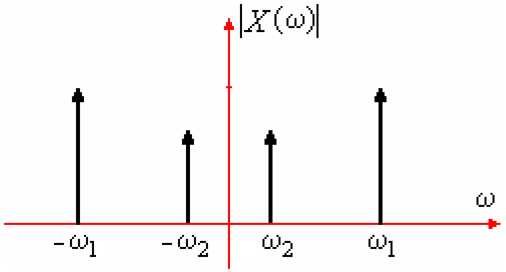


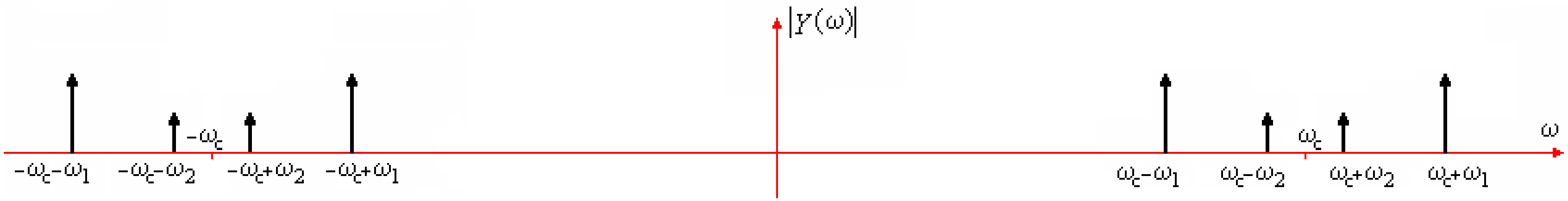
Lower Side Band

Upper Side Band

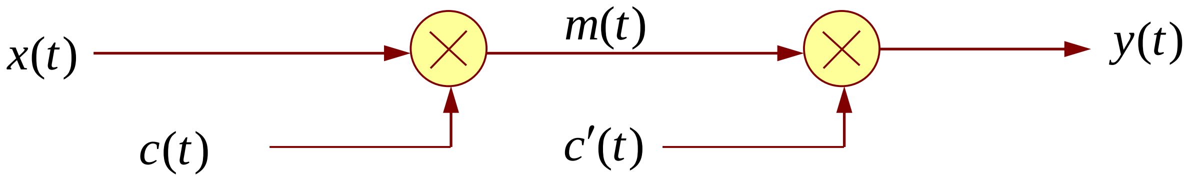
Example : Find the modulated signal $m(t)$ and its Fourier spectrum for $x(t) = A_1\cos(\omega_1 t) + A_2\cos(\omega_2 t + \phi)$

and $c(t) = A_c\cos(\omega_c t)$





Let us apply the same multiplication operation on the modulated signal $m(t)$



$$m(t) = x(t)c(t)$$

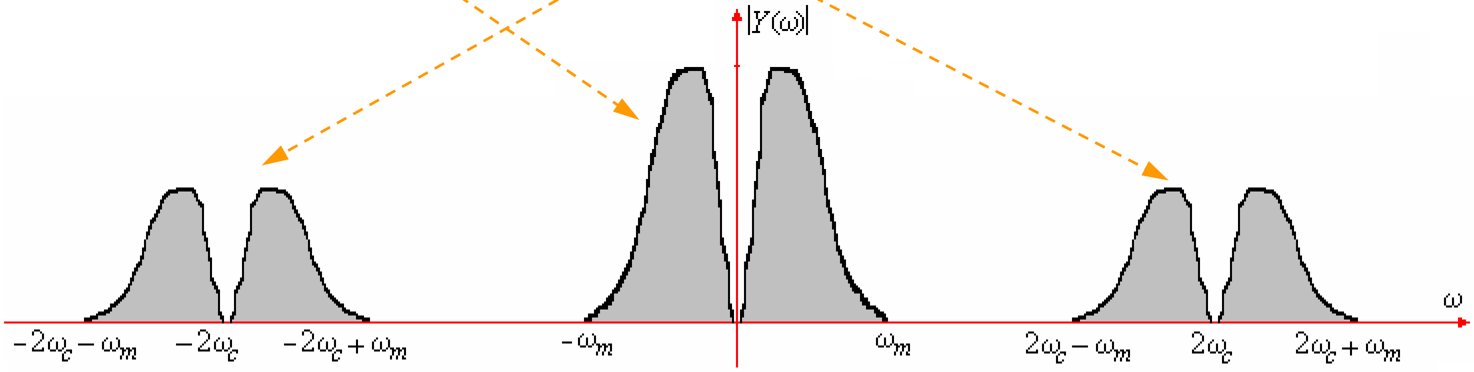
$$y(t) = m(t)c'(t)$$

if $c(t) = A_c \cos(\omega_c t)$ and $c'(t) = A_c' \cos(\omega_c t)$

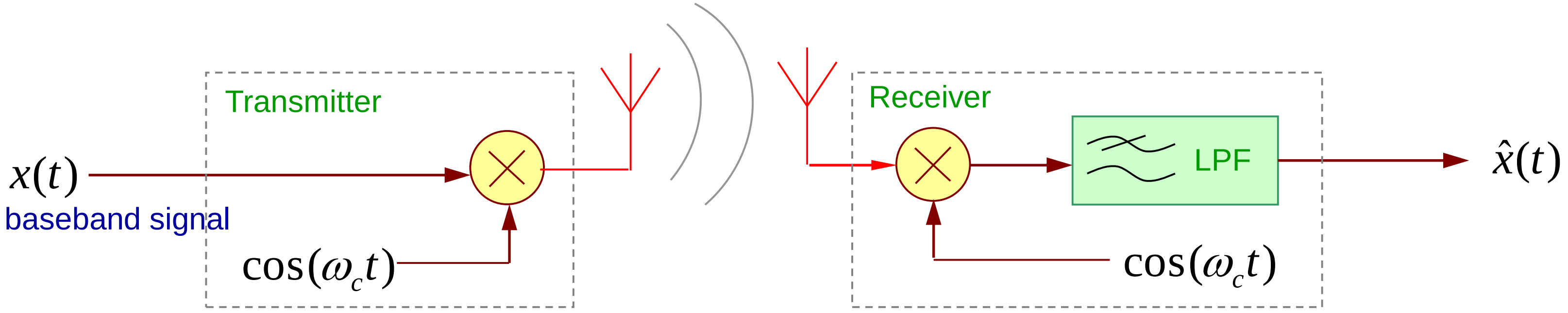
then $y(t) = x(t)A_c A_c' \cos^2(\omega_c t)$

$$= \frac{1}{2} A_c A_c' \left(x(t) + \underline{x(t)\cos(2\omega_c t)} \right)$$

(use $\cos^2(x) = (1 + \cos(2x))/2$)

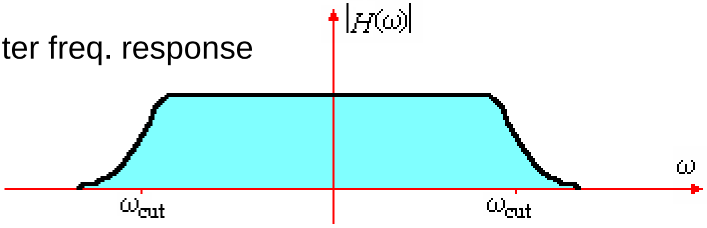


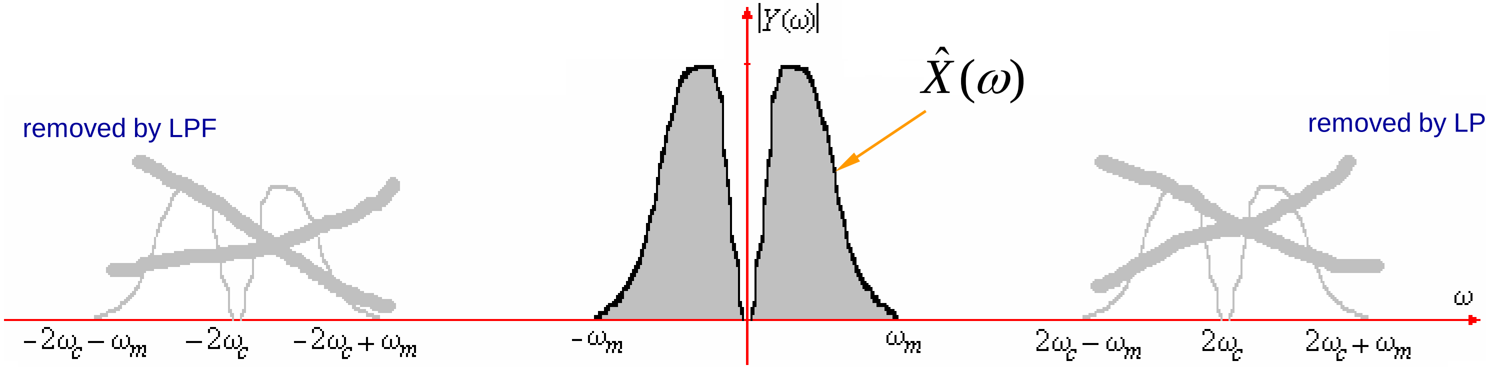
Basic AM Modulator, Transmitter and Synchronous Receiver



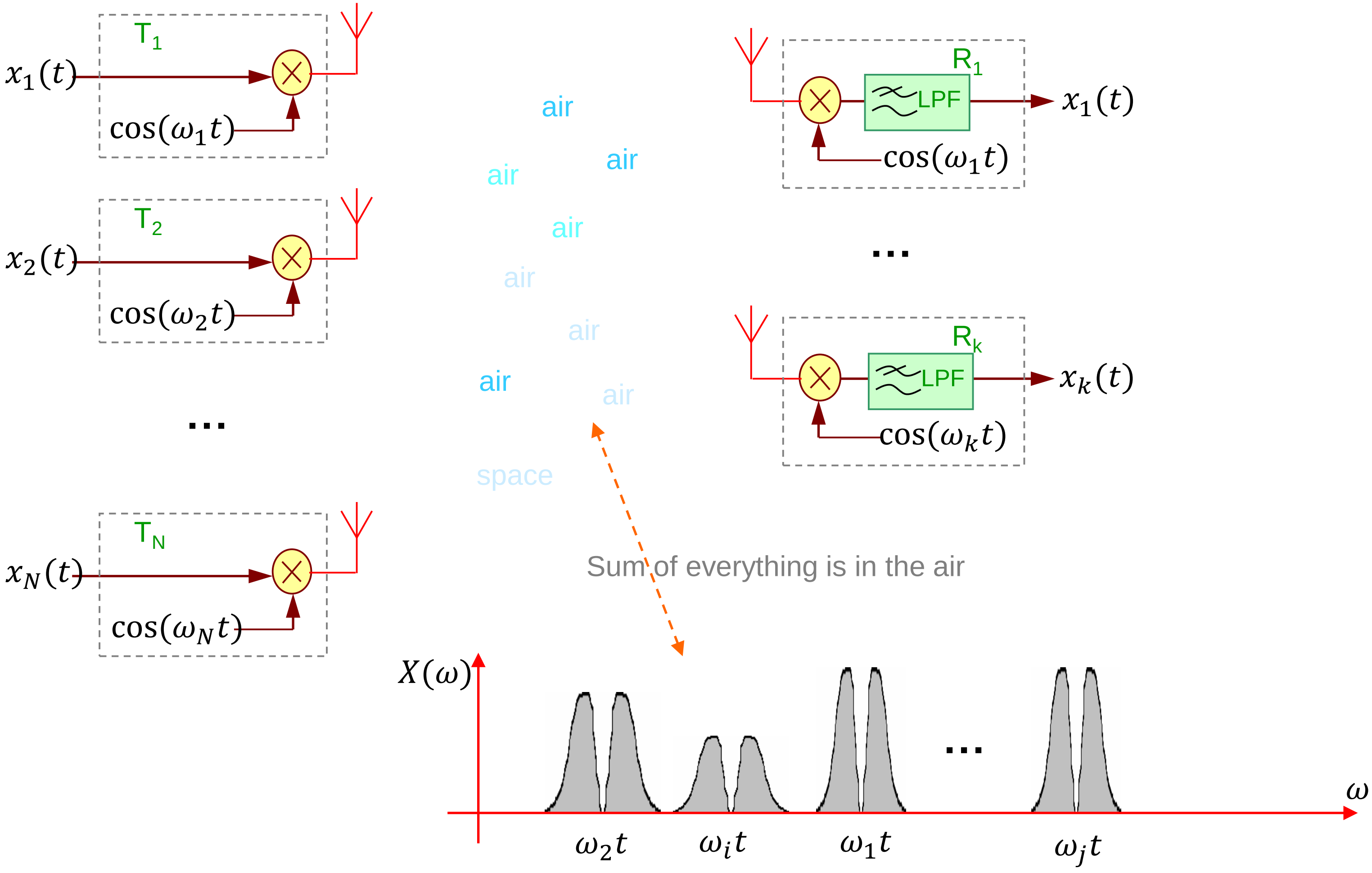
This is called synchronous demodulation

typical Low Pass Filter freq. response





Multiple Transmitters and Receivers

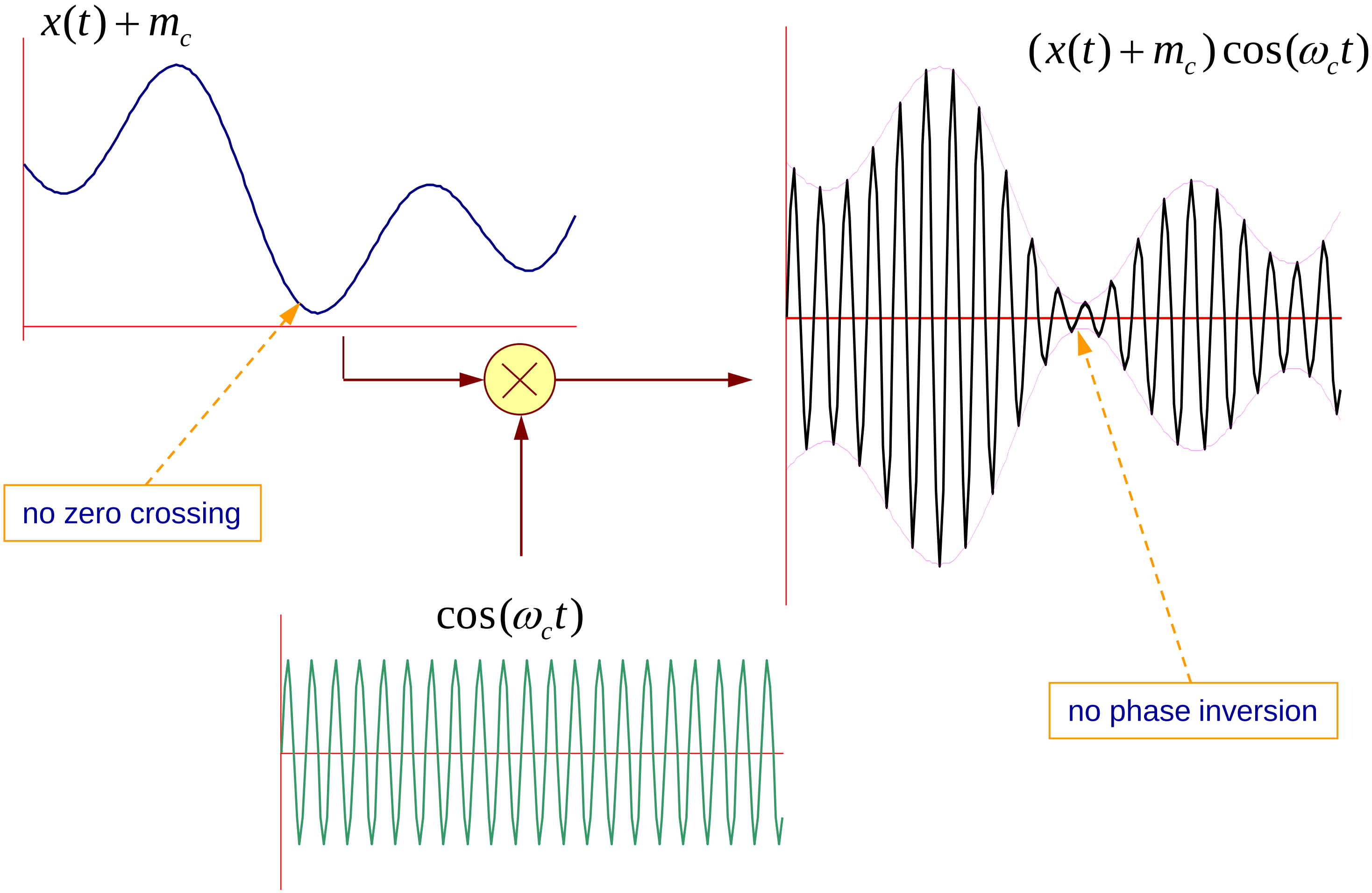


Sum of everything is in the air

Each receiver can adjust its own center frequency to pick up anyone of the signals

Problem is : how to create $\cos(\omega_c t)$ at the receiver in phase with the transmitter oscillator

Let us use $(x(t)+m_c)\cos(\omega_c t)$ instead of $x(t)\cos(\omega_c t)$ at the transmitter where $m_c > \left|\min\{x(t)\}\right|$



no zero crossing

no phase inversion

Synchronous demodulation is easier now since we have a carrier signal to extract from input and use

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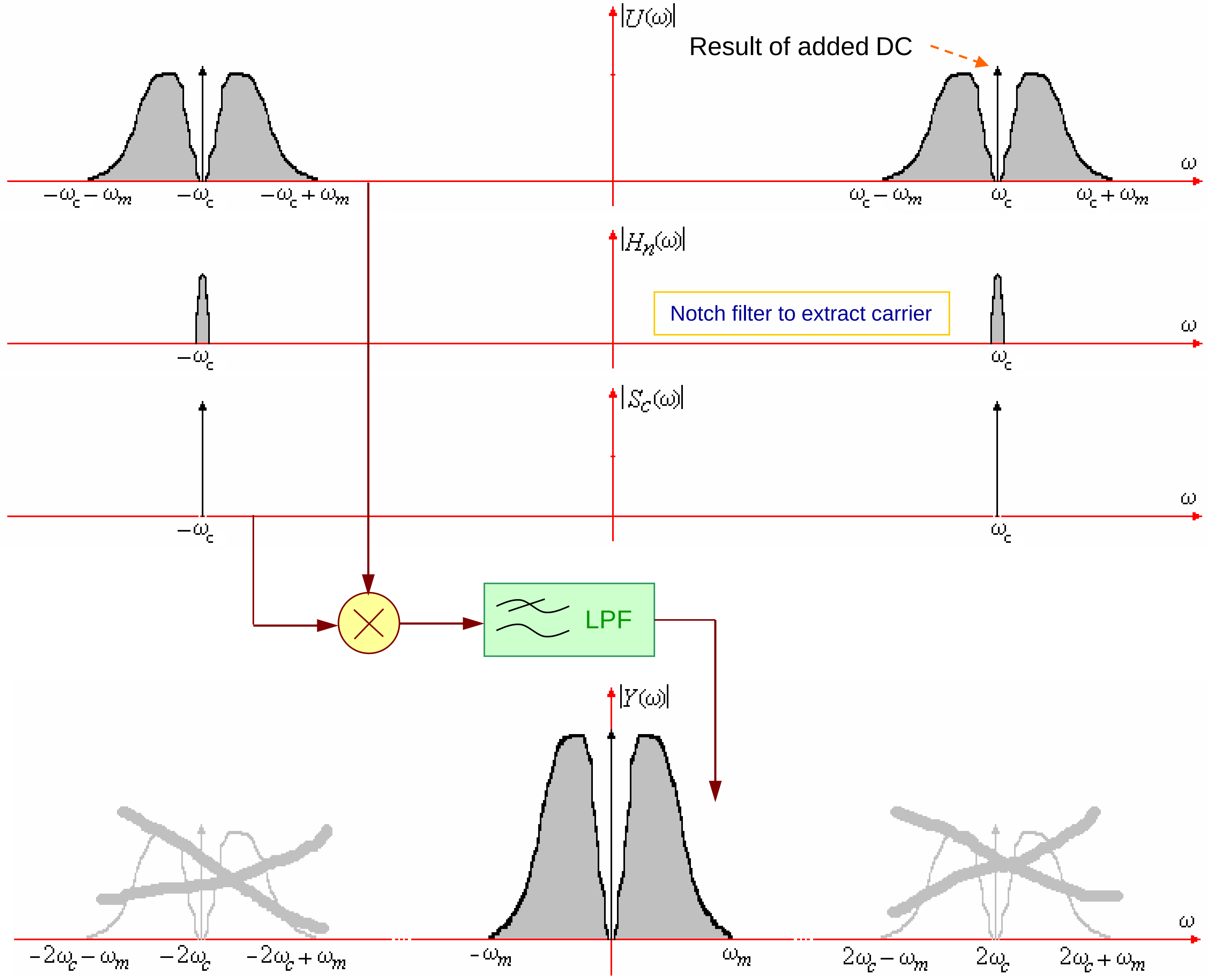
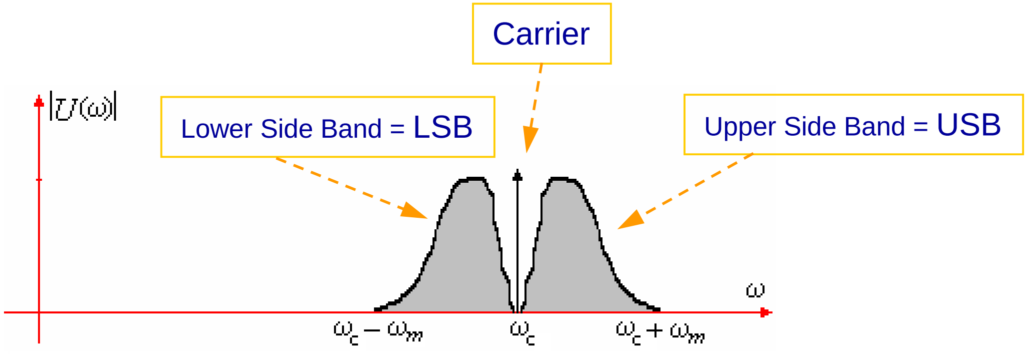


Result of added DC

Notch filter to extract carrier

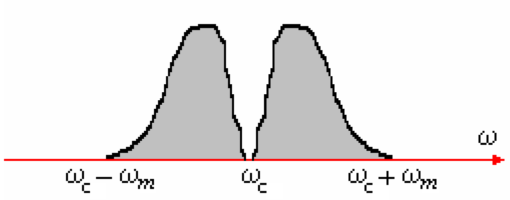
LPF



Carrier

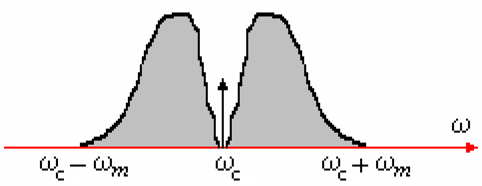
Lower Side Band = LSB

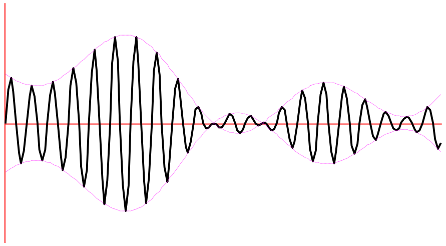
Upper Side Band = USB



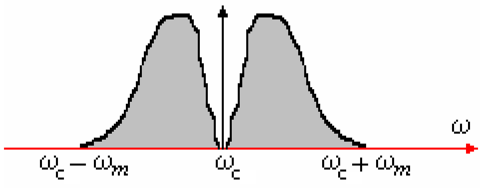
Double Side Band, Suppressed Carrier = DSB-SC AM

Can we have single?



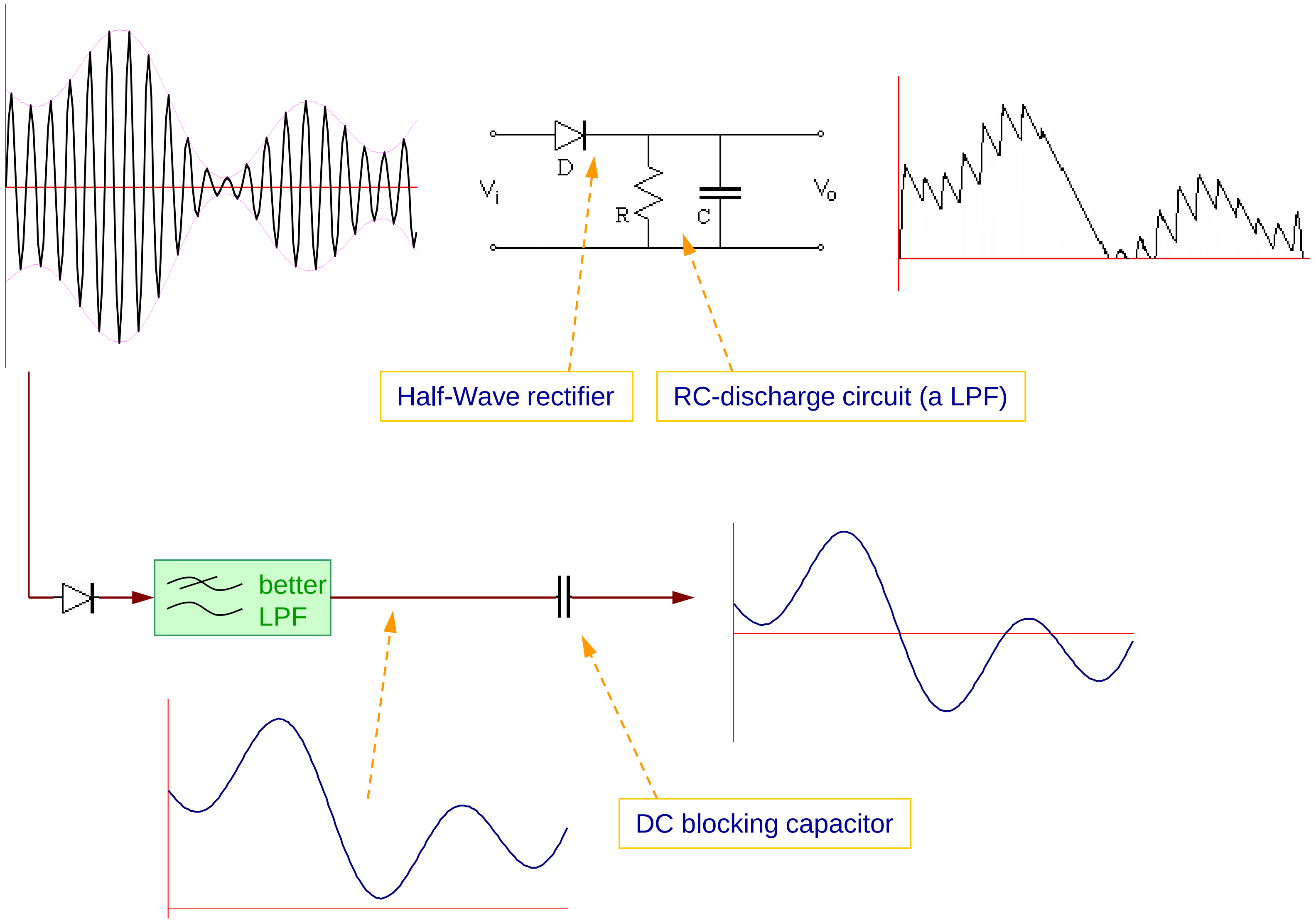


$$m_c < \left|\min\{x(t)\}\right|$$



Conventional Amplitude Modulation

Another Way to Demodulate Conventional AM Signal



Half-Wave rectifier

RC-discharge circuit (a LPF)

DC blocking capacitor

Note : Better LPF may not be enough. Much higher carrier frequency than illustrated would clearly improve the performance

In general $y(t) = A(1 + a_m x_n(t))\cos(\omega_c t + \phi)$

modulation index

normalized signal

$$x_n(t) = \frac{x(t)}{\max|x(t)|}$$ so that $-1 < x_n(t) < 1$

or

$$y(t) = K(m_c + x(t))\cos(\omega_c t + \phi)$$

in order for $x(t) + m_c > 0$ $m_c > |\min\{x(t)\}|$

$$a_m = \frac{|\min\{x(t)\}|}{m_c}$$

larger a_m smaller carrier power per signal power

Carrier Power $P_c =$ mean square of $m_c \cos(\omega_c t) = \dfrac{m_c^2}{2}$

Sideband Power $P_s =$ mean square of $x(t)\cos(\omega_c t) =$ ½ mean square of $x(t) = \dfrac{1}{2}\overline{x^2(t)}$

Power of a single sideband $P_u = P_L = \dfrac{1}{4}\overline{x^2(t)}$

Total Power $P_T = P_s + P_c = \dfrac{1}{2}\left(m_c^2 + \overline{x^2(t)}\right)$

define $\eta = \dfrac{P_s}{P_s + P_c}(\times 100) = \dfrac{\overline{x^2(t)}}{m_c^2 + \overline{x^2(t)}}(\times 100)$ as efficiency

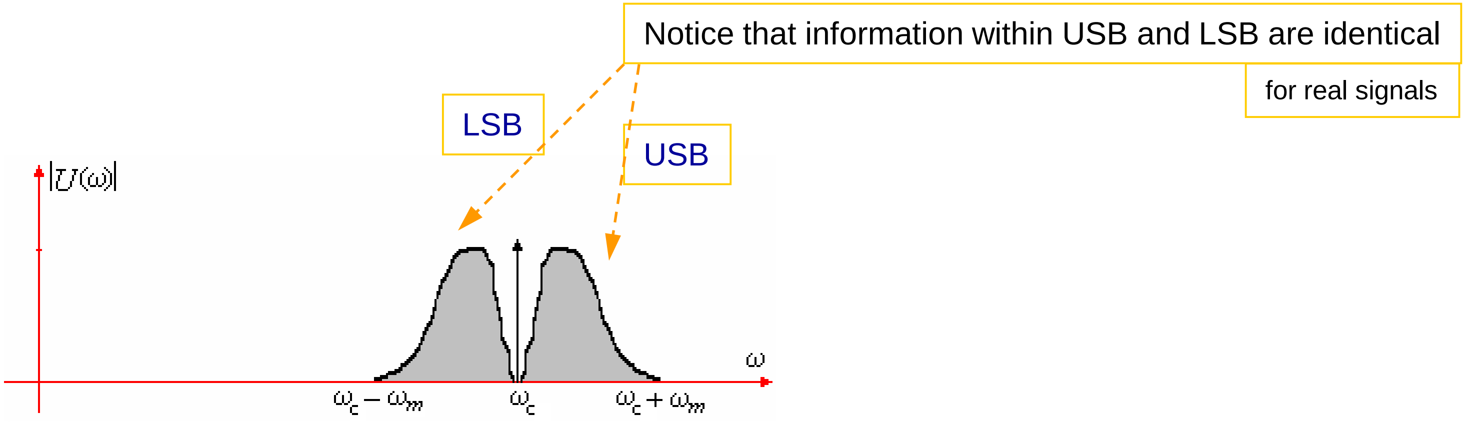
for pure sinusoidal message signal $x(t) = a_m m_c \cos(\omega_m t)$ $\overline{x^2(t)} = \dfrac{(a_m m_c)^2}{2}$

and $\eta = \dfrac{a_m^2}{2 + a_m^2}(\times 100), \quad a_m \le 1$

at $a_m = 1$ (best case) $\eta = \eta_{\max} = 33\%$

For conventional AM two thirds of power is wasted at best. That is, if we want to send 1 then we have to spend additional 2. So, why do we use conventional AM instead of other versions of AM (DSB-SC for example)?

Question : Why do we use conventional AM instead of other versions of AM (DSB-SC for example) even though we know the power disadvantage ?
Simple Answer : Receiver is simpler and cheaper (explain)



Notice that information within USB and LSB are identical

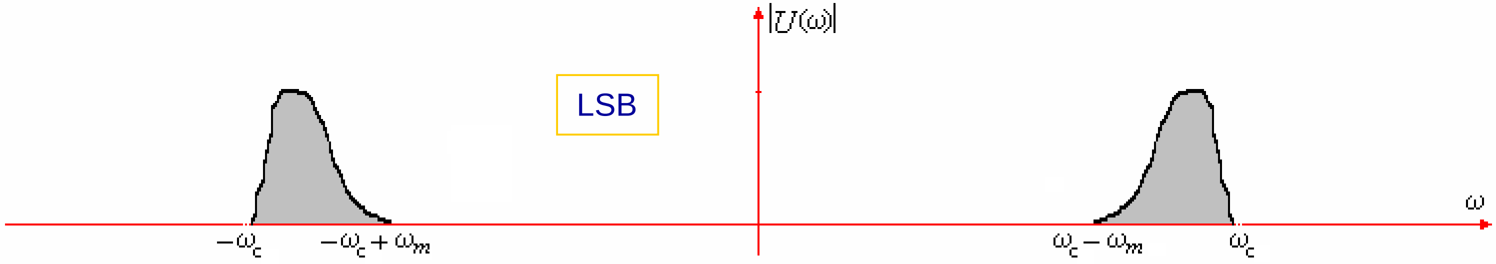
for real signals

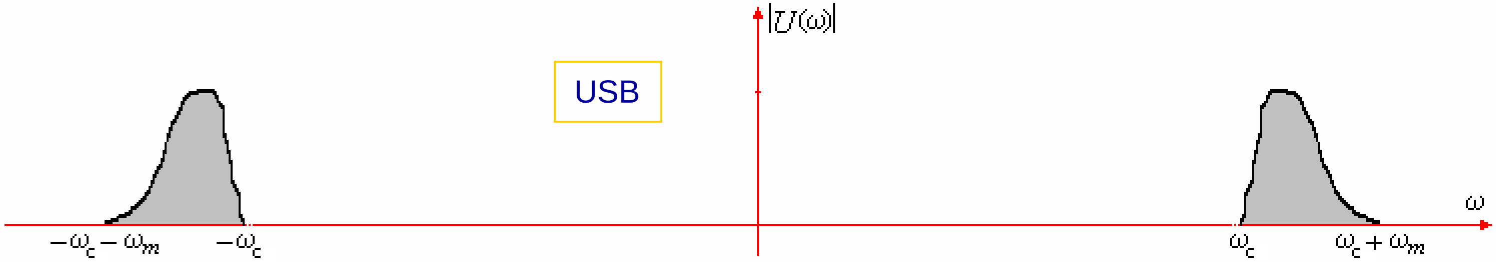
LSB

USB

Then, is it enough to transmit only one side to save power (and have the info transmitted of course)?

Single Side Band Suppressed Carrier AM

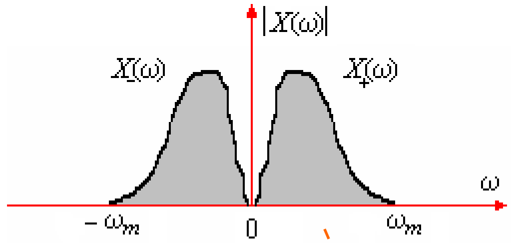




The power advantages are obvious. The question is; **how** do we generate these **SSB-SC AM** signals?

Let us assume that $x(t) = x_-(t) + x_+(t)$

so that $x_-(t) = x_+^*(t)$



It can be written that $x_-(t) = \frac{1}{2}\left[x(t) - jx_h(t)\right]$

and $x_+(t) = \frac{1}{2}\left[x(t) + jx_h(t)\right]$

$$X_+(\omega) = X(\omega)U(\omega)$$

$$X_+(\omega) = \tfrac{1}{2}X(\omega) + \tfrac{1}{2}X(\omega)\operatorname{sgn}(\omega)$$

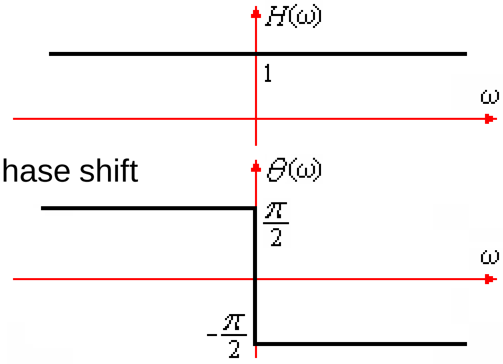
$jx_h(t) \Leftrightarrow X(\omega)\operatorname{sgn}(\omega)$ or $X_h(\omega) = -jX(\omega)\operatorname{sgn}(\omega)$

We know that (from tables) $\frac{j}{\pi t} \Leftrightarrow \operatorname{sgn}(\omega)$ and $\mathcal{F}\{x(t) * y(t)\} = X(\omega)Y(\omega)$ (convolution)

Hilbert Transform

$$x_h(t) = \mathcal{F}^{-1}\{-jX(\omega)\operatorname{sgn}(\omega)\} = \frac{1}{\pi}\int_{-\infty}^{\infty}\frac{x(\alpha)}{t-\alpha}\,d\alpha$$

ideal $\frac{\pi}{2}$ phase shift



$$y_{\mathrm{USB}}(t) = A_c x(t)\cos(\omega_c t) - A_c\hat{x}(t)\sin(\omega_c t)$$

$$y_{\mathrm{LSB}}(t) = A_c x(t)\cos(\omega_c t) + A_c\hat{x}(t)\sin(\omega_c t)$$

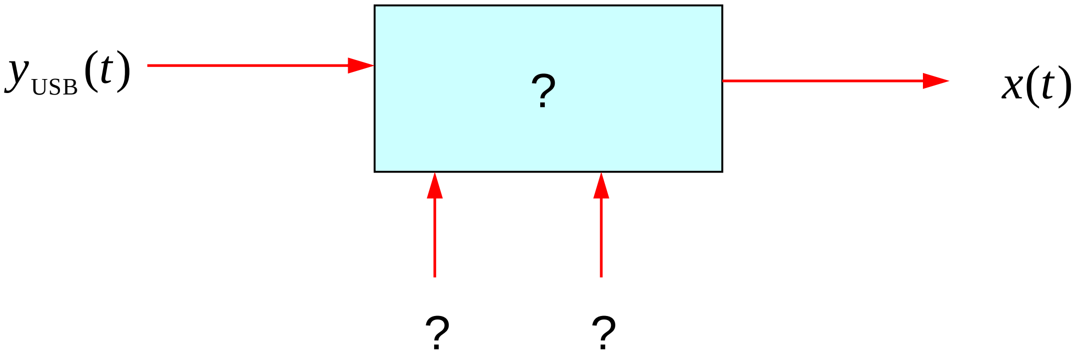
Example : Find $y_{USB}(t)$ for $x(t) = \cos(\omega_x t)$ where $\omega_c >> \omega_x$

Solution $\hat{x}(t)$ is 90 degrees phase shifted version of $x(t)$ So $\hat{x}(t) = \sin(\omega_x t)$

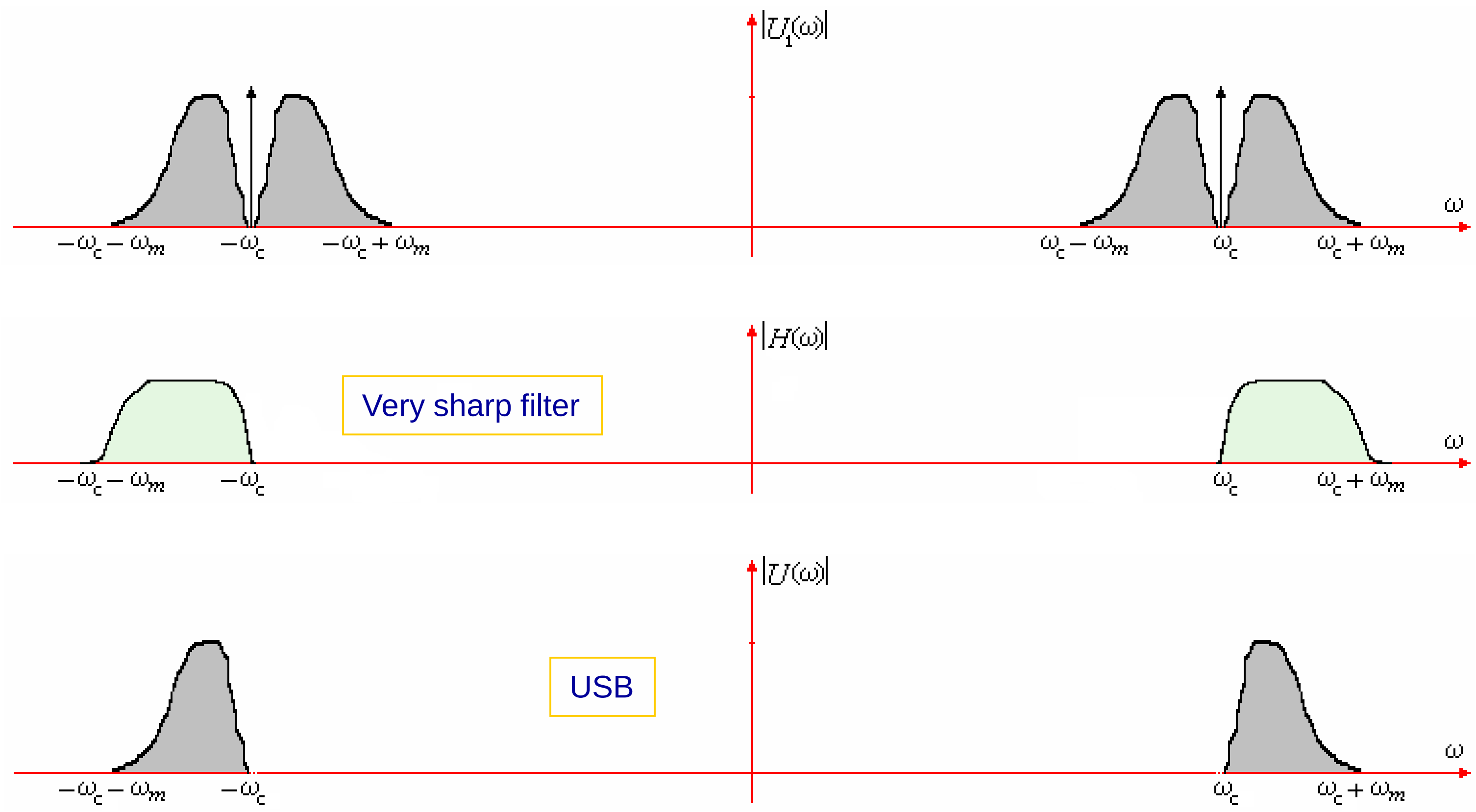
$$y_{\mathrm{USB}}(t) = A_c \cos(\omega_x t)\cos(\omega_c t) - A_c \sin(\omega_x t)\sin(\omega_c t)$$

$y_{\mathrm{USB}}(t) = A_c \cos((\omega_c + \omega_x)t)$ and $y_{\mathrm{LSB}}(t) = A_c \cos((\omega_c - \omega_x)t)$

Homework How can we demodulate SSB-AM signals?



Another way to generate SSB



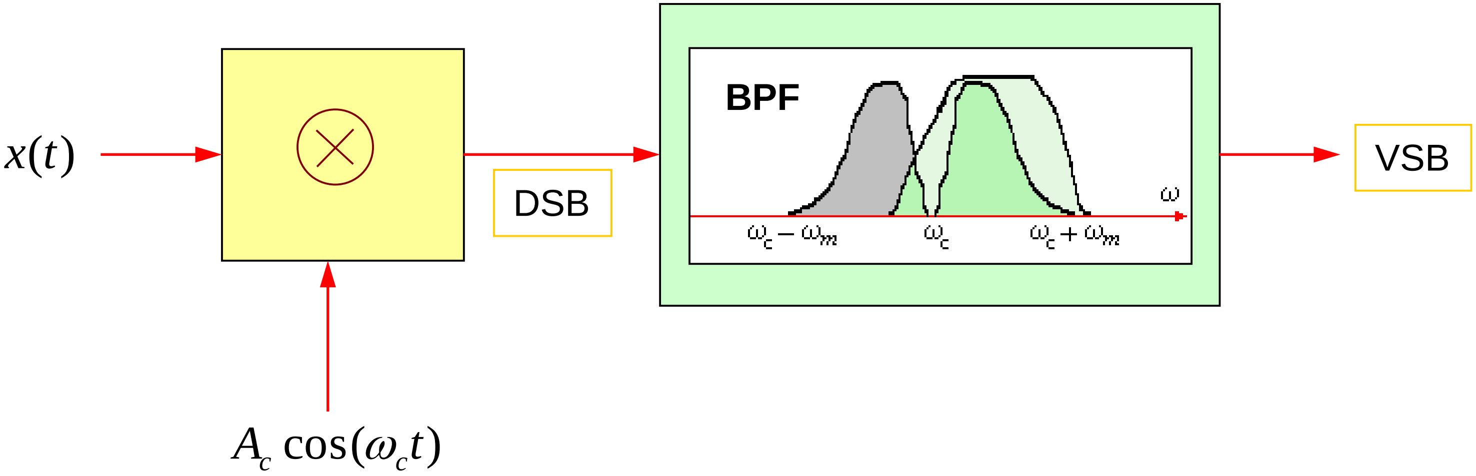
Very sharp filter

USB

We need to have very sharp filters to achieve this.!

VSB : Vestigial Side Band

Instead we can allow a little bit of other sideband to pass; which means a relaxed version of the filter (cheaper)

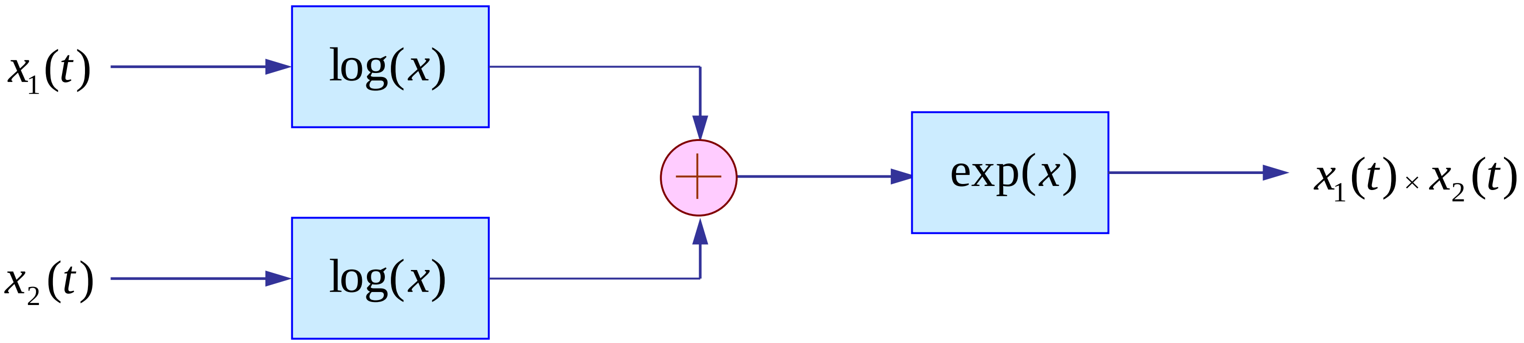


VSB – AM is used in modulation of analog monochrome television picture signals

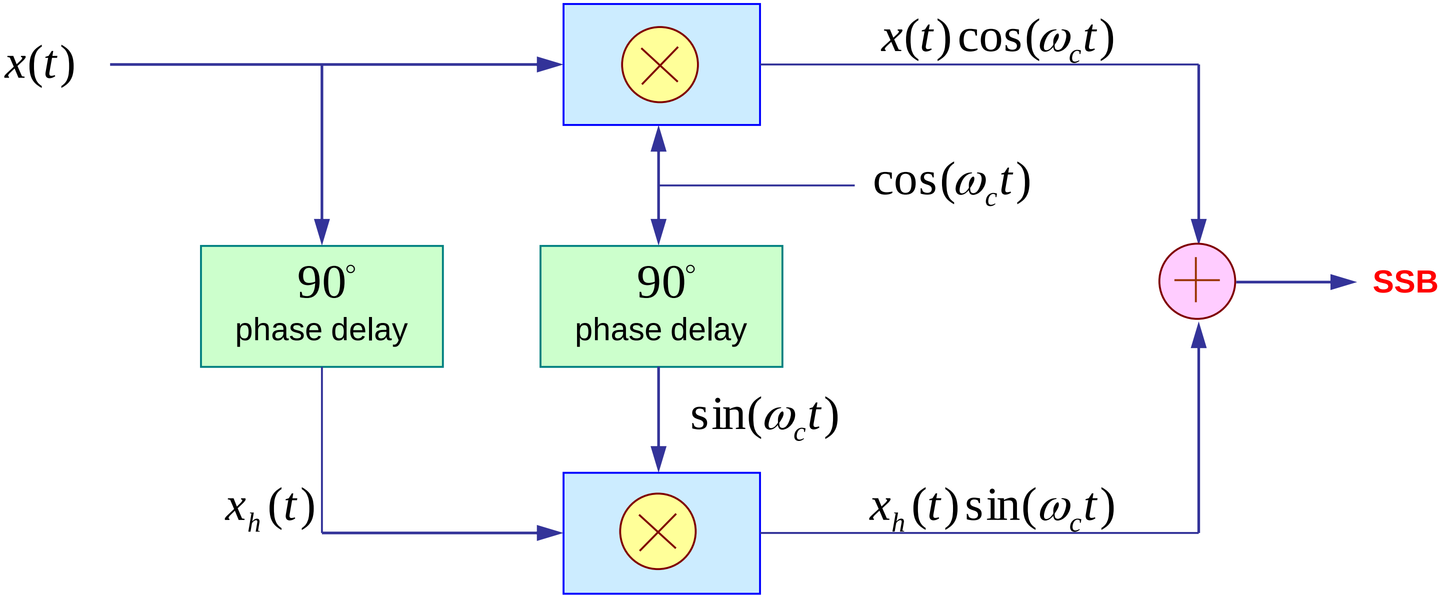
modulation of analog color television picture/color signals is another story

Use of Nonlinear Circuits to Realize Multiplication

(Since Multiplication cannot be realized by Linear Circuits)



Generation of SSB

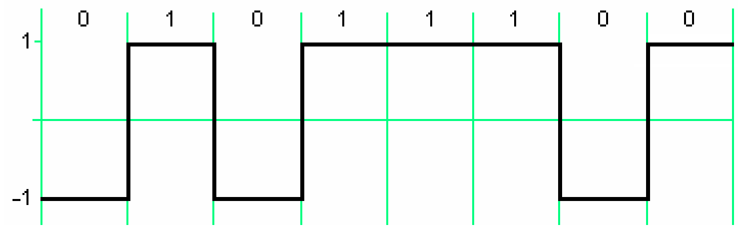


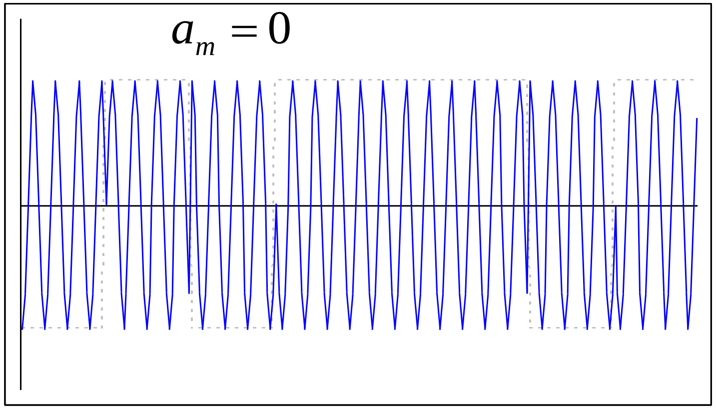
Example : Draw $y(t)=(x(t)+a_m)\cos(\omega_c t)$

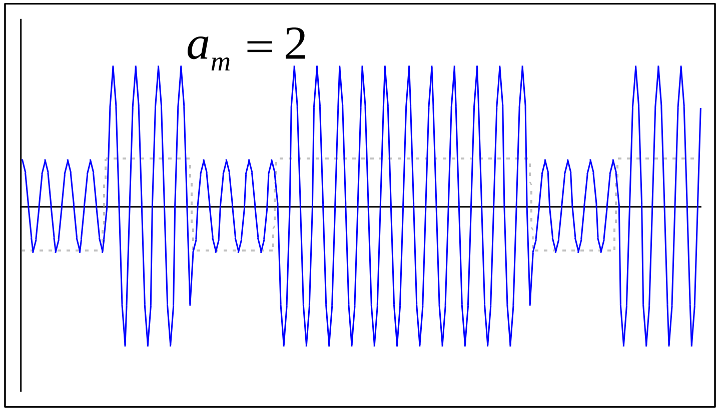
for $a_m=0$, $a_m=2$ and $a_m=1$

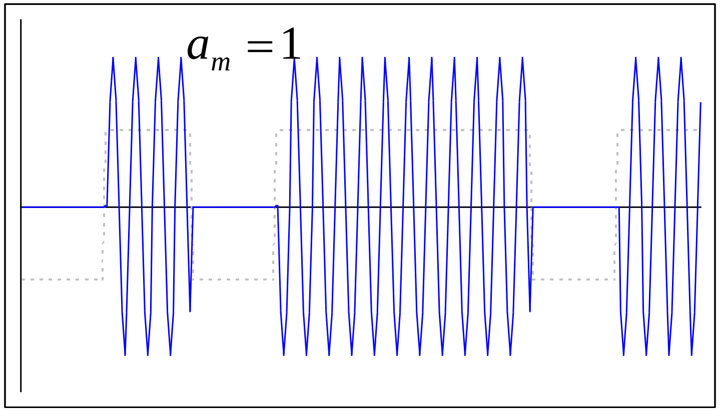
if the binary message signal is given as shown below.

Assume that carrier has high enough frequency that at least 3 cycles fit into a binary period.

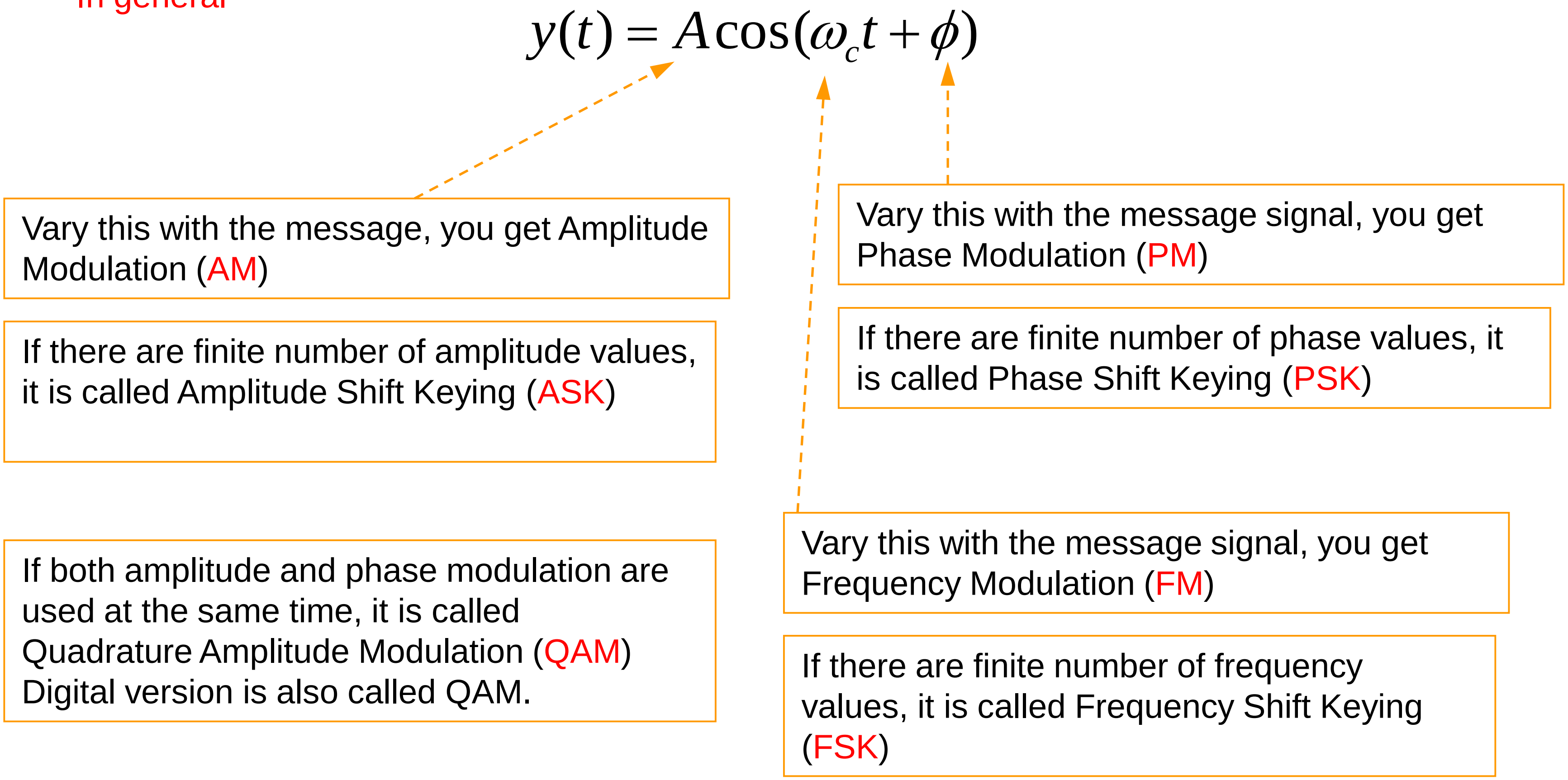








In general



$$y(t) = A\cos(\omega_c t + \phi)$$

Vary this with the message, you get Amplitude Modulation (AM)

If there are finite number of amplitude values, it is called Amplitude Shift Keying (ASK)

If both amplitude and phase modulation are used at the same time, it is called Quadrature Amplitude Modulation (QAM) Digital version is also called QAM.

Vary this with the message signal, you get Phase Modulation (PM)

If there are finite number of phase values, it is called Phase Shift Keying (PSK)

Vary this with the message signal, you get Frequency Modulation (FM)

If there are finite number of frequency values, it is called Frequency Shift Keying (FSK)

In AM, amount of carrier and sidebands in the frequency spectrum determines the modulation type : SSB, SSB-SC, DSB, DSB-SC, Conventional AM, VSB and their sub-types.

END