Angle Modulation (FM/PM)

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For the course "Communications"

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Changing the frequency and phase angle of the carrier signal with the message signal are called “Frequency Modulation” (FM) and “Phase Modulation” (PM) respectively. The class of such techniques is called “Angle Modulation”.

$$u(t) = A_c \cos(\omega(t)t + p(t)) \quad \text{or} \quad u(t) = A_c \cos(\theta(t))$$

Phase is only meaningful with a particular given frequency, cannot be thought of independent entities

$$\omega_i(t) = \frac{d\theta(t)}{dt}$$

instantaneous frequency

$$f_i(t) = \frac{1}{2\pi}\frac{d\theta(t)}{dt}$$

$$u(t) = A_c \cos(\omega_c t + \phi(t))$$

center frequency ($\omega_c t$)

deviation around center ($\phi(t)$)

so

$$\omega_i(t) = \omega_c + \frac{d\phi(t)}{dt} \quad \text{or} \quad f_i(t) = f_c + \frac{1}{2\pi}\frac{d\phi(t)}{dt}$$

Let $m(t)$ be the message signal

$$\phi(t) = k_p m(t)$$

phase deviation constant for PM (k_p)

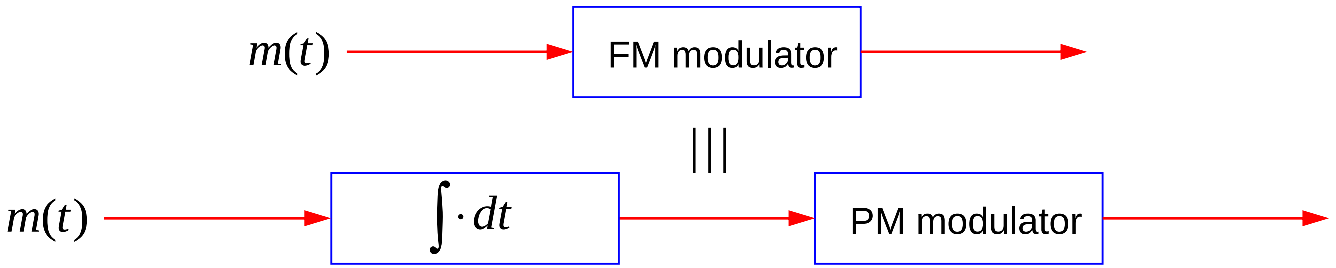
$$\omega_i(t) - \omega_c = k_\omega m(t) = \frac{d\phi(t)}{dt}$$

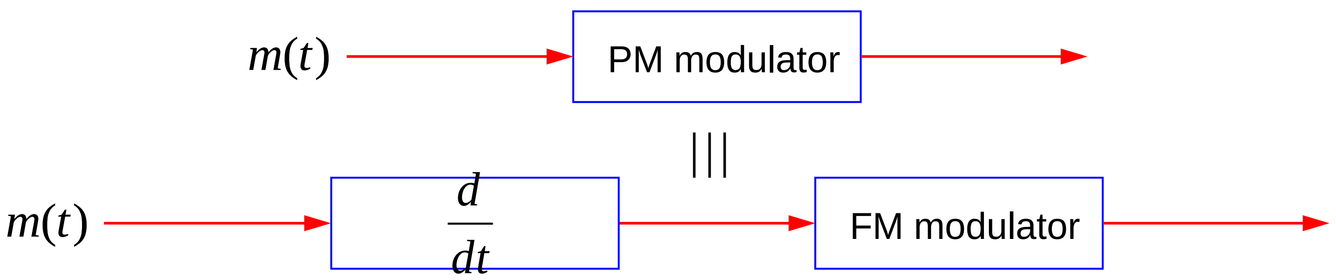
frequency deviation constant for FM (k_ω)

$$\phi(t) = \begin{cases} k_p m(t) & , \quad PM \\ 2\pi k_f \int_{-\infty}^{t} m(\tau) d\tau & , \quad FM \end{cases}$$

or

$$\frac{d}{dt}\phi(t) = \begin{cases} k_p \frac{d}{dt} m(t) & , \quad PM \\ 2\pi k_f m(t) & , \quad FM \end{cases}$$





Example : Tone signal $x(t) = A\cos(\omega_m t)$ is given. Find FM and PM waveforms.

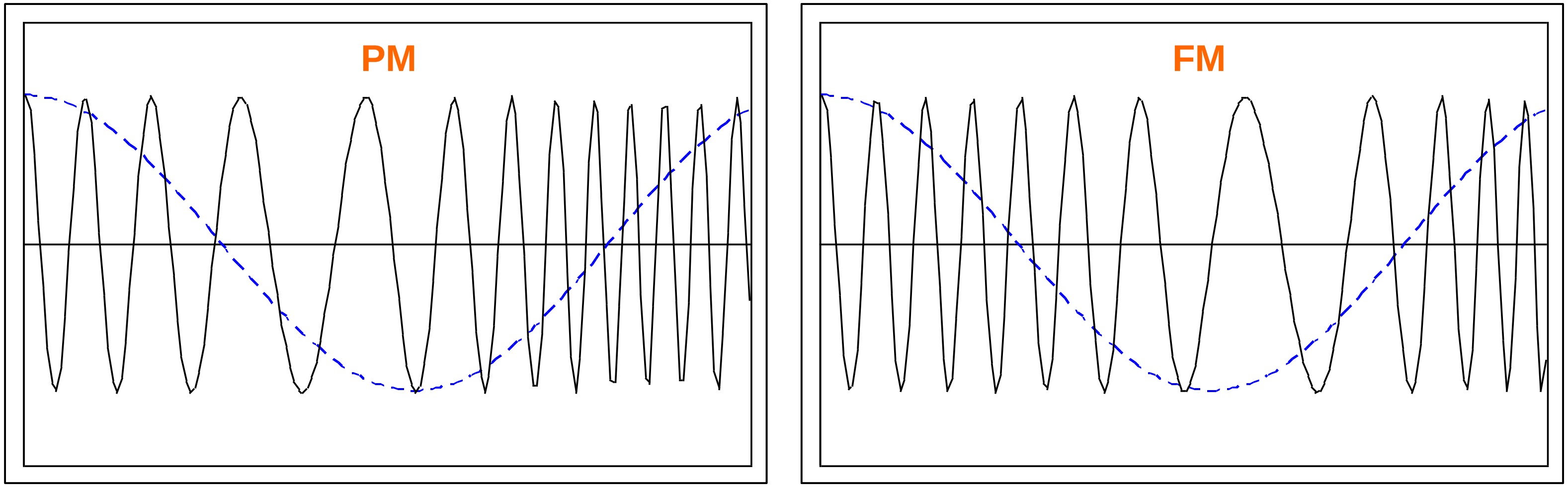
$$\phi_{PM}(t) = k_p x(t) = k_p A\cos(\omega_m t) \qquad \text{(PM)}$$

$$\phi_{FM}(t) = k_\omega \int_{-\infty}^{t} x(\tau)d\tau = \frac{k_\omega A}{\omega_m}\sin(\omega_m t) \qquad \text{(FM)}$$

therefore

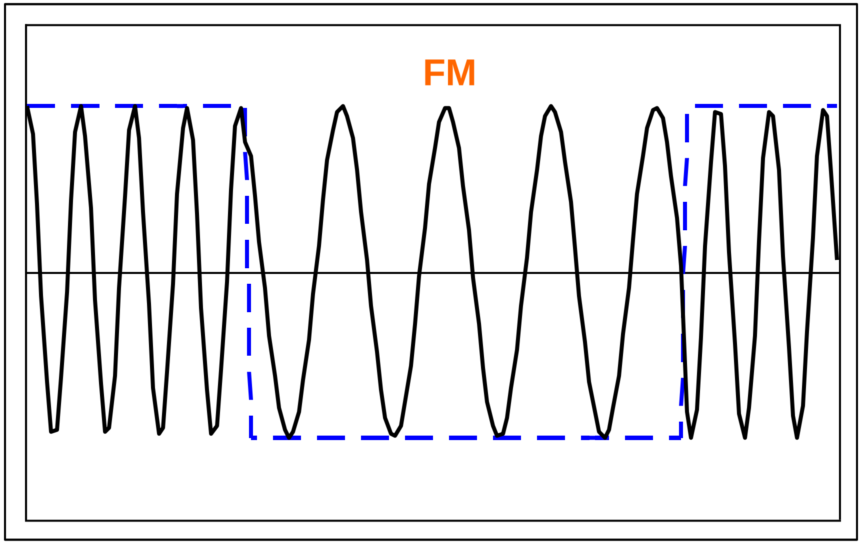
$$u(t) = A_c \cos(\omega_c t + k_p A\cos(\omega_m t)) \qquad \text{(PM)}$$

$$u(t) = A_c \cos\left(\omega_c t + \frac{k_\omega A}{\omega_m}\sin(\omega_m t)\right) \qquad \text{(FM)}$$



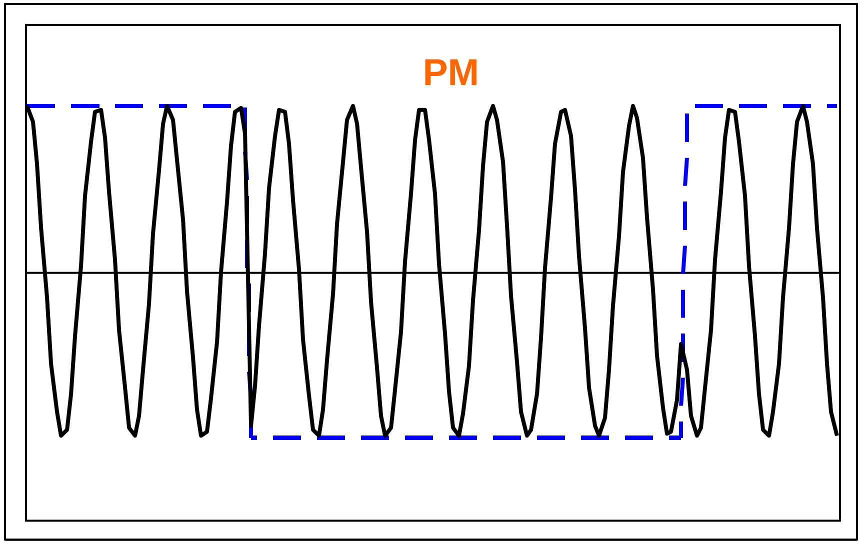
Example :

Rectangular wave as message signal.



Note that neither FM nor PM changes the amplitude of the carrier.

FM changes the frequency
PM changes the phase of the carrier



Remember that when there is only a finite number of frequency values, it is FSK.
When there is only a finite number of phase values, it is called PSK.

Spectral Characteristics of Angle Modulated Signals

Let $u(t) = A_c \cos(2\pi f_c t + \beta \sin(2\pi f_m t))$

Modulation index (for β)

$$\beta = \begin{cases} k_p \max\{|m(t)|\} & PM \\ \dfrac{k_f \max\{|m(t)|\}}{2\pi f_m} & FM \end{cases}$$

$$u(t) = \mathrm{Re}\{A_c e^{j2\pi f_c t} e^{j\beta \sin(2\pi f_m t)}\}$$

Since $u(t)$ is periodic it can be represented by Fourier series

$$C_n = f_m \int_0^{1/f_m} e^{j\beta \sin(2\pi f_m t)} e^{-jn2\pi f_m t} dt$$

Let $u = 2\pi f_m t$

$$C_n = \frac{1}{2\pi} \int_0^{2\pi} e^{j(\beta \sin(u) - u)} du$$

This integral is called the Bessel function of the first kind of order n $J_n(\beta)$

The Fourier series is $$e^{j\beta \sin(2\pi f_m t)} = \sum_{n=-\infty}^{\infty} J_n(\beta) e^{j2\pi n f_m t}$$

Therefore $$u(t) = \mathrm{Re}\left\{ A_c \sum_{n=-\infty}^{\infty} J_n(\beta) e^{j2\pi n f_m t} e^{j2\pi f_c t} \right\}$$

$$u(t) = \sum_{n=-\infty}^{\infty} A_c J_n(\beta) \cos(2\pi(f_c + n f_m)t)$$

Actual bandwidth of FM/PM is infinite. So we usually define a finite "effective bandwidth"

Bessel function can be represented by a series expansion

$$J_n(\beta) = \sum_{k=0}^{\infty} \frac{(-1)^k \left(\frac{\beta}{2}\right)^{n+2k}}{k!(k+n)!}$$

For small β $$J_n(\beta) \cong \frac{\beta^n}{2^n n!}$$ (approximation)

For $n > 2$ $J_n(\beta)$ is negligible.

Therefore only the first sideband corresponding to $n=1$ is of importance.

The effective bandwidth of an angle modulated signal is, in general, given by

$$B_c = 2(\beta + 1) f_m$$

which contains the 98% of the signal power.

Modulation index (→ β)

Frequency of the sinusoidal message signal (or BW of $m(t)$) (→ f_m)

For the message signal $m(t) = a\cos(2\pi f_m t)$

$$B_c = 2(\beta + 1) f_m = \begin{cases} 2(k_p a + 1) f_m & , \quad PM \\ 2(\dfrac{k_f a}{f_m} + 1) f_m & , \quad FM \end{cases}$$

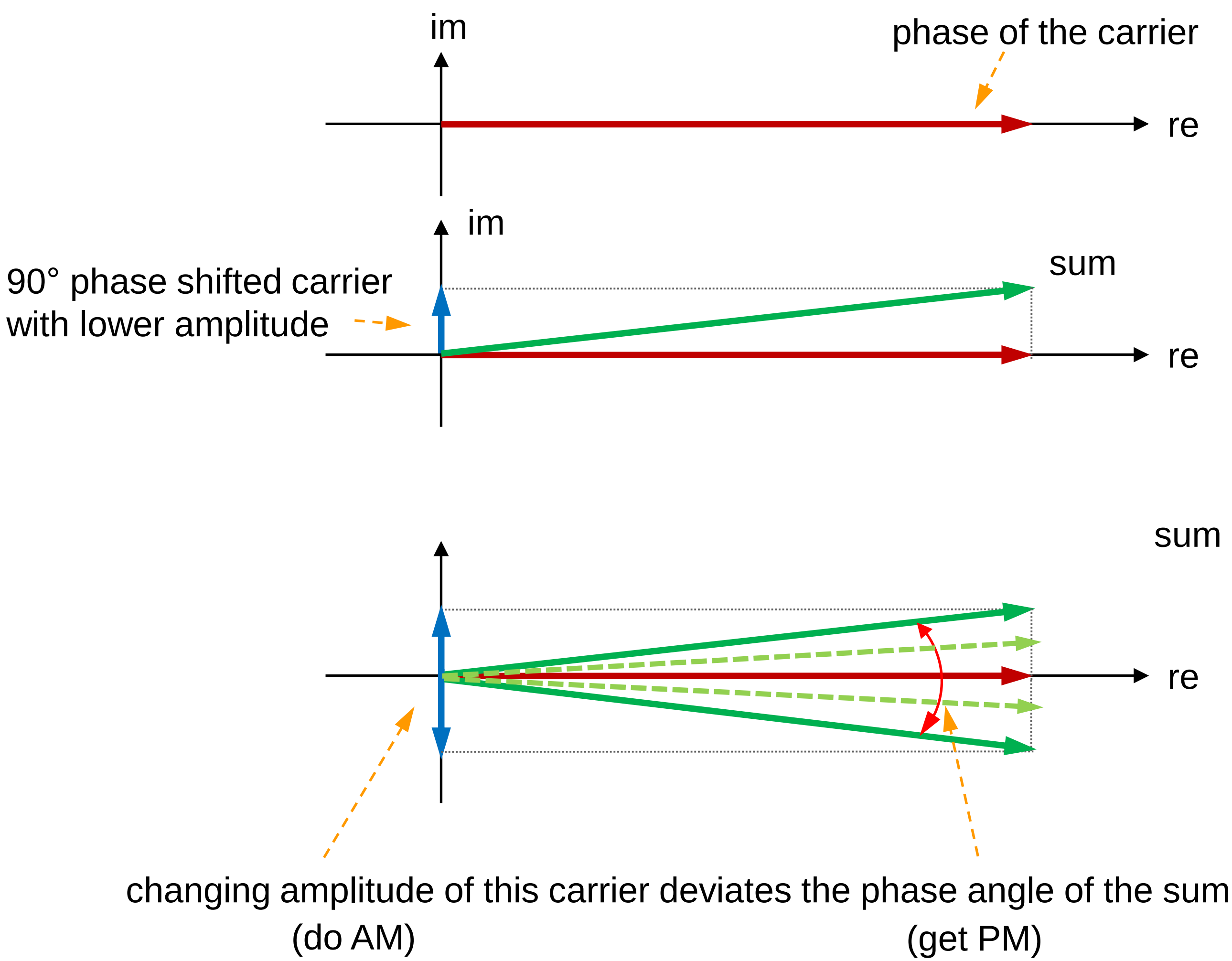
increasing a → increases the effective bandwidth

increasing f_m → increases the bw.

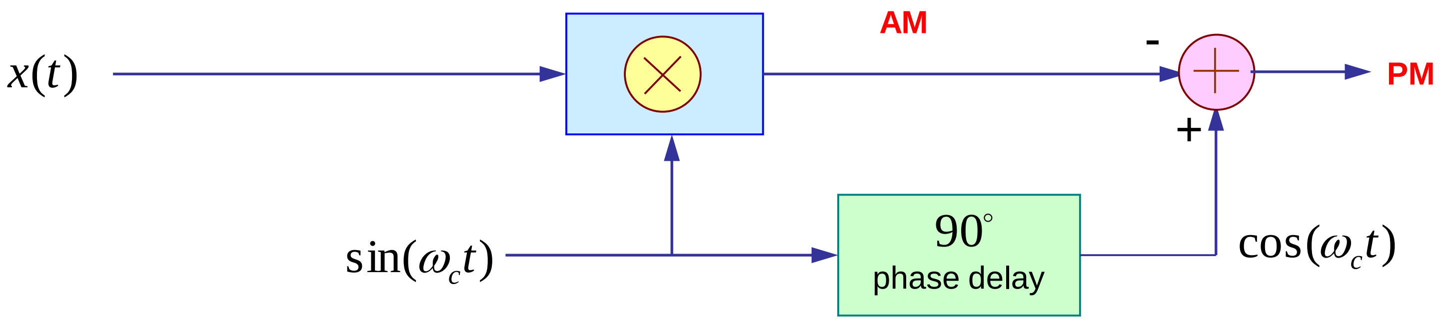
→ proportional increase in PM (large)

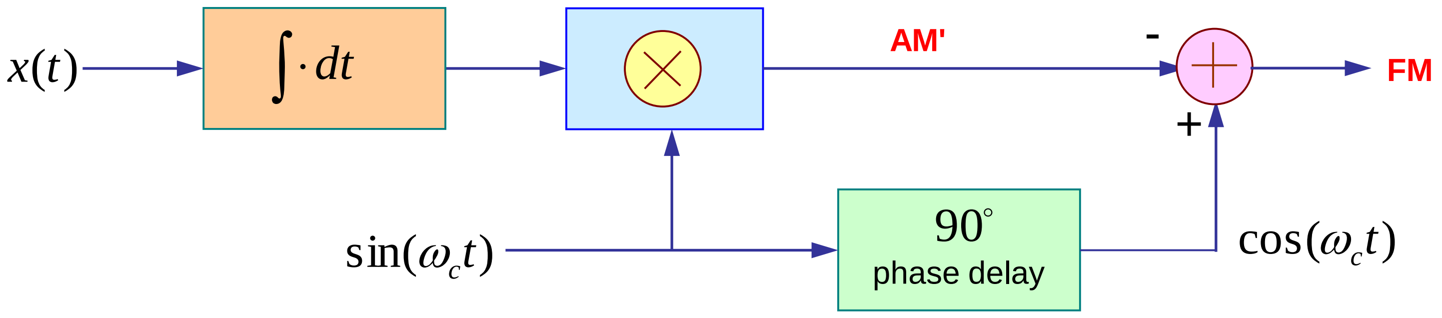
→ additive increase in FM (small)

Generation of Narrowband FM/PM (Phasor View)

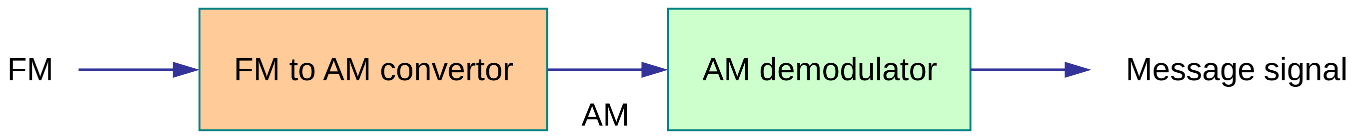


Generation of Narrowband FM/PM



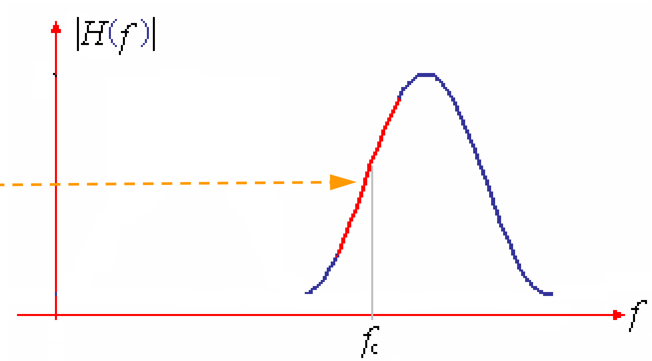


Demodulation

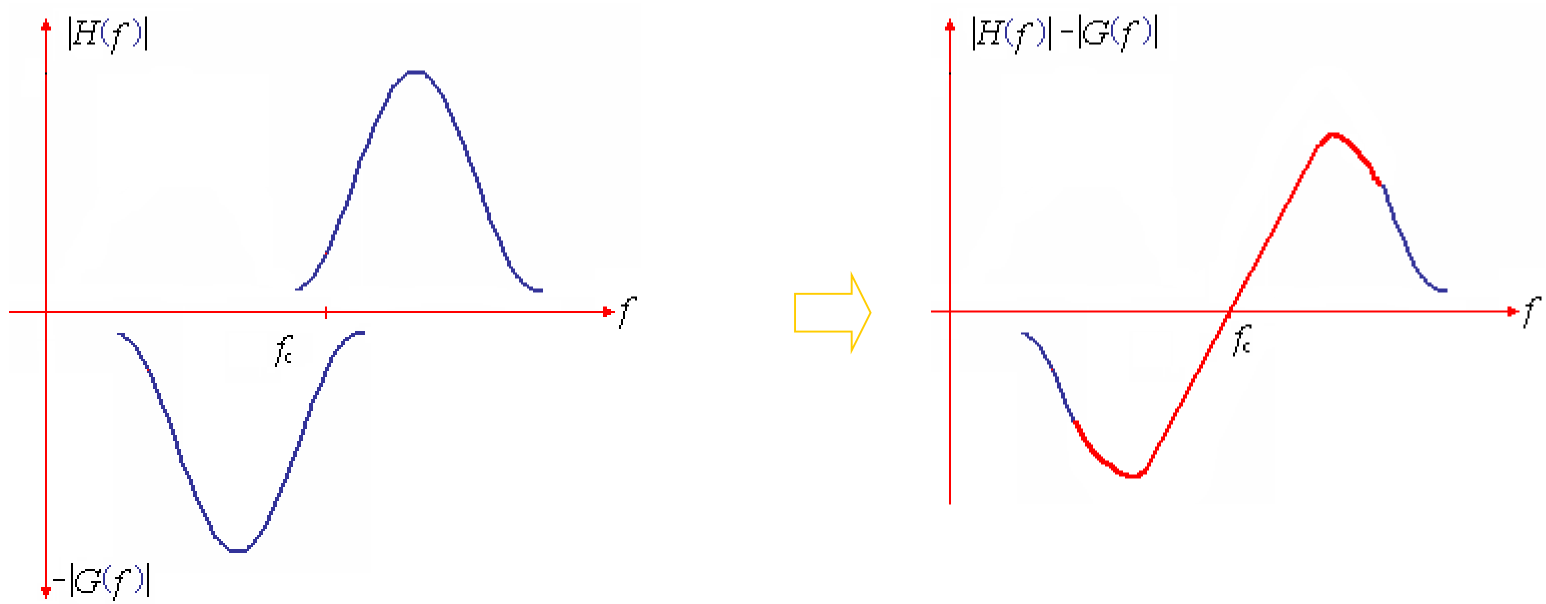


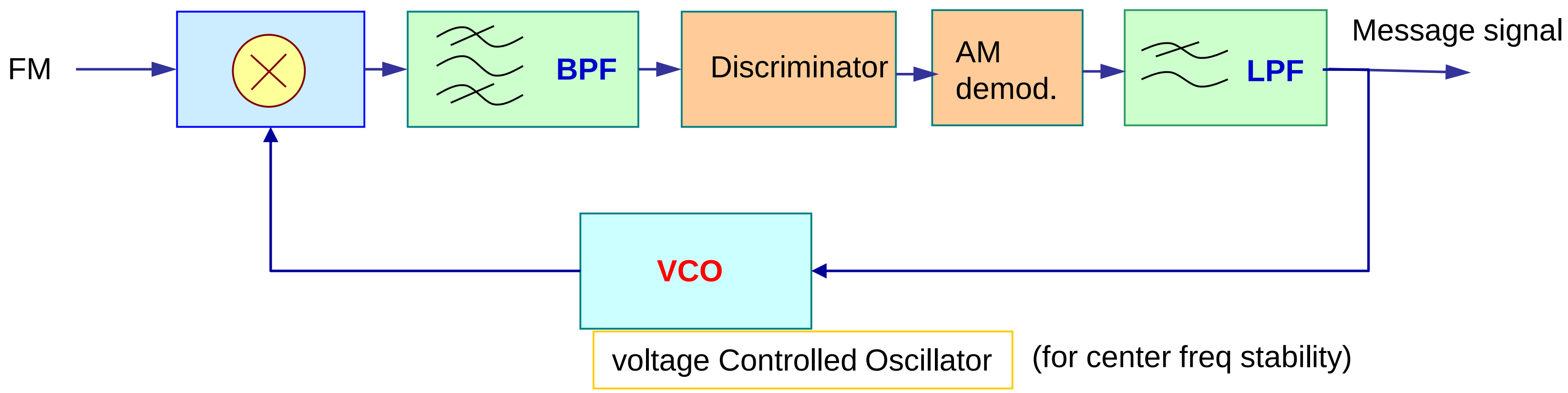
This is usually an LTI system whose response curve is approximately an increasing linear line in the related band.

Approximately linear region



Balanced Discriminator



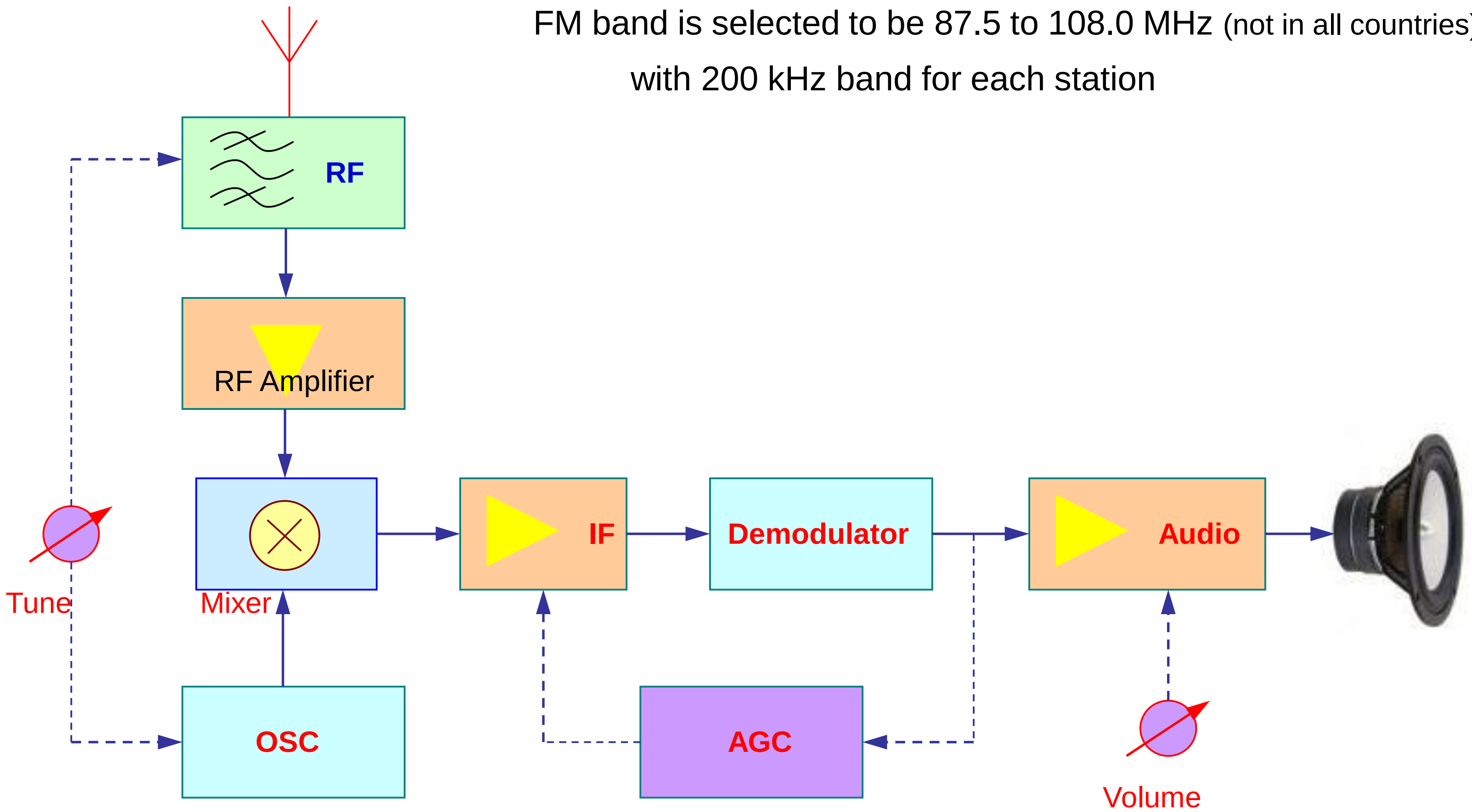


(for center freq stability)

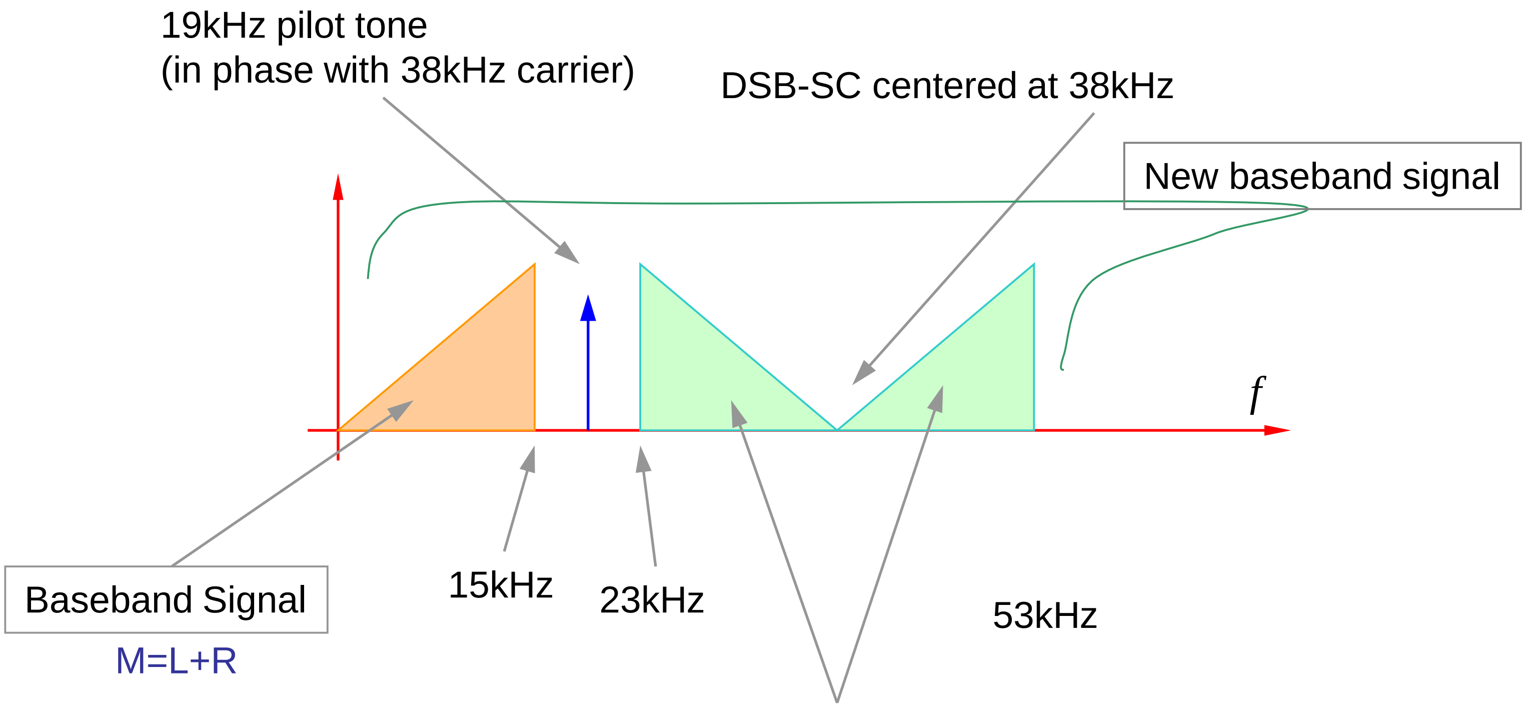
General Commercial Receiver

FM band is selected to be 87.5 to 108.0 MHz (not in all countries)

with 200 kHz band for each station



Stereo FM



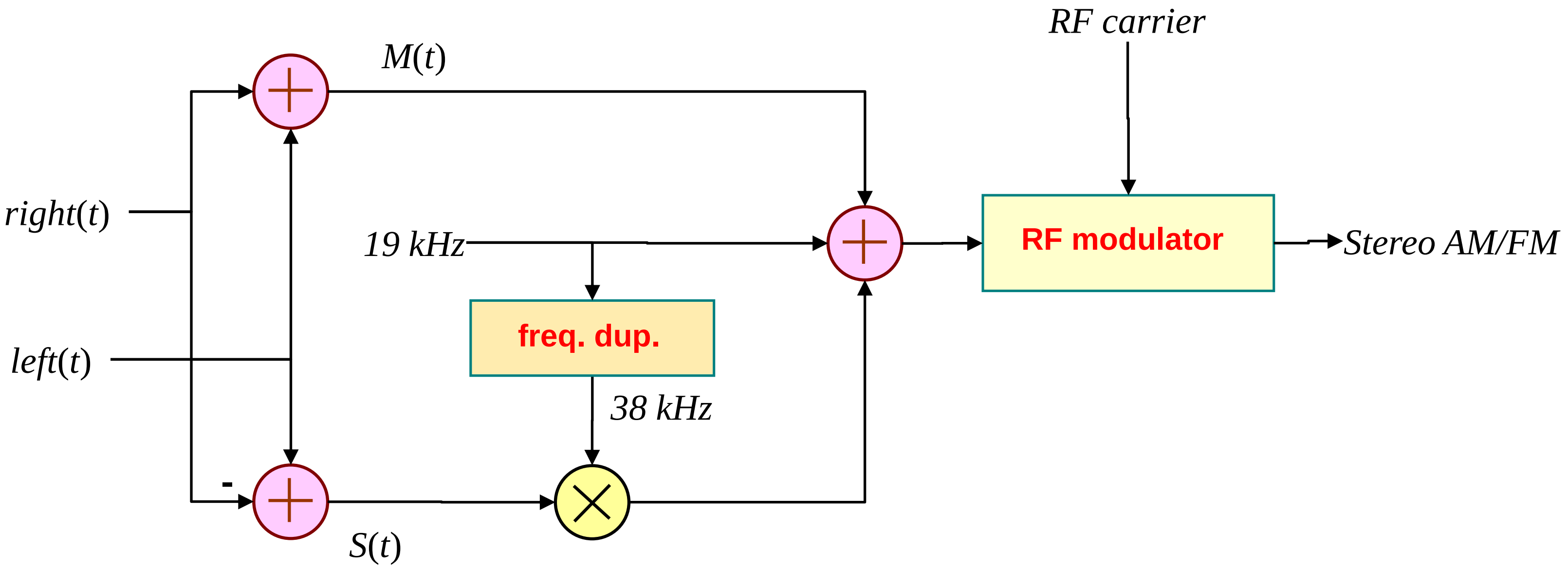
Mono FM receivers will only receive L+R

S=L-R

Stereo receivers will do: $L=(M+S)/2$ and $R=(M-S)/2$

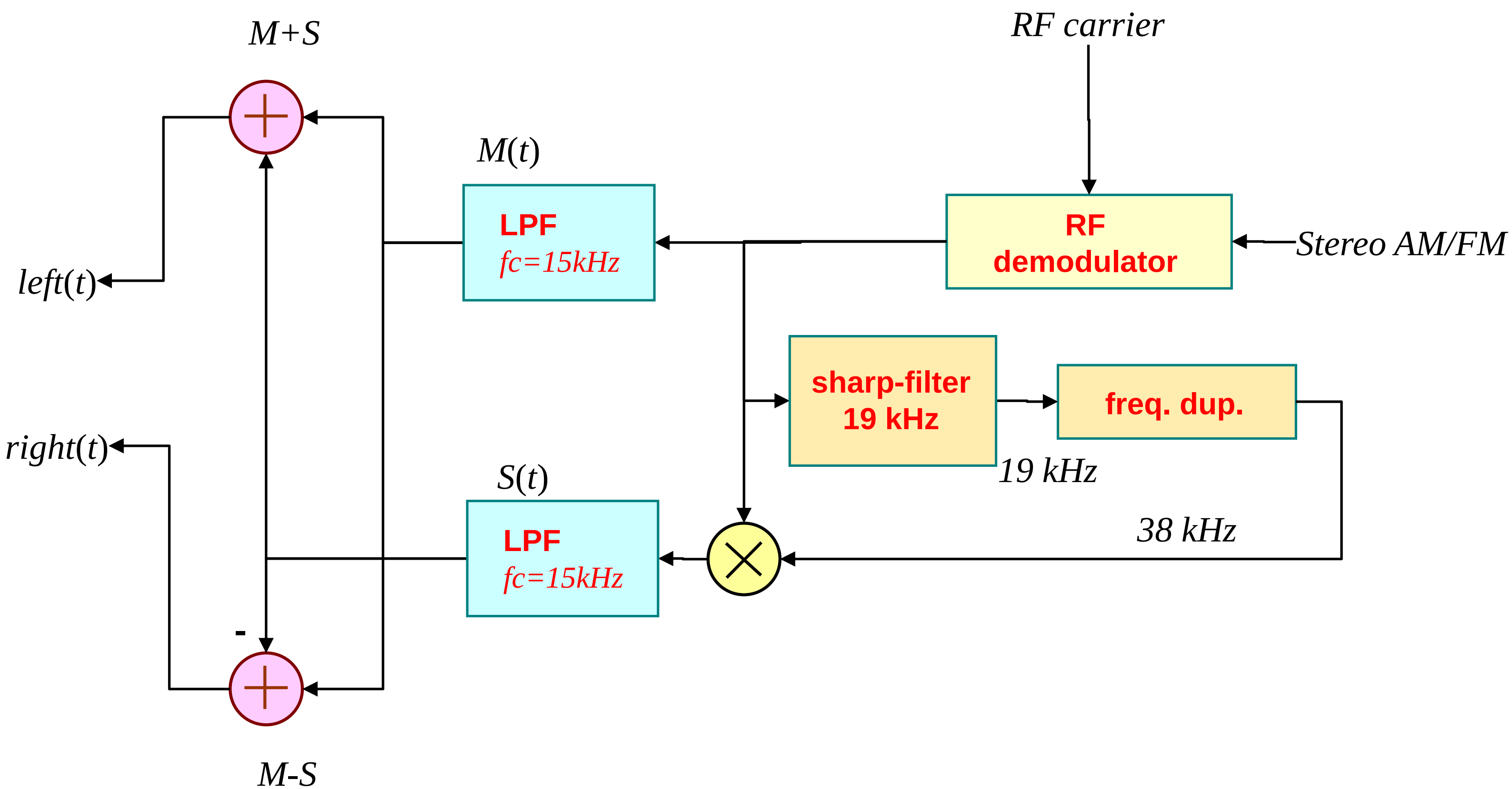
Example

Block diagram of stereo AM/FM transmitter



Example

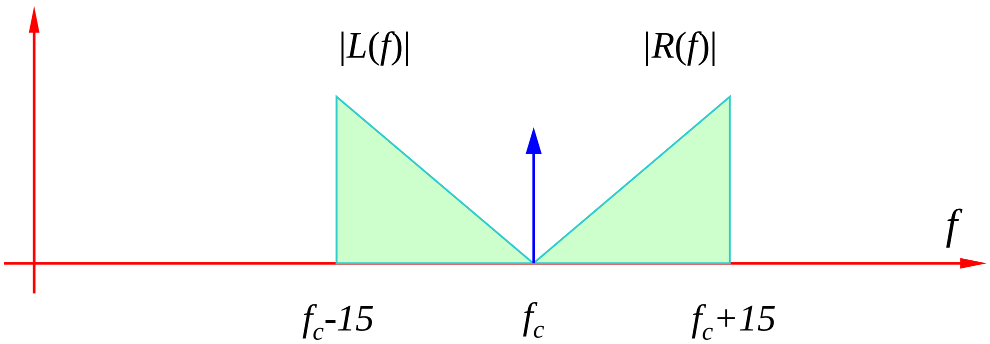
Block diagram of stereo AM/FM receiver



Hmw : Add some blocks for Mono-transmitter reception

Homework

Block diagram to generate the following spectrum



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