Baseband Comm.

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For the course "**Communications**"

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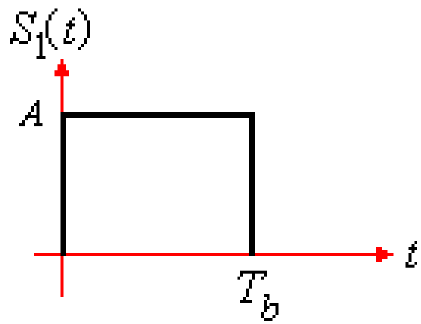
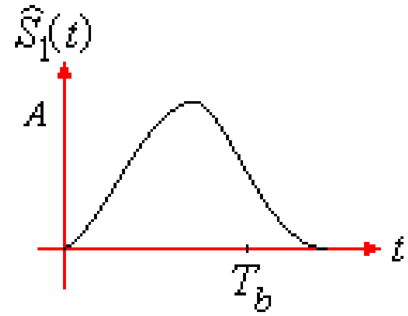
Pulse Amplitude Modulation (PAM)

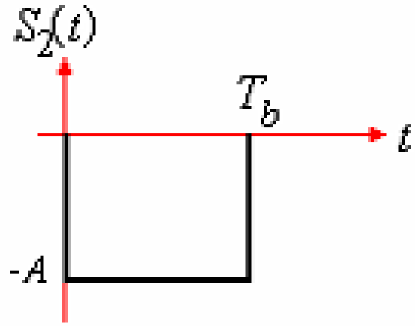
Simplest PAM : binary antipodal signaling

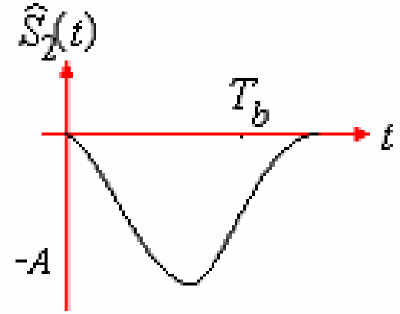
Binary 1





Binary 0





T_b = Bit interval Bit Rate = $1/T_b$

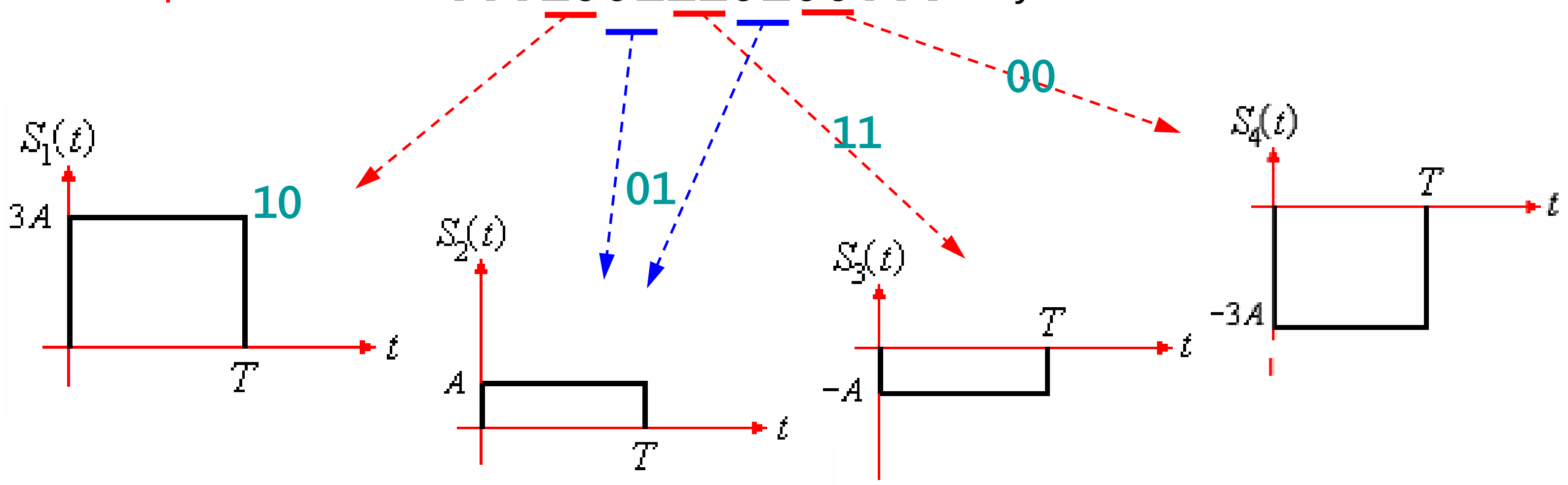
M-ary PAM

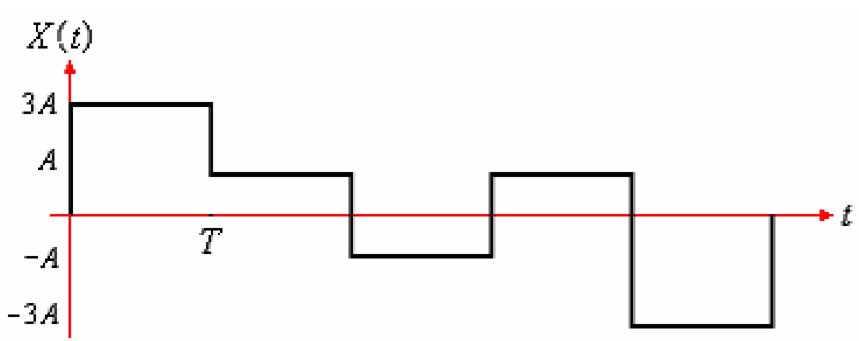
Instead of 2 levels (binary) M levels (M-ary) can be used.

M is selected so that

$$M = 2^k$$

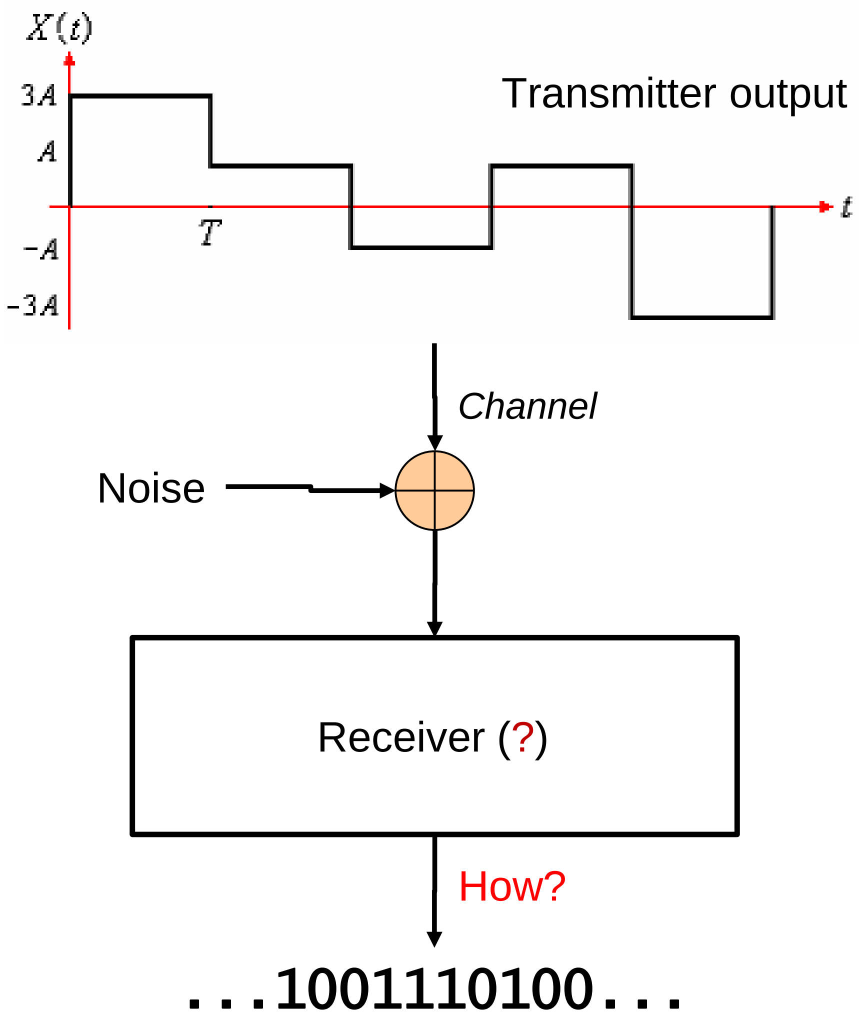
Example : k =2 . . . 1001110100 . . . Symbols are 2 bits





Symbol interval = 2 Bit interval
since 1 change transfers 2 bits

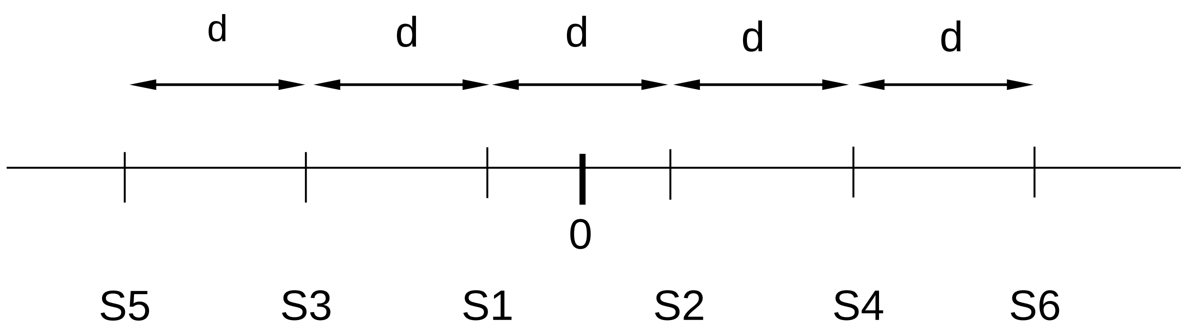
PAM Receiver Side



Now we are faced with the receiver-side problems:

- How will the receiver tell individual pulses apart?
- How will it recognize/decide the symbols (bits)?
- Any optimal waveform for easing up the receiver's job?
- Spectral characteristics?
- What are the effects of noise?
- What is the optimal T ?
- Power/energy considerations?

If we are given an amplitude range for PAM signals we would obviously place the amplitude points as far from each other as they can be in order to minimize the decision errors

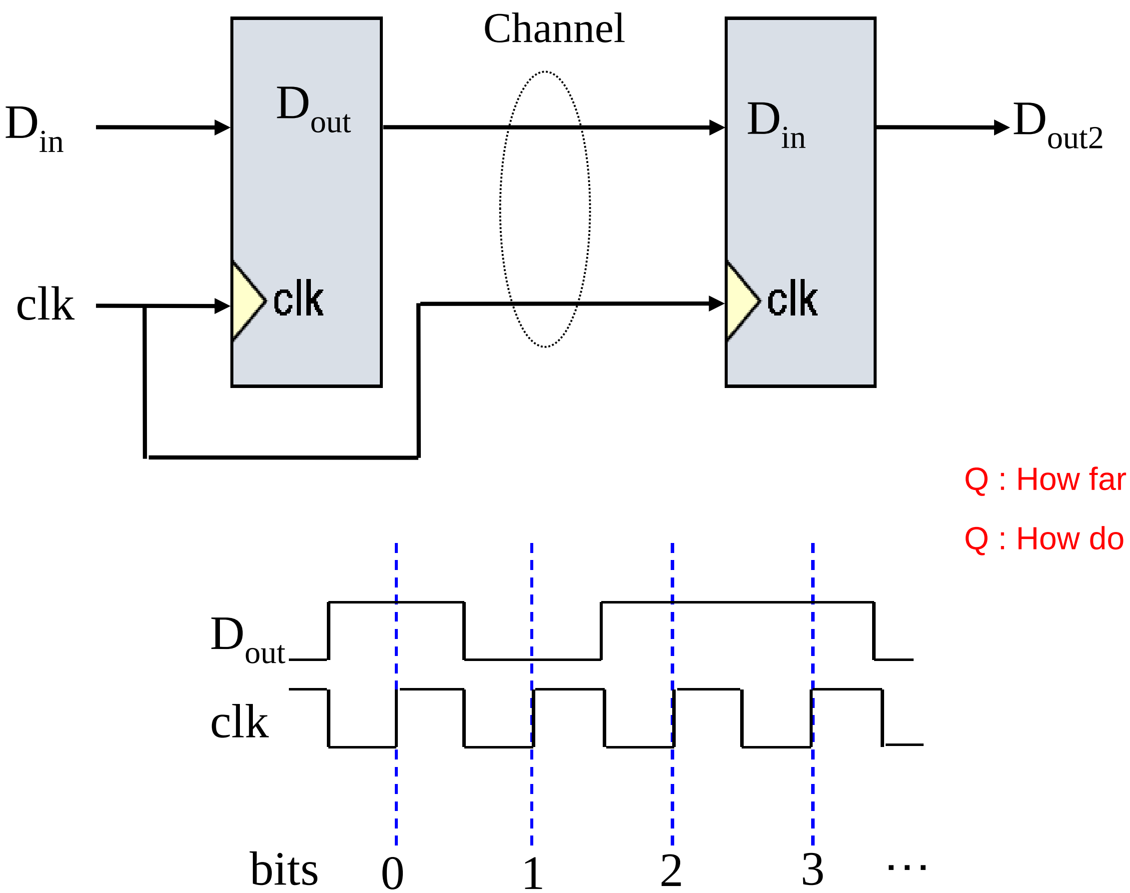


Constellation diagram for symmetric PAM

Similar placement for other waveforms/pulses?

FF-to-FF Transmission

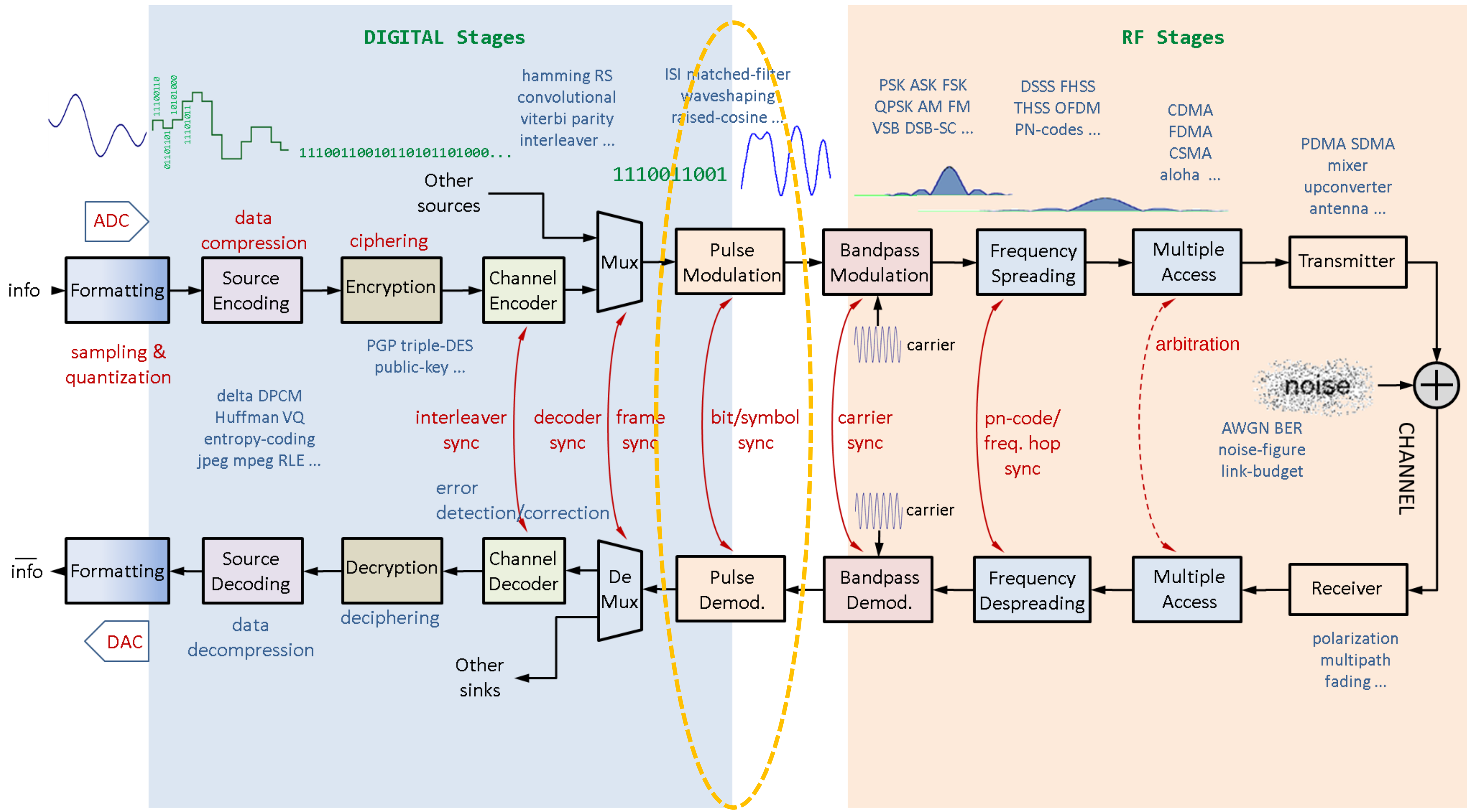
Synchronous : clock pulses manage everything



Q : How far can we transmit data this way?

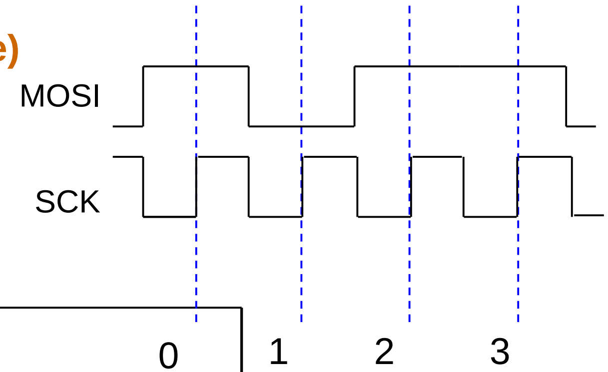
Q : How do we transmit clock?

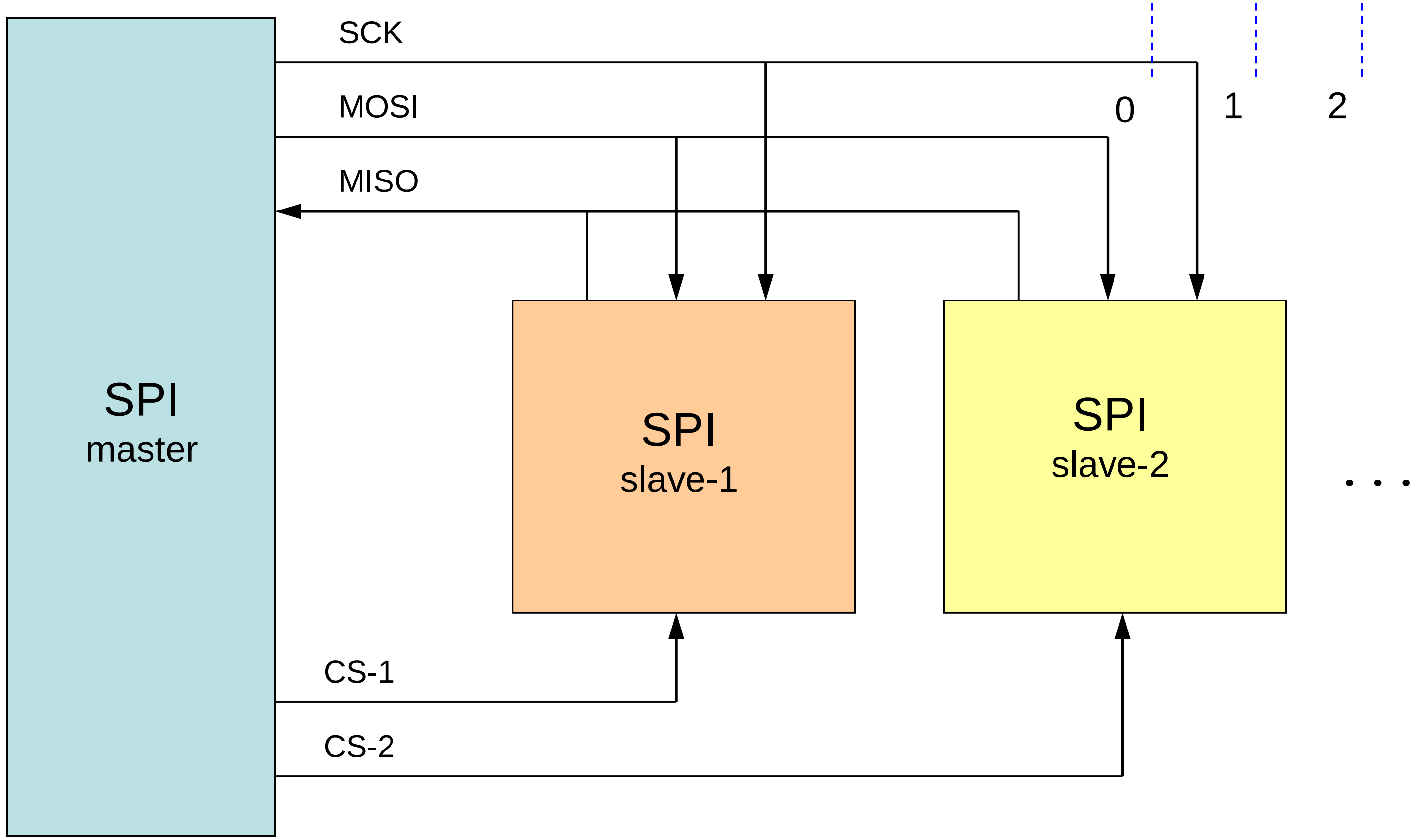
General Communication System



you are not responsible from these in exams

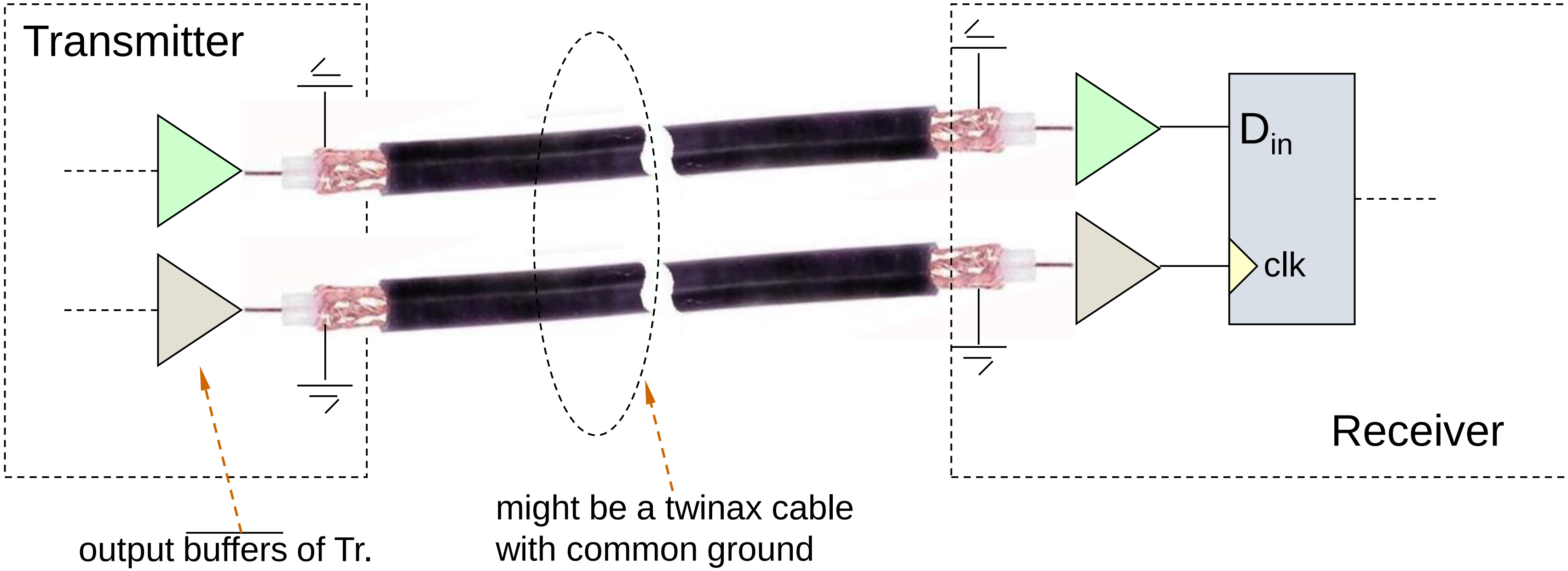
An Example : SPI (Serial Peripheral Interface)

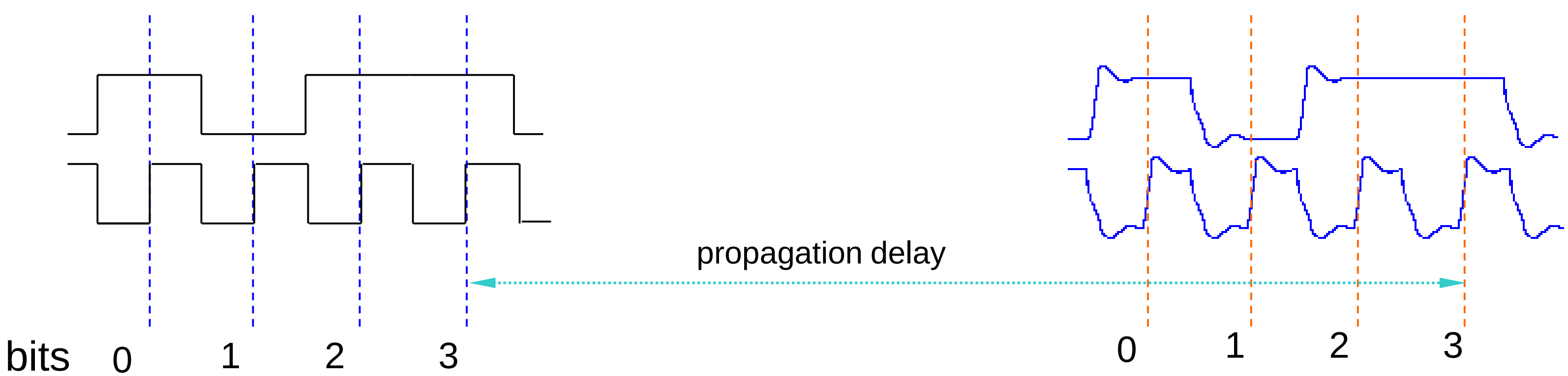




SPI is a modern way to communicate between master and slave IC's on a single PC-board

First Design (Synchronous)



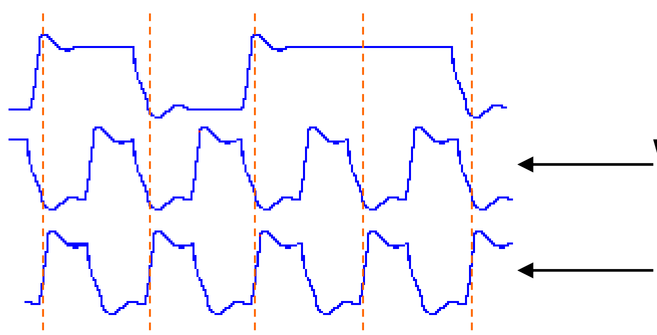


Let us assume that data and clock line lengths differ by 10 cm

one of the signals arrive 0.5 picoseconds late.

(speed of e.m. wave on copper is about 2×10^8 m/s)

Problem is : for a 1GHz clock, 0.5 ps is about half a clock cycle.



what you need/want

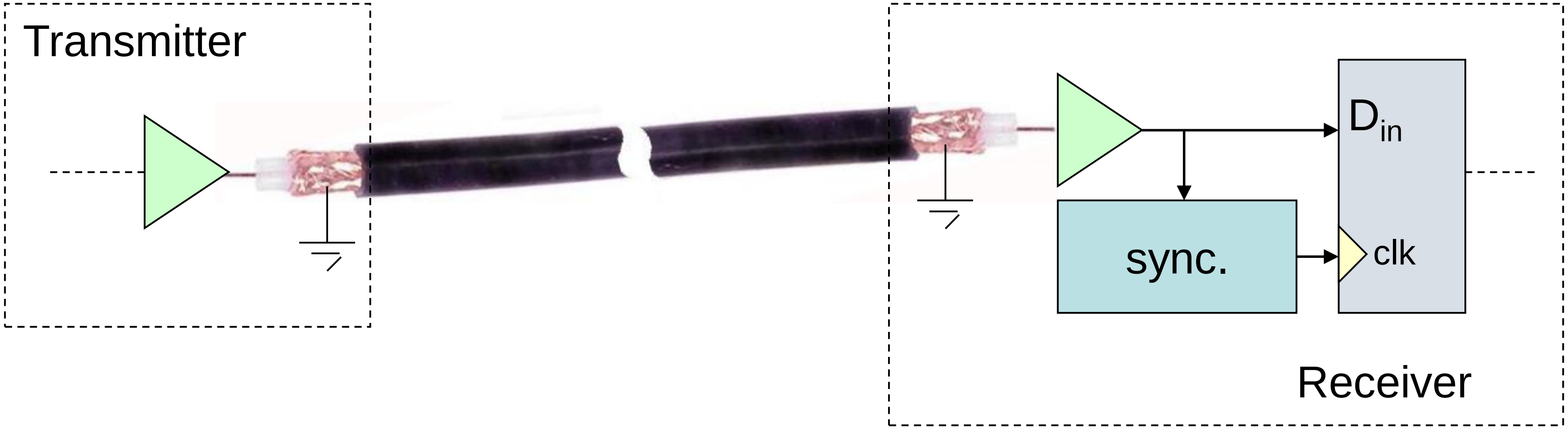
what you get

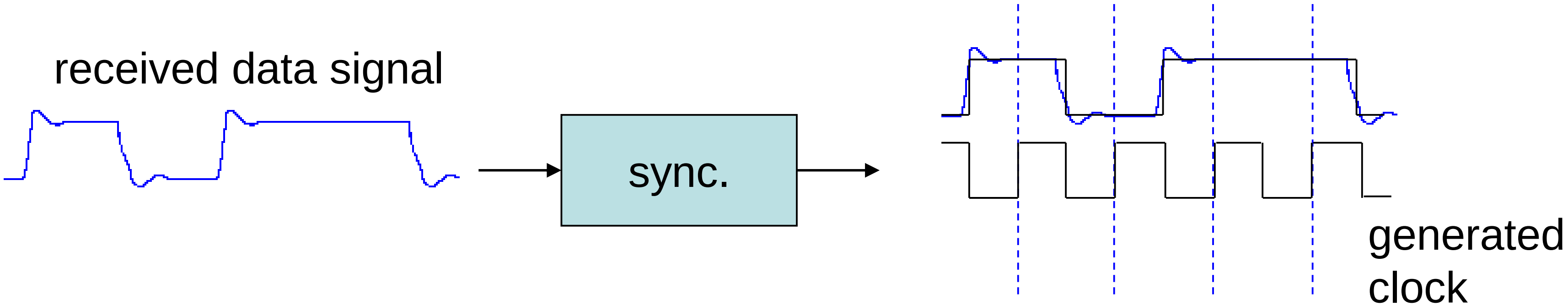
(Note: having speed of wave-travel 3×10^8 m/s or any other does not change the point here)

Solution

Solution is to generate clock from data at the receiver.

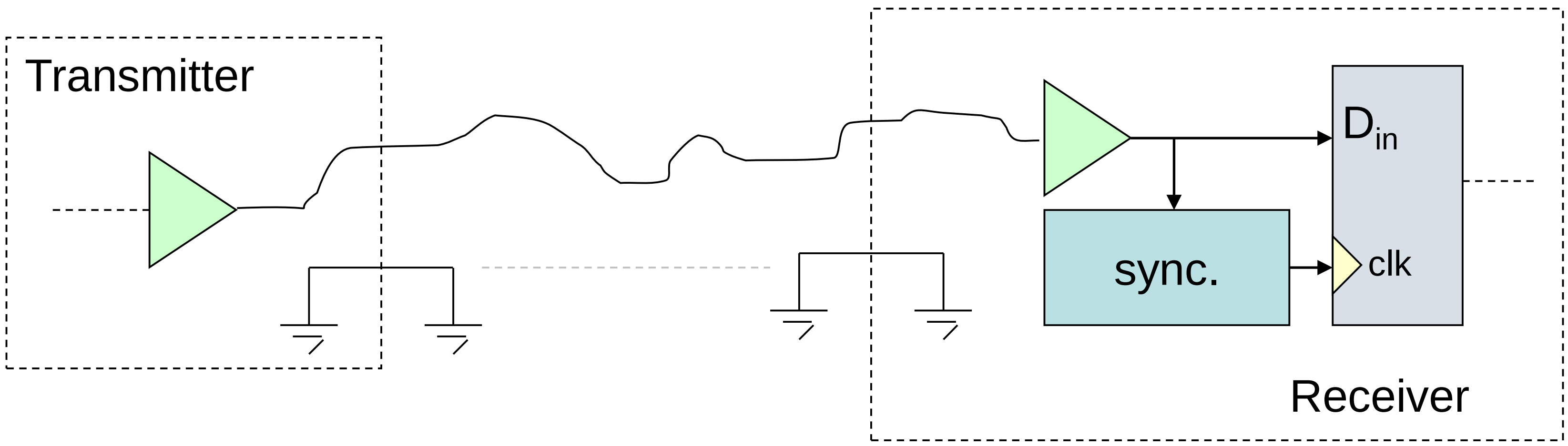
The data signal should necessarily be designed to help perform such an operation





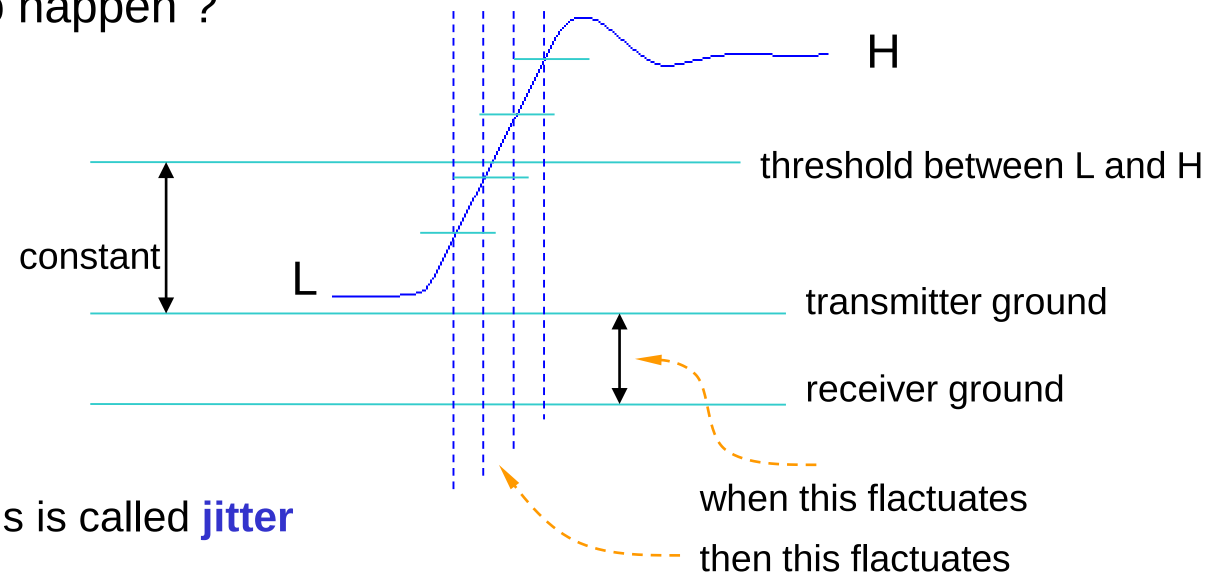
A Phase Locked Loop (PLL) can be used if there are enough transitions in the signal

Use of Ground as signal return



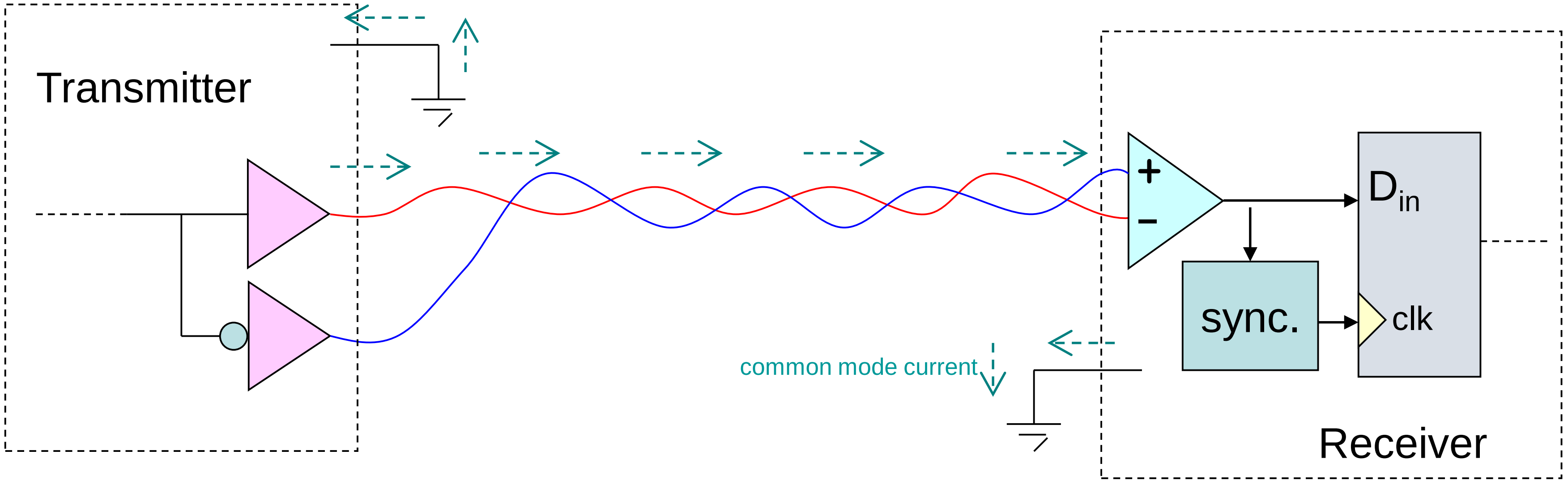
This requires that grounds at both side must have the same potential which we cannot guarantee

What is likely to happen ?



This is called **jitter**

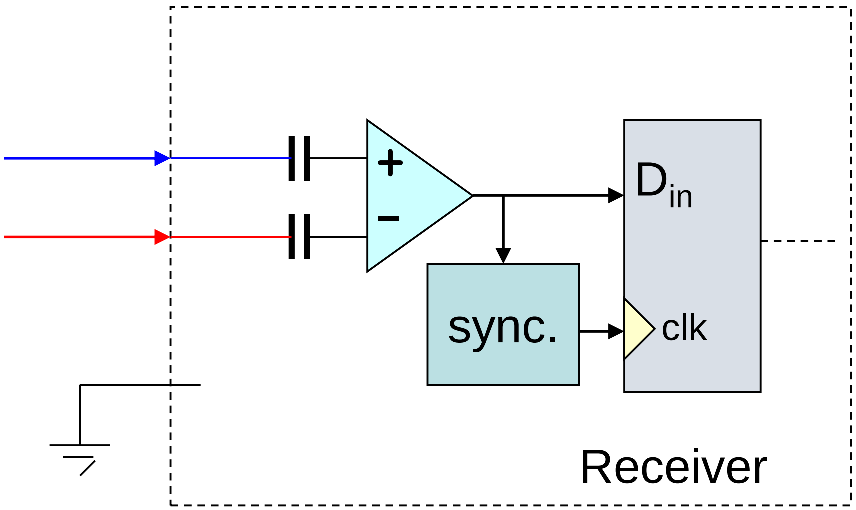
Differential Signaling



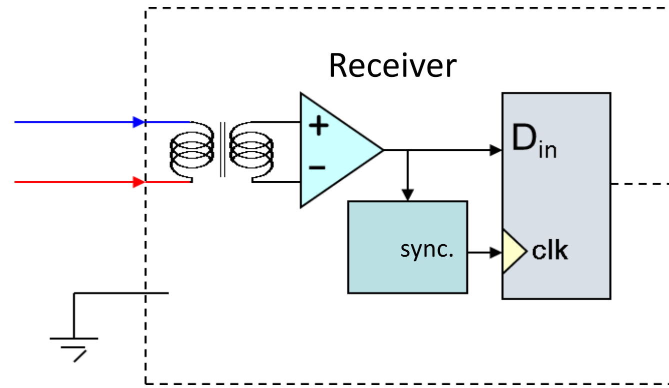
Receiver uses the voltage difference between two inputs.
The voltage between a signal line and the ground is not in the formula here, but we have another problem.

If there is a voltage difference between two grounds then there will be a *common mode current* on the signal lines returning from ground.

Capacitors to prevent CMC



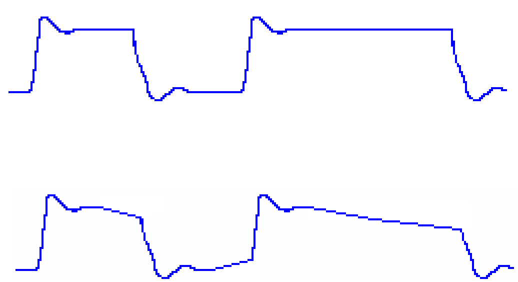
A transformer will also serve



Problem is different this time.

received signal

measured signal



if there are long runs of 0s or 1s in the signal the receiver might loose the synchronization and/or cannot read the data.

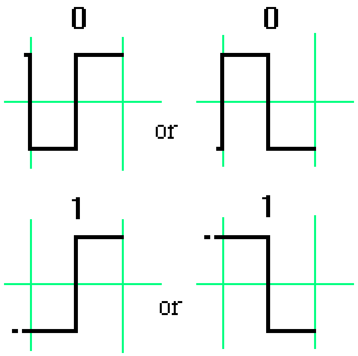
Solution to Long Runs Problem

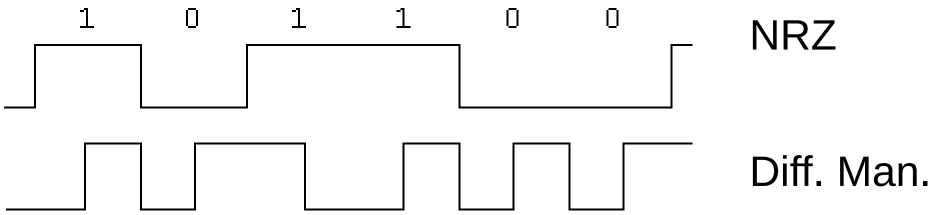
Data are coded in such a way that there never be long runs of **H**s or **L**s in the transmitted signal

A Popular Solution : *Bi-Phase* Encoding Techniques

Example : Differential Manchester

Inversion at the middle of each interval.
Transition (inversion) at the beginning means 0
No transition at the beginning means 1

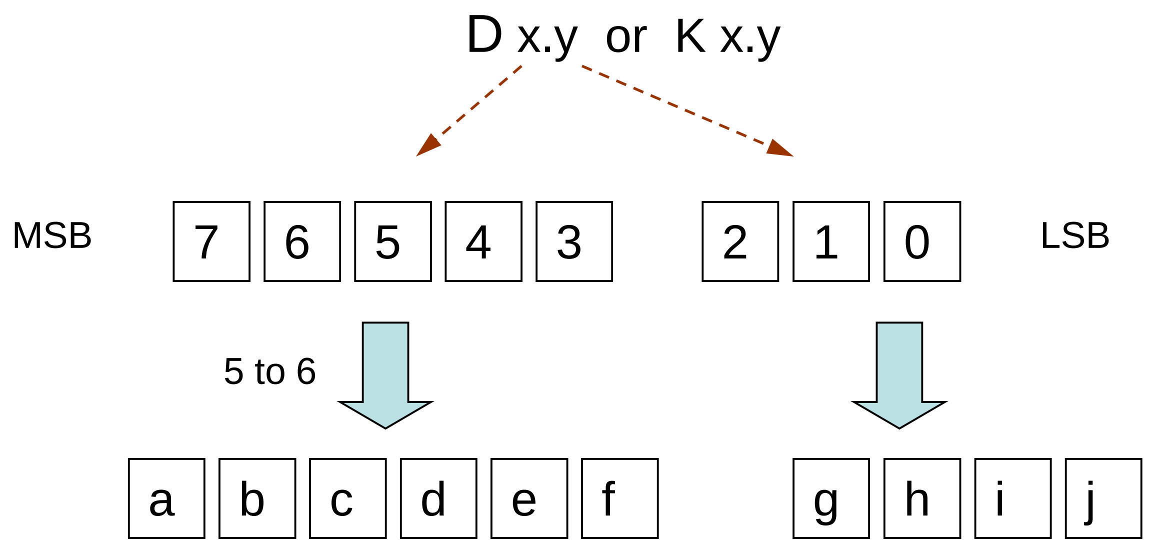




Bi-phase encodings somewhat increase the signal bandwidth

8B10B (Widmer-Franaszek 1983)

Uses specially selected 10 bit codes out of 1024 possible, to represent 8 bit values



Conversion table entries are selected with minimum *disparity* in mind

Disparity : Number of 1s minus Number of 0s

Code table for y (3 bits in 4 bits out)

y	-D	+D
000	0100	1011
001	1001	
010	0101	
011	0011	1100
100	0010	1101
101	1010	
110	0110	
111	0001 or 1000	1110 or 0111

For codes with multiple possibilities, the one that reduces total disparity after combining with the code of x (5 -> 6 bits) is selected.

i

	5b in	6b out (abcdef)
0	00000	100111 or 011000
1	00001	011101 or 100010
2	00010	101101 or 010010
3	00011	110001
4	00100	110101 or 001010
5	00101	101001
6	00110	011001
7	00111	111000 or 000111
8	01000	111001 or 000110
9	01001	100101
10	01010	010101
11	01011	110100
12	01100	001101
13	01101	101100
14	01110	011100
15	01111	010111 or 101000
16	10000	011011 or 100100
17	10001	100011
18	10010	010011
19	10011	110010
20	10100	001011
21	10101	101010
22	10110	011010
23	10111	111010 or 000101
24	11000	110011 or 001100
25	11001	100110
26	11010	010110
27	11011	110110 or 001001
28	11100	001110
29	11101	101110 or 010001
30	11110	011110 or 100001
31	11111	101011 or 010100

Example :
Consider D2.6
that is 110 00010

The code for 00010 is either
101101 or 010010

and the code for 110 is 0110

We have two possibilities for output
101101 0110 or 010010 0110

if the disparity of the previous codes is
\+ : select 010010 0110
− : select 101101 0110

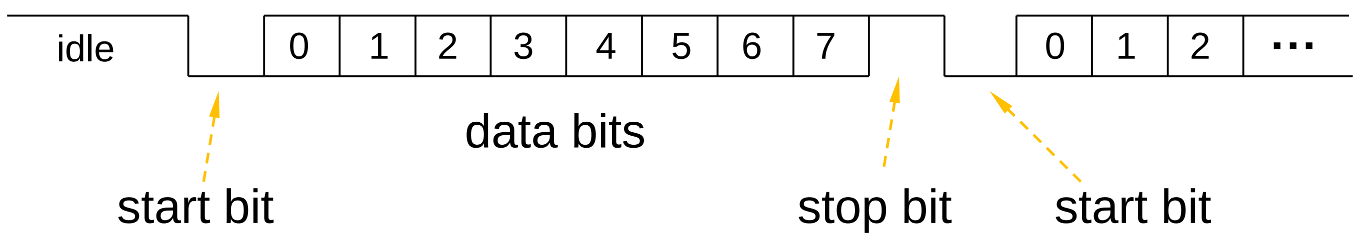
Disparity of the bits output so far is called the *Running Disparity*

there is more in 8b10b than touched here but let us skip it

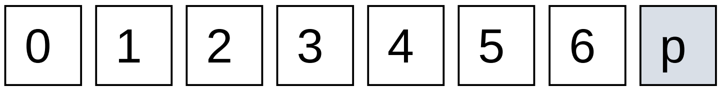
Where is 8B10B used?

USB 3.0
DVI and HDMI
Fibre Channel
PCI Express
IEEE 1394b
Serial ATA
Gigabit Ethernet
SAS
SSA
HyperTransport
Common Public Radio Interface (CPRI)
InfiniBand
XAUI
Serial RapidIO
DVB Asynchronous Serial Interface (ASI)
DisplayPort

Async. Serial Comm. Signal



If the data is 7-bit ASCII then the 8^{th} bit is usually a parity bit

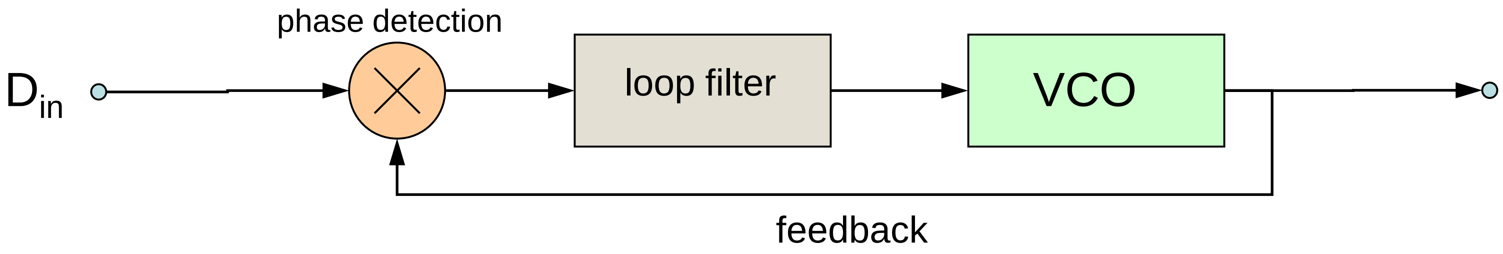


Start bit indicates that, for the receiver, it is time to start reading data bits
Stop bit identifies the end of the 8 bit sequence.
Start-Stop bits are together used for synchronization

Bit Synchronization

Bit synchronization is to generate a clock signal with transitions at correct times at the receiver end, using only the incoming serial data signal.

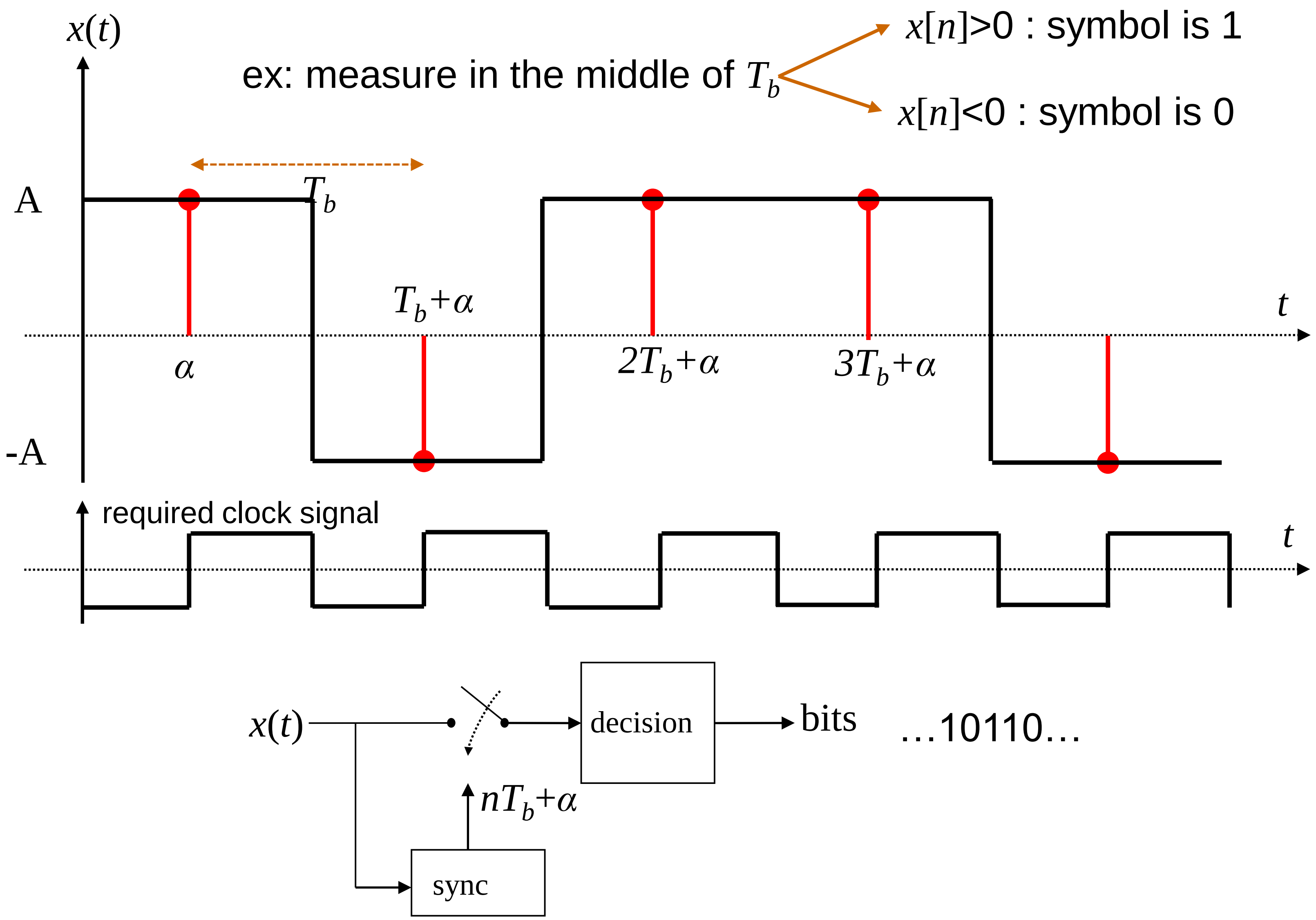
This is usually achieved by Phase Locked Loops



Phase detector outputs a signal proportional to phase difference between internally generated clock and incoming signal.
Loop Filter is usually a low-pass filter (or integrator) which provides a long duration voltage, that represents the phase difference, to VCO.
VCO generates a clock signal centered at the fundamental frequency of incoming signal.
Clock frequency increments or decrements very small amounts according to the phase difference.

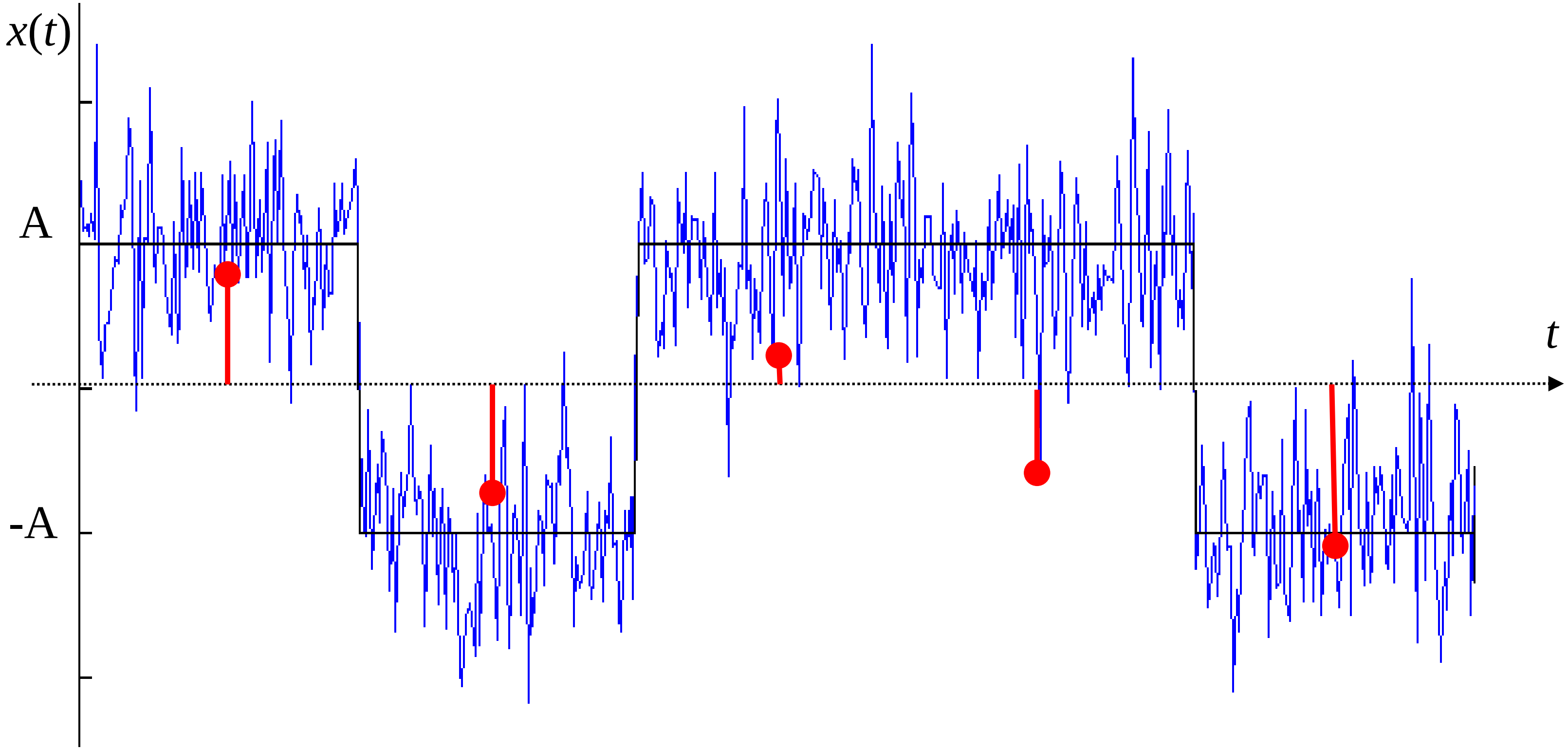
We will get back to synchronization later

Receiver Side Considerations

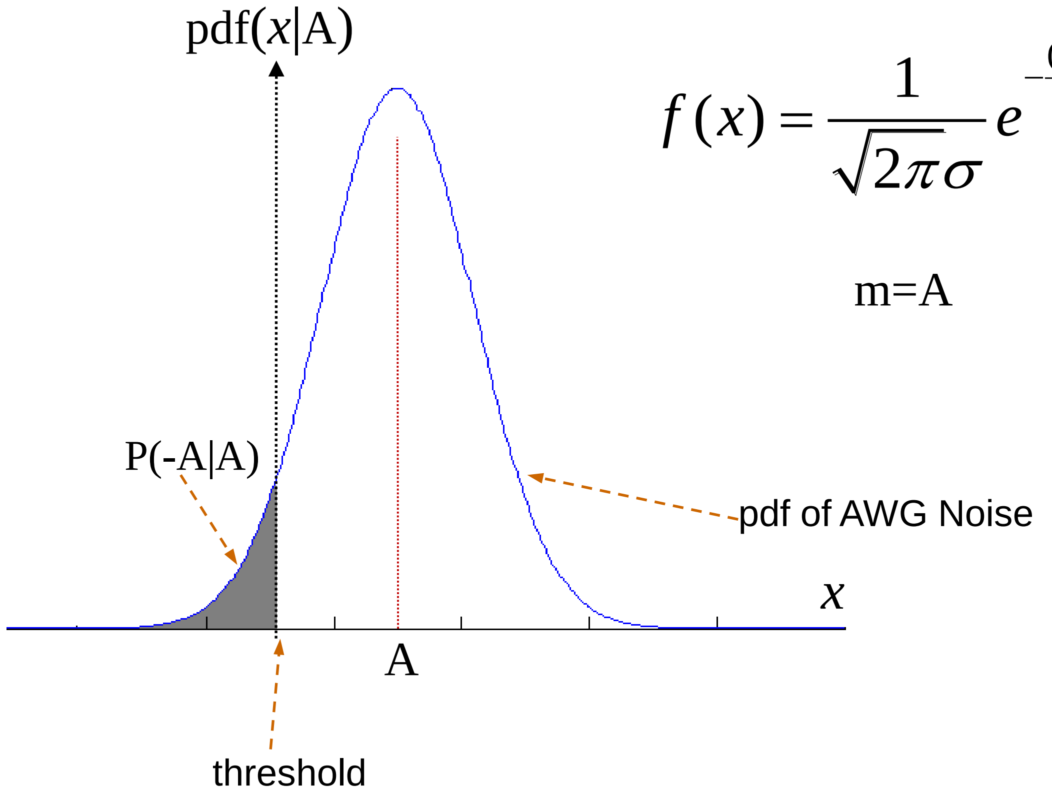


Effects of Noise

on binary PAM (±A antipodal signals)



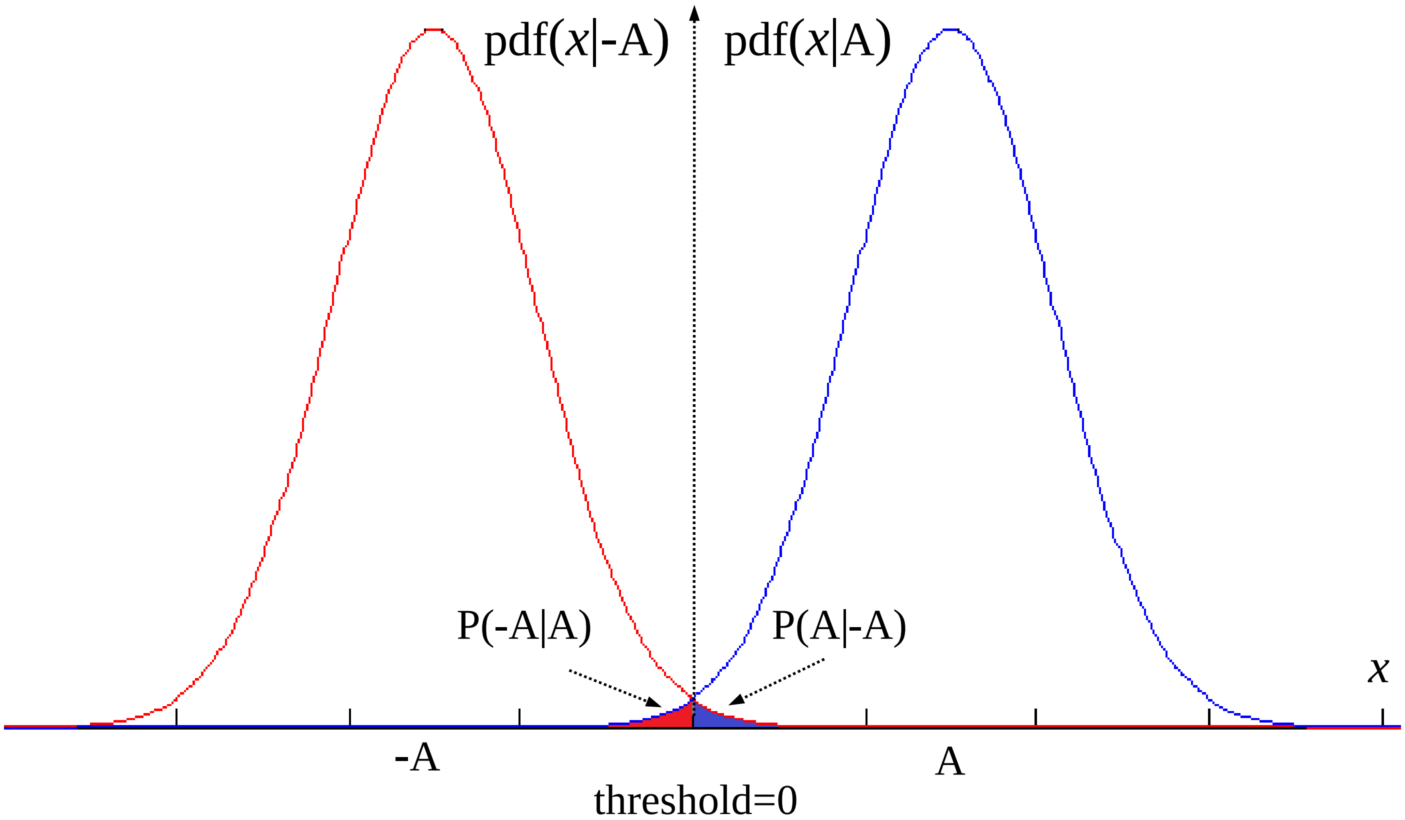
Decision Errors



$$f(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-m)^2}{2\sigma^2}}$$

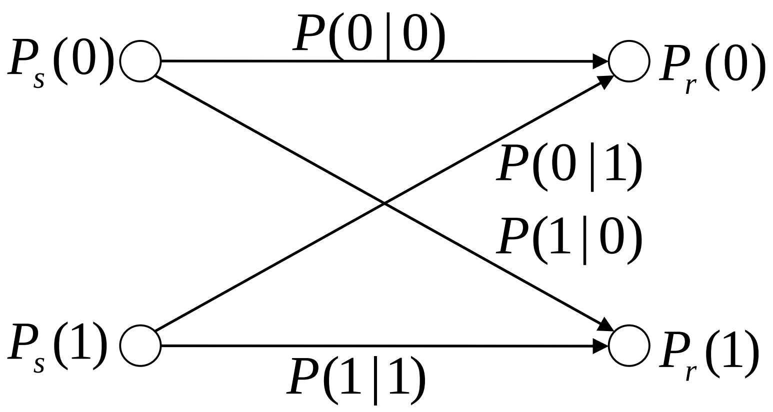
m=A

ML Decision Errors for Binary Antipodal Signaling



Total error $$p_e = \sum_{i=0}^{M-1} p(a_i) p_e(a_i)$$

Binary Symmetric Channel



$$p_e = P(0|1) = P(1|0)$$

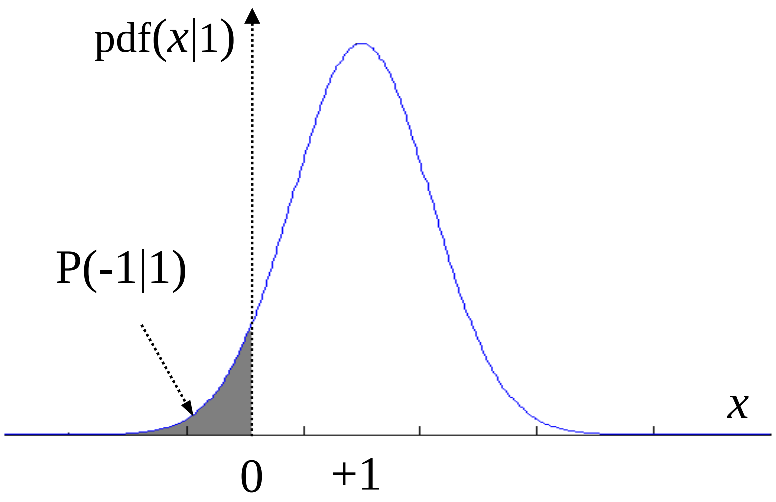
$$p_e = \int_{-\infty}^{V_t} \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-m)^2}{2\sigma^2}} dx$$

Example

$$A = 1 \qquad V_t = 0$$

$$P(-1\,|\,+1) = \int_{-\infty}^{0} N(m,\sigma^2)dx = \int_{-\infty}^{0} \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-m-A)^2/2\sigma^2} dx = \int_{-\infty}^{0} \frac{1}{\sqrt{2\pi}} e^{-(x-1)^2/2} dx$$

$$p_e = P(-1\,|\,+1) = P(+1\,|\,-1)$$



Approximation

Since the integral $Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^{\infty} e^{-t^2} dt$ cannot be calculated analytically

we either use tables or approximations

example $erf(x) \cong 1 - 1/(1 + c_1 x + c_2 x^2 + c_3 x^3 + c_4 x^4)^4$ for $erf(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt$

where $c_1 = 0.278393$

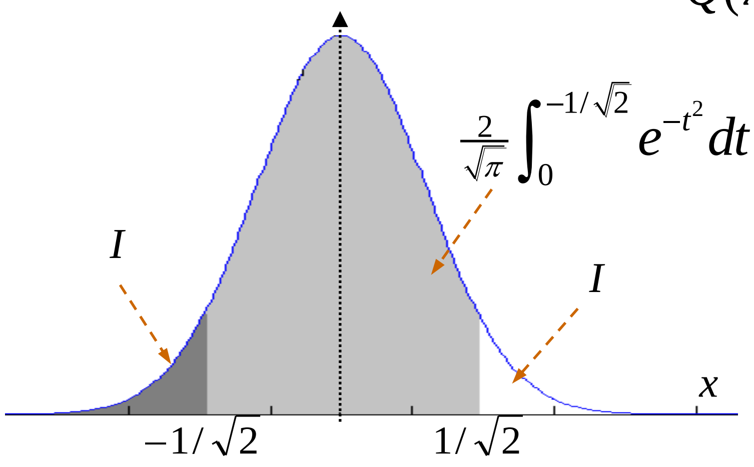
$c_2 = 0.230389$

$c_3 = 0.000972$

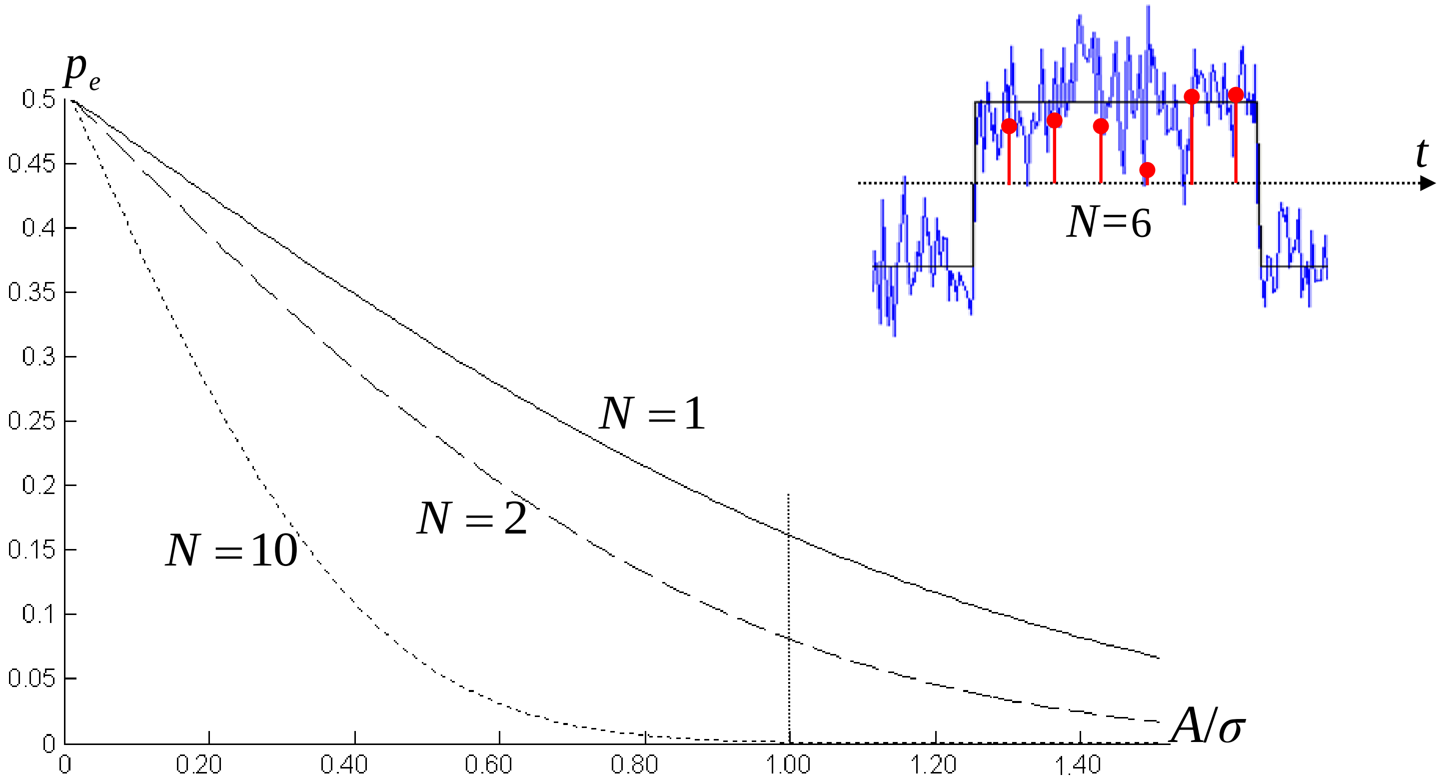
$c_4 = 0.078108$

$$erf(x) = 1 - 2Q(\sqrt{2}x)$$

$$Q(x) = \tfrac{1}{2}(1 - erf(x/\sqrt{2}))$$



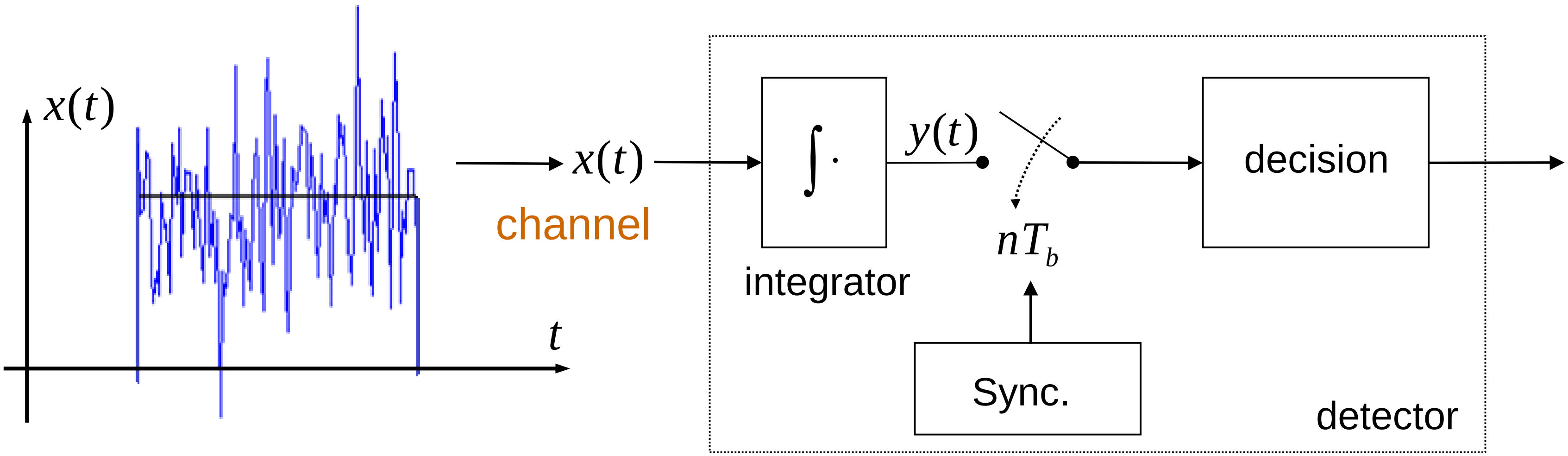
Averaging Multiple Samples Within T_b



$$\bar{x} = \frac{1}{N}\sum_{i=0}^{N-1} x[n] \longrightarrow \bar{x} = \frac{1}{T_b}\int_0^{T_b} x(t)dt$$

In binary case, division by N is not important for decision

Binary Antipodal PAM Error



$$x(t) = s_i(t) + \eta(t)$$
$$s_0(t) = +A$$
$$s_1(t) = -A$$

at decision instant

$$y_d = y_x + y_\eta = \int_0^{T_b} (\pm A + \eta(t))dt$$

$$y_x = \int_0^{T_b} \pm A dt = \pm A T_b \qquad y_\eta = \int_0^{T_b} \eta(t) dt$$

So, if $|y_x| > |y_\eta|$ we make a correct decision

otherwise we may have an incorrect decision

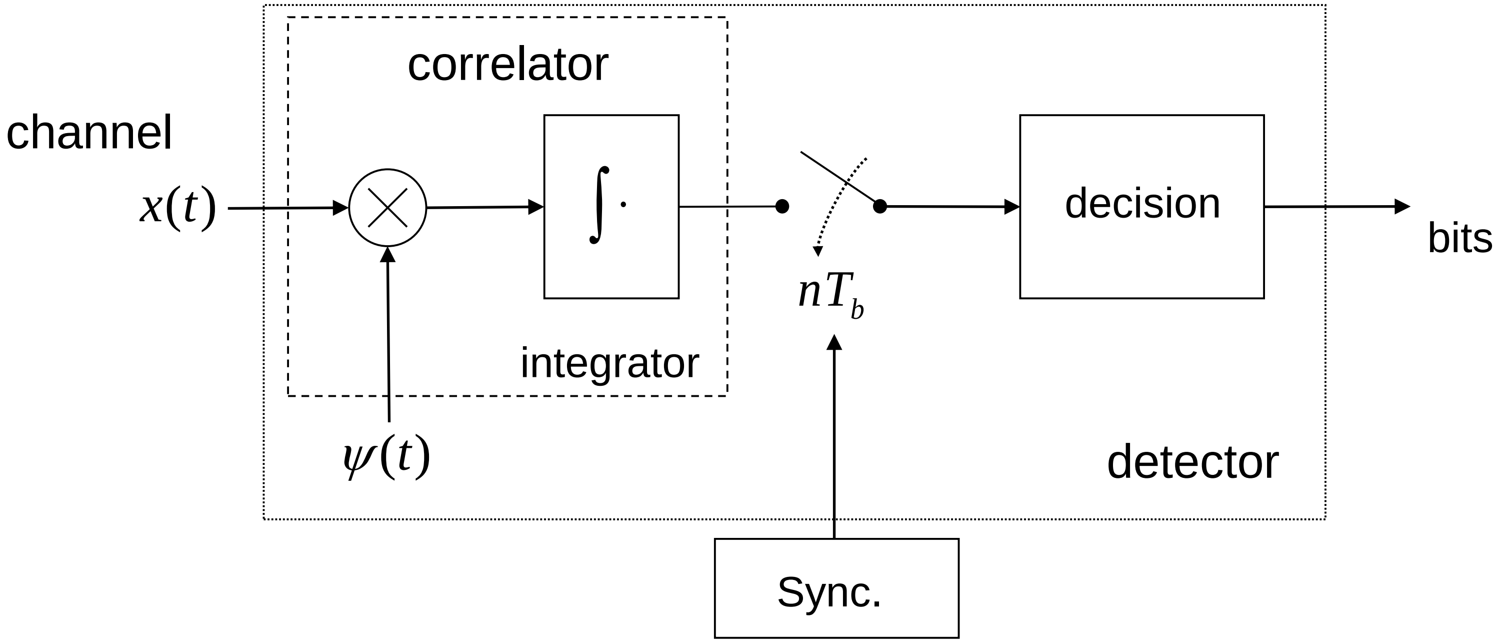
Arbitrary waveforms

If $x(t)$ is an arbitrary waveform instead of $\pm A$

we need to measure the similarity : $R(\tau) = \int_{\tau}^{\tau+T_b} x(t)\psi(t)dt$ $\psi(t)$: waveform

received expected

Therefore, the receiver becomes



$\pm A$ is a special case of arbitrary waveforms for which the multiplier is not required

$$R(nT_b) = \int_{(n-1)T_b}^{nT_b} x(\tau)\psi(\tau)d\tau$$

For antipodal case, this can be either $\psi(\tau)$ or $-\psi(\tau)$ (plus noise, of course)

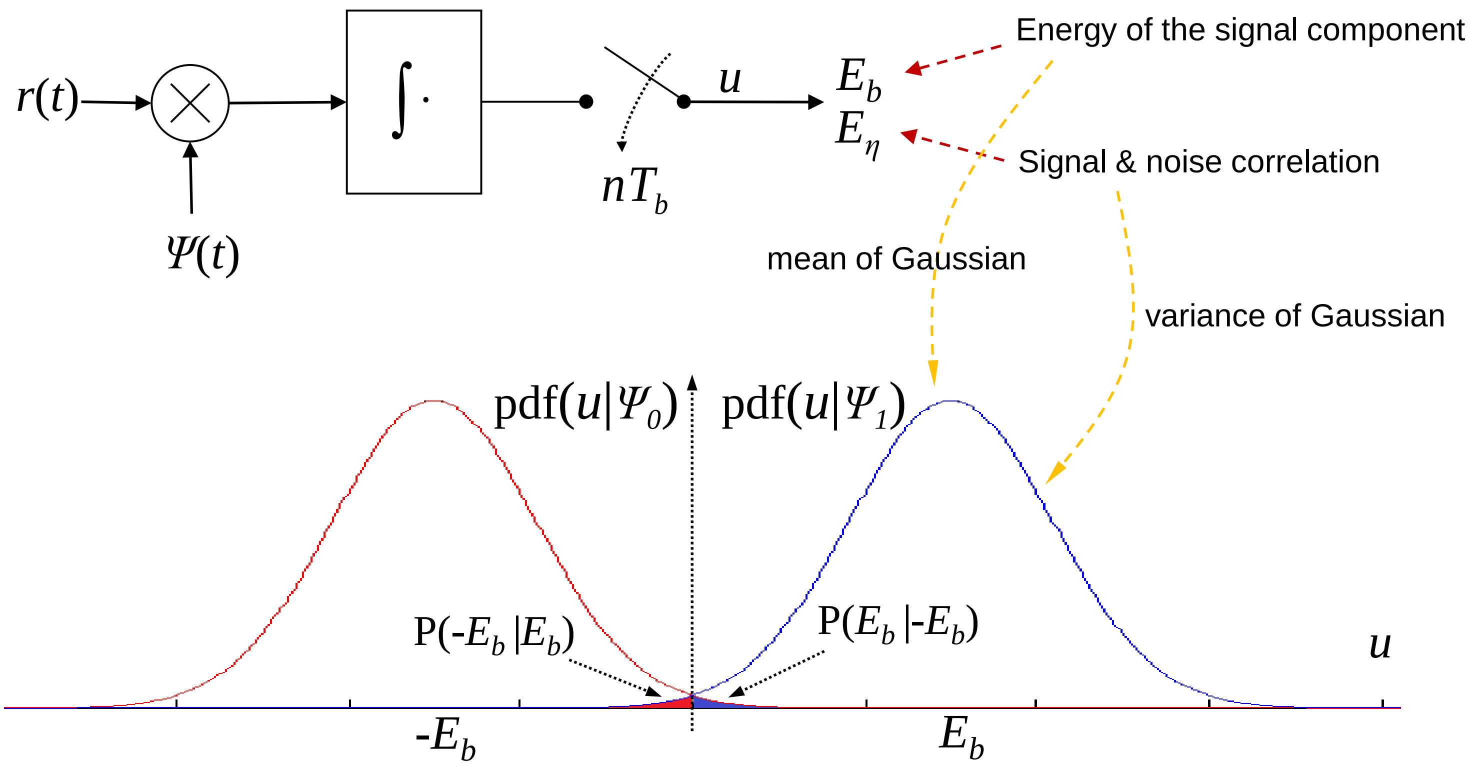
When $x(\tau) = \psi(\tau)$ then $R(nT_b) = \int_{(n-1)T_b}^{nT_b} \psi^2(\tau)d\tau = E_b$ + noise part

When $x(\tau) = -\psi(\tau)$ then $R(nT_b) = -E_b$ + noise part

provided that the local $\psi(\tau)$ is synchronously generated at the receiver

p_e for Binary Antipodal Waveforms under AWGN

The received signal is either Ψ_0 or Ψ_1 representing binary 0 or 1. (Ψ_0 =-Ψ_1)
Consequently the correlator output, at the end of T_b, is either E_b or $-E_b$.



$$\sigma_\eta^2 = ExpectedValue\left(E_\eta^2\right) \quad \text{where} \quad E_\eta = \int_0^{T_b} \psi(t)\eta(t)dt$$

Summary for Binary Antipodal Waveforms under AWGN so far

- Signal portion of correlator output, at the end of T_b, is either E_b or $-E_b$ for antipodal waveforms.
- Noise portion of the correlator output has also Gaussian distribution. Because linear operations do not change the shape of the distribution, but the variance.
- Variance of the noise portion, at the end of T_b, is the expected value of the E_η^2

where $$E_\eta = \int_0^{T_b} \psi(t)\eta(t)dt$$ (cross-correlation term)

(expected value of E_η is zero because noise and signal are uncorrelated)

- Probability of decision error is therefore, the area shown in the previous figure.

$$p_e = \frac{1}{\sigma\sqrt{2\pi}} \int_{E_b}^{\infty} e^{-t^2/2\sigma^2} dt$$

assuming that $+\Psi$ is sent and the decision threshold is zero (hmw: is it reasonable?)

- If the system is symmetric (antipodal and probabilities of sending 0 and 1 are equal), then it is not necessary to also calculate for $-\Psi$. (hmw: Think. what if otherwise is true?)

now, we can focus on calculating p_e

Variance of AWGN at the correlator output

Since $\sigma_\eta^2 = ExpectedValue(E_\eta^2)$ and $E_\eta = \int_0^{T_b} \psi(t)\eta(t)dt$

$$\sigma_\eta^2 = \frac{1}{T_b}\int_0^{T_b}\left[\int_0^{T_b} \psi(t)\eta(t)dt\right]^2 d\tau$$

(from the definition of variance)

$$\sigma_\eta^2 = \frac{1}{T_b}\int_0^{T_b}\left[\int_0^{T_b} \psi(t)\eta(t)dt \int_0^{T_b} \psi(v)\eta(v)dv\right] d\tau$$

(used Fubini's theorem here)

$$\sigma_\eta^2 = \frac{1}{T_b}\int_0^{T_b}\left[\int_0^{T_b}\int_0^{T_b} \psi(v)\eta(v)\psi(t)\eta(t)dtdv\right] d\tau$$

$$\sigma_\eta^2 = \frac{1}{T_b}\int_0^{T_b}\left[\int_0^{T_b}\int_0^{T_b} \psi(v)\eta(t)dtdv \int_0^{T_b}\int_0^{T_b} \eta(v)\psi(t)dtdv\right] d\tau$$

$$\sigma_\eta^2 = \frac{1}{T_b}\int_0^{T_b}\left[\int_0^{T_b} \psi^2(t)dt \int_0^{T_b} \eta^2(t)dt\right] d\tau = \frac{1}{T_b}\int_0^{T_b} E_b\frac{N_0}{2} d\tau$$

$\sigma_\eta^2 = \frac{E_b N_0}{2}$ or $\sigma_\eta = \sqrt{\frac{E_b N_0}{2}}$

since we know the variance now, we can calculate the probability of making an errorenous decisions for the symbols by looking at the output of the correlator at the end of the symbol duration

p_e for Binary Antipodal Waveforms under AWGN

Since we have tables or approximations for $Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^\infty e^{-t^2/2} dt$

we try to make $p_e = \frac{1}{\sigma\sqrt{2\pi}} \int_{E_b}^{\infty} e^{-t^2/2\sigma^2} dt$ look like it

Putting $\sigma_\eta^2 = \frac{E_b N_0}{2}$ and $\sigma_\eta = \sqrt{\frac{E_b N_0}{2}}$ into their places in the p_e integral

and doing necessary arrangements, we get

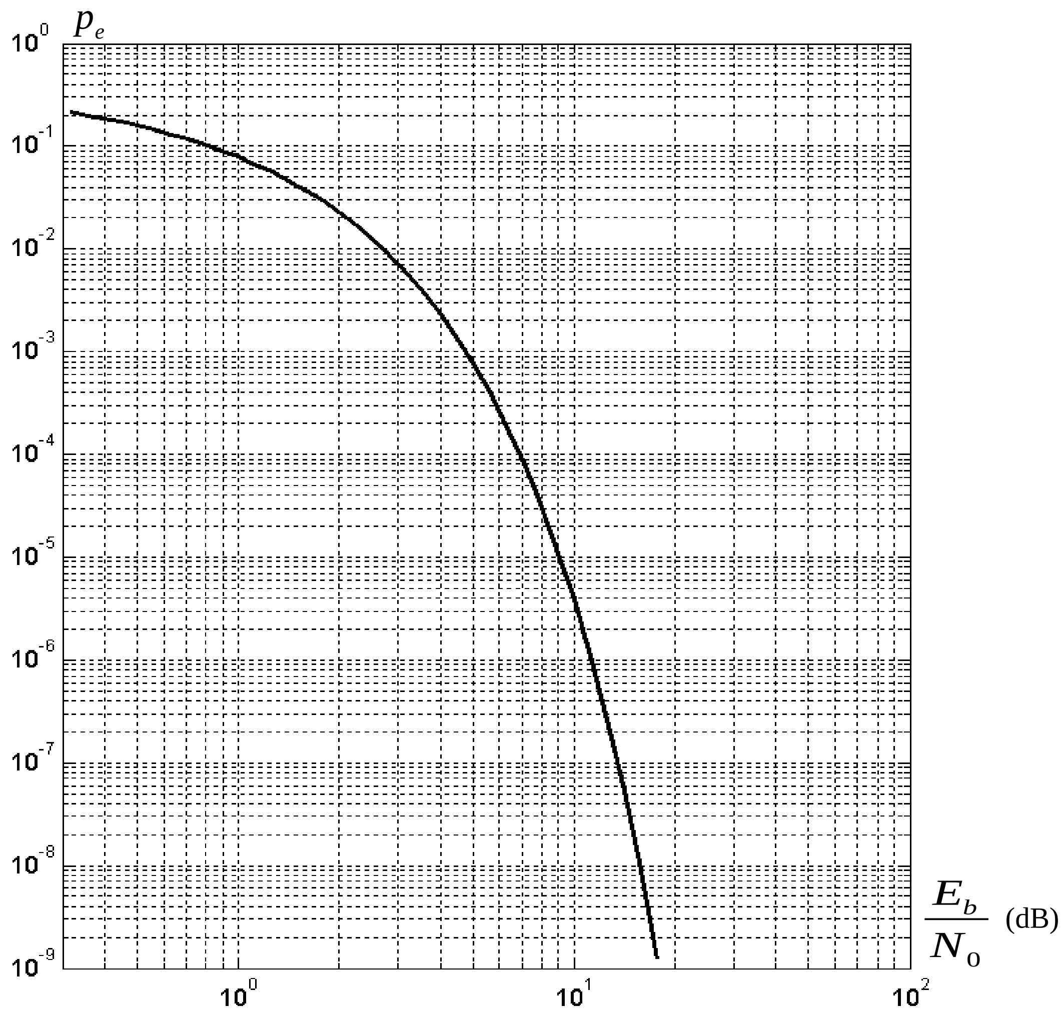
$$p_e = \frac{1}{\sqrt{2\pi}} \int_{\sqrt{2E_b/N_0}}^{\infty} e^{-t^2/2} dt$$

(t is just a variable, not time here)

That is $p_e = Q\left(\sqrt{\frac{2E_b}{N_0}}\right)$

The probability of making an error at the output of the correlator through measurement at the end of T_b, for binary systems that 0 and 1 are represented by two antipodal finite and equal duration waveforms (pulses)

(since we did not assume rectangular pulses, we can say that this is valid for any antipodal waveform pairs. p_e is only dependent on the energy of the pulse, not its shape)

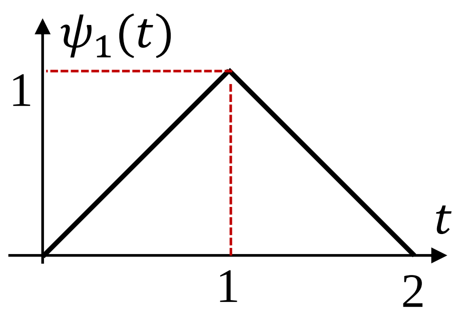


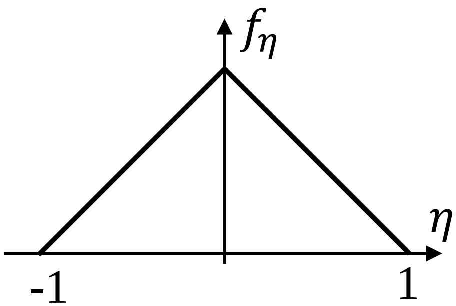
p_e
10^0
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
10^{-7}
10^{-8}
10^{-9}
10^0
10^1
10^2
$\frac{E_b}{N_o}$ (dB)

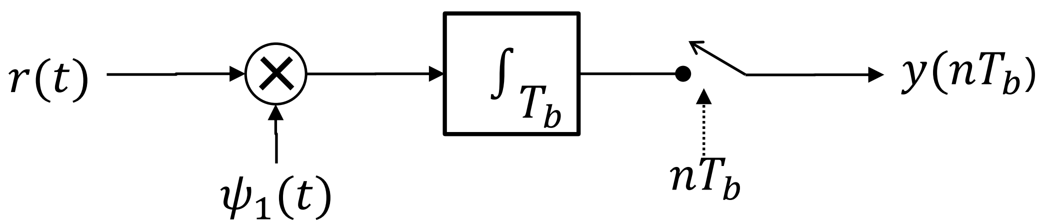
Example

A binary transmission system uses the following waveform and its antipodal counterpart to represent binary 1 and 0 symbols respectively. On the receiver, a correlator receiver is used as shown. The correlator output signal at the fully synchronous measurement times is $y[nT_b] = R[nT_b] + \eta$

η is the noise component whose pdf is also given below.







Calculate the probability of decision error p_e

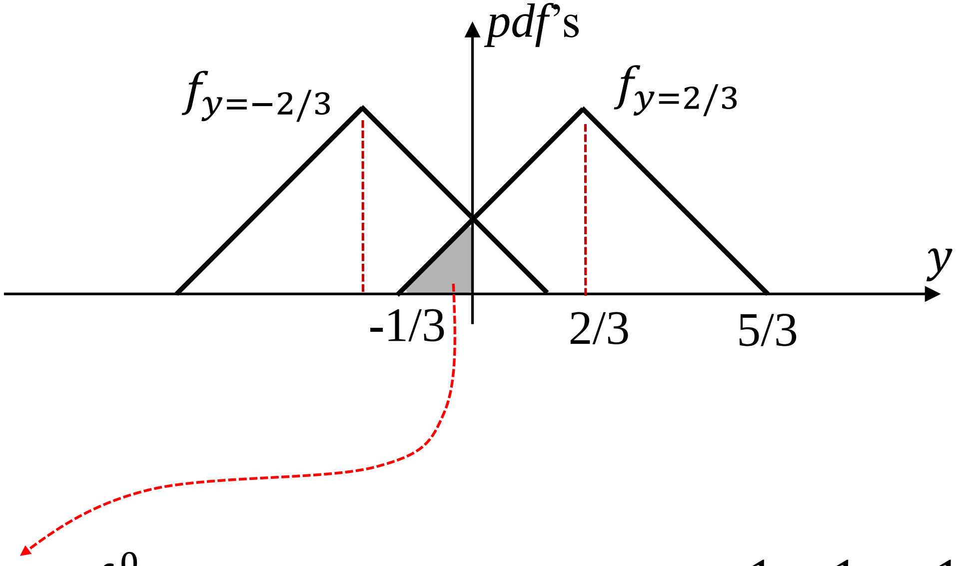
assuming that the system is in full synchronization, symbol transmission probabilities are equal and the channel has no ISI (intersymbol interference).

Solution

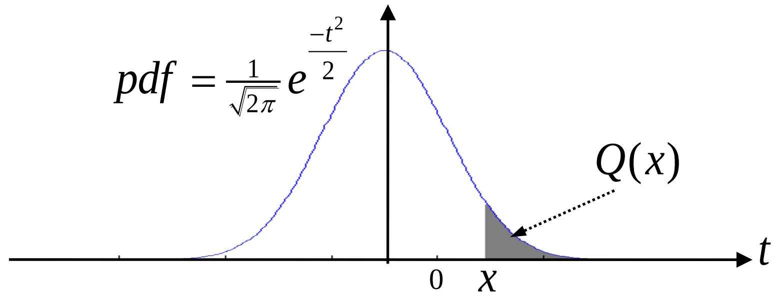
In full synchronization

$$R[nT_b] = \mp\int_0^{T_b}\psi_1^2(t)dt = \mp 2\int_0^1 t^2 dt = \mp\frac{2t^3}{3} = \mp\frac{2}{3}$$

At the decision instant, pdf's of two output possibilities will be as shown



$$p_e = \int_{-1/3}^{0}\left(u+\tfrac{1}{3}\right)du = \left[\tfrac{u^2}{2}+\tfrac{u}{3}\right]_{-1/3}^{0} = \frac{-1}{18}+\frac{1}{9} = \frac{1}{18}$$



$$pdf = \frac{1}{\sqrt{2\pi}} e^{\frac{-t^2}{2}}$$

x	1.	1.5	2.	2.5	3.	3.5	4.	4.5	5.
000	15865525	06680720	02275013	00620967	00134990	00023263	00003167	00000340	00000029
025	15268159	06362955	02143368	00578491	00124317	00021174	00002849	00000302	00000025
050	14685906	06057076	02018222	00538615	00114421	00019262	00002561	00000268	00000022
075	14118736	05762822	01899327	00501200	00105251	00017511	00002301	00000238	00000019
100	13566606	05479929	01786442	00466119	00096760	00015911	00002066	00000211	00000017
125	13029452	05208128	01679331	00433245	00088903	00014448	00001854	00000187	00000015
150	12507194	04947147	01577761	00402459	00081635	00013112	00001662	00000166	00000013
175	11999736	04696712	01481506	00373646	00074918	00011892	00001490	00000147	00000011
200	11506967	04456546	01390345	00346697	00068714	00010780	00001335	00000130	00000010
225	11028761	04226374	01304062	00321507	00062986	00009766	00001195	00000115	00000009
250	10564977	04005916	01222447	00297976	00057703	00008842	00001069	00000102	00000008
275	10115462	03794894	01145296	00276009	00052831	00008000	00000956	00000090	00000007
300	09680048	03593032	01072411	00255513	00048342	00007235	00000854	00000079	00000006
325	09258558	03400051	01003598	00236403	00044209	00006539	00000763	00000070	00000005
350	08850799	03215677	00938671	00218596	00040406	00005906	00000681	00000062	00000004
375	08456572	03039636	00877448	00202014	00036908	00005331	00000607	00000054	00000004
400	08075666	02871656	00819754	00186581	00033693	00004810	00000541	00000048	00000003
425	07707860	02711468	00765419	00172228	00030740	00004336	00000482	00000042	00000003
450	07352926	02558806	00714281	00158887	00028029	00003908	00000429	00000037	00000003
475	07010627	02413407	00666181	00146494	00025543	00003519	00000382	00000033	00000002

END