Modulation

part 1

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For the course “**Communications**”

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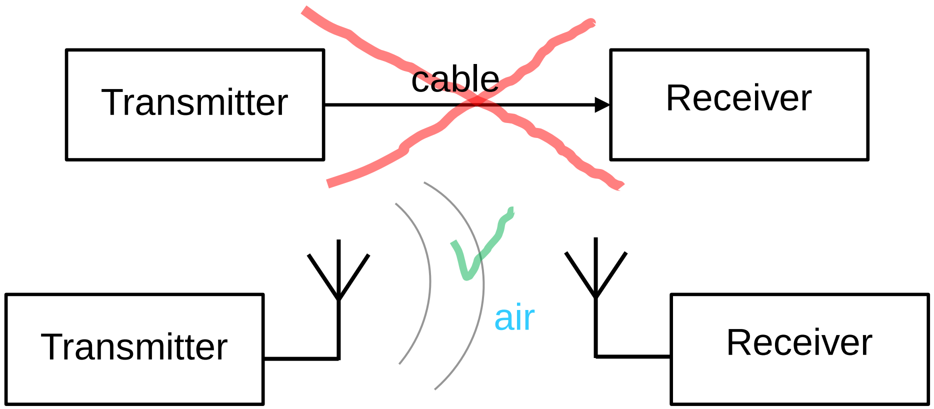
Initial Problem : Carry message signal over distances without using cable

The Solution : Radiate its electromagnetic wave through air
and pick this up at the receiver locations



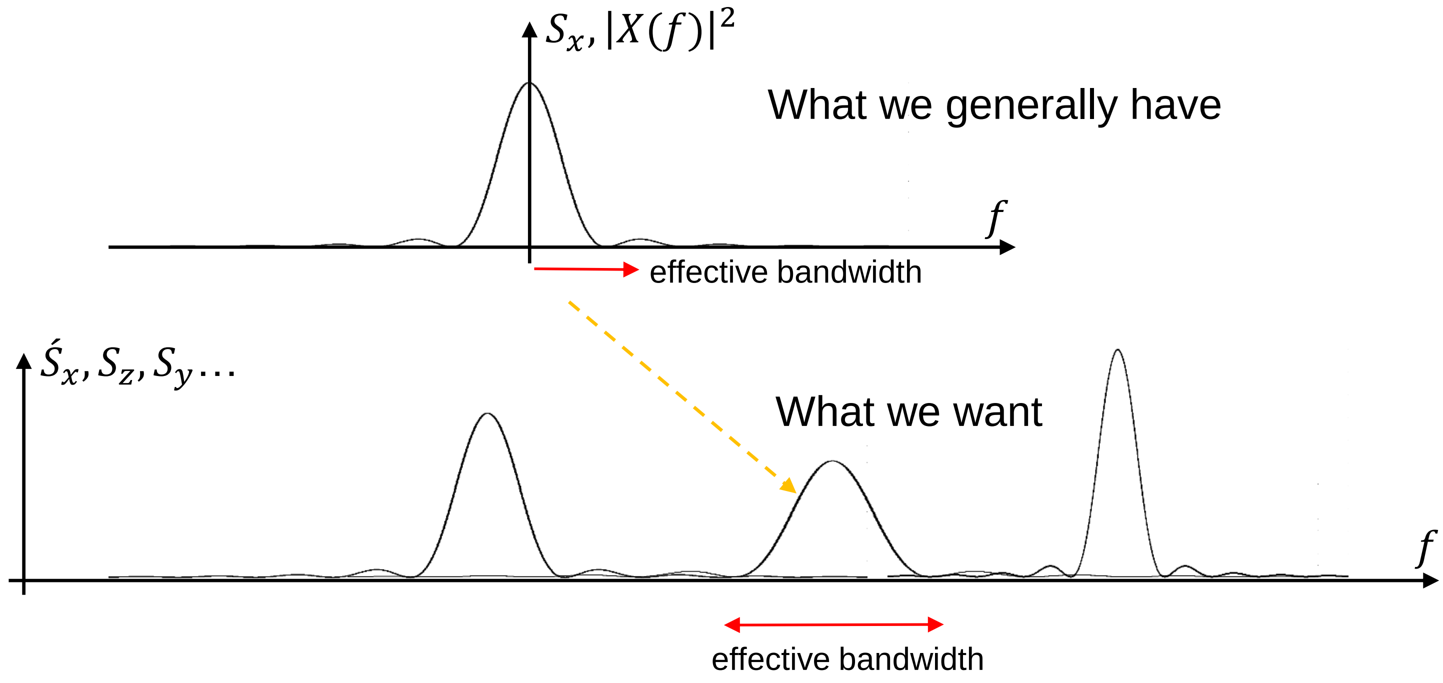
Resulting Problems :

1. There can only be one EM-wave in the air since the receiver(s) pick(s) up all.
2. Transmitter antenna must be very long when message signal has low frequencies.

Hints : EM-spectrum is much larger compared to effective bandwidth of the signal. So, the spectrum can be shared by multiple transmitters, but all baseband signals are initially have the same/overlapping frequency band.

Solution : Move frequency bands of the signals to unused spectrum area.
How : modulation

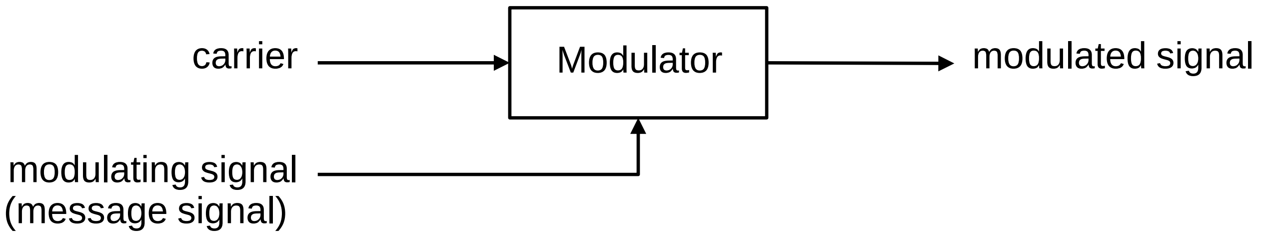
Example



So that, the receiver can extract the signal using filters.
Inherently, one media can be used by multiple message signals, albeit some interference from neighboring signals.

Modulation

Modulation : “controlling a physical quantity using another physical quantity”



How : Change one or more properties of EM-wave by the message signal

modulated signal

carrier

$$\psi(r,t) = A cos(kr + \omega t + \varphi)$$

Polarity

Amplitude

Frequency

Phase

Possible Changes

$$y(t) = A\cos(\omega_c t + \phi)$$

Vary this with the message, you get Amplitude Modulation (AM)

If there are finite number of amplitude values, it is called Amplitude Shift Keying (ASK)

If both amplitude and phase modulation are used at the same time, it is called Quadrature Amplitude Modulation (QAM) Digital version is also called QAM.

Vary this with the message signal, you get Phase Modulation (PM)

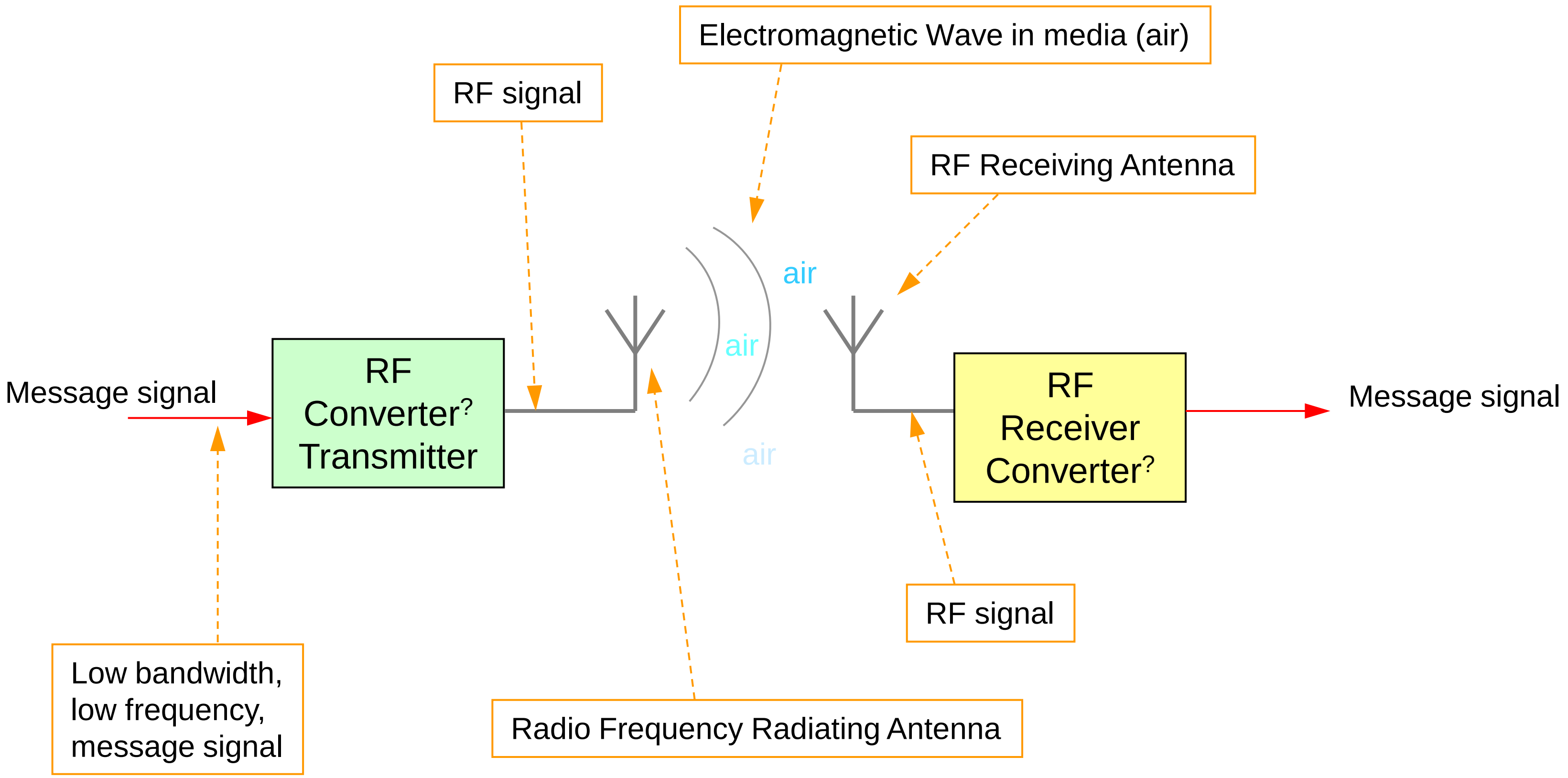
If there are finite number of phase values, it is called Phase Shift Keying (PSK)

Vary this with the message signal, you get Frequency Modulation (FM)

If there are finite number of frequency values, it is called Frequency Shift Keying (FSK)

In AM, amount of carrier and sidebands in the frequency spectrum determines the modulation type : SSB, SSB-SC, DSB, DSB-SC, Conventional AM, VSB and their sub-types. Other modulation schemas have similar variations too.

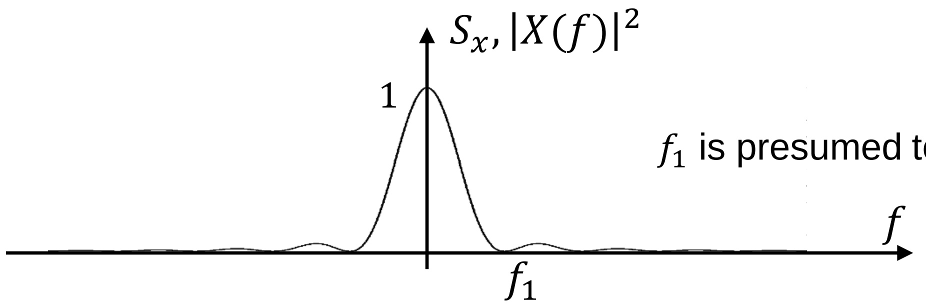
Summary of AM Transmission



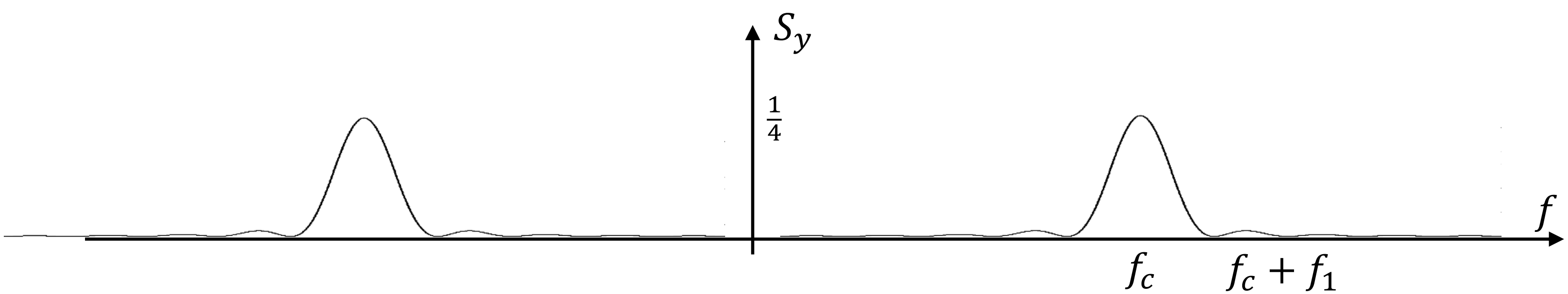
Modulation property of Fourier Transform

in general, if $x(t) \Leftrightarrow X(f)$ and $c(t) \Leftrightarrow C(f)$ then $x(t)c(t) \Leftrightarrow X(f) * C(f)$

specifically, if $c(t) = \cos(2\pi f_c t)$ then $x(t)\cos(2\pi f_c t) \Leftrightarrow \frac{1}{2}X(f - f_c) + \frac{1}{2}X(f + f_c)$

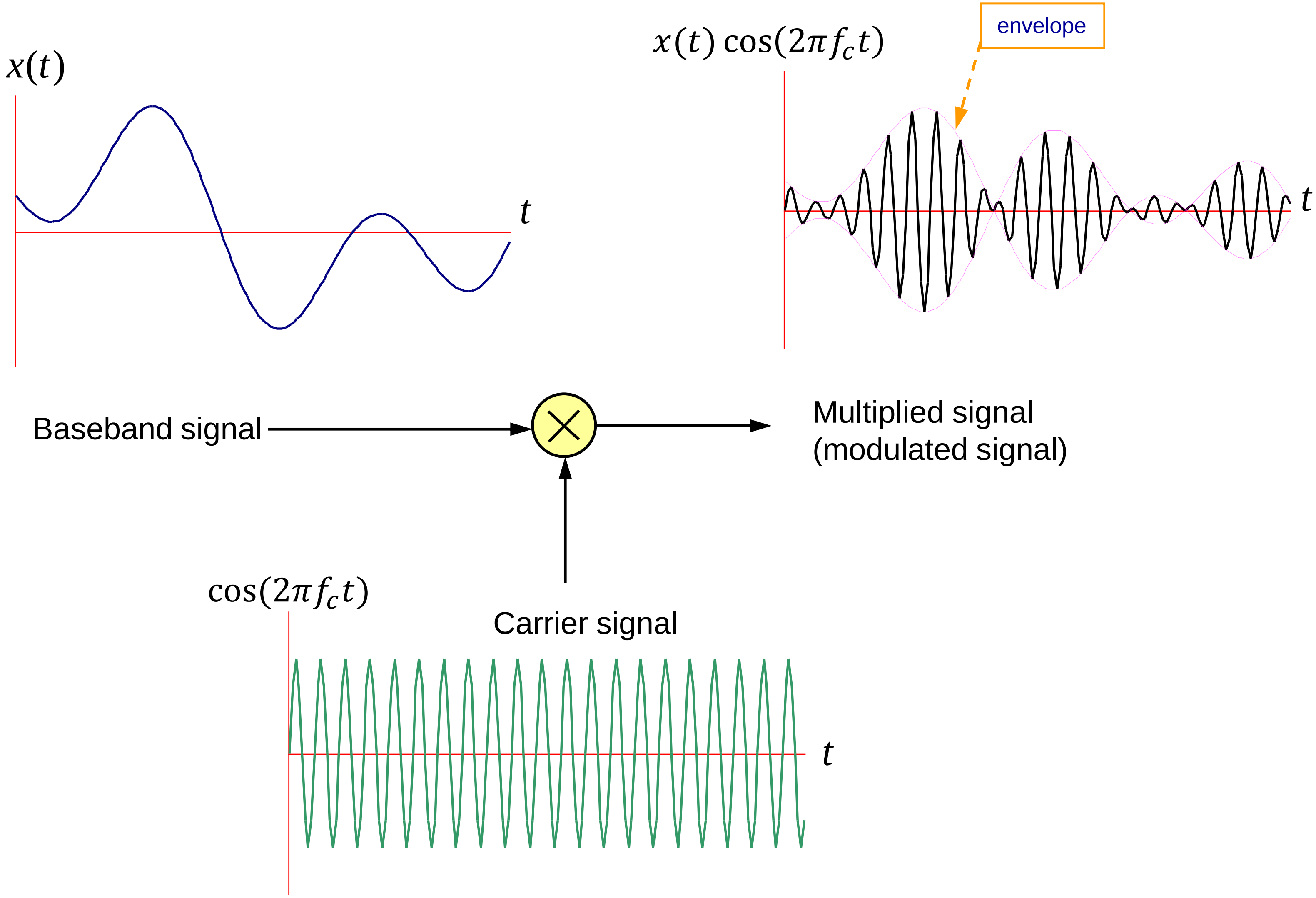


f_1 is presumed to be cutoff freq. of $x(t)$



$\cos(2\pi f_c t)$ is called the carrier signal or carrier. f_c is called carrier frequency

In time domain with a bandlimited signal



Example : Find/draw $\mathcal{F}\{x(t)\cos(2\pi f_c t)\}$ for $x(t) = \sin(2\pi f_m t)$ where $f_c \gg f_m$

Solution $X(f) = \mathcal{F}\{\sin(2\pi f_m t)\} = \frac{j}{2}\left(\delta(f + f_m) - \delta(f - f_m)\right)$

$$Y(f) = \mathcal{F}\{x(t)\cos(2\pi f_c t)\} = \tfrac{1}{2}X(f + f_m) + \tfrac{1}{2}X(f - f_m)$$

$$Y(f) = \underbrace{\frac{j}{2}\delta(f - f_c + f_m)}_{\text{I}} - \underbrace{\frac{j}{2}\delta(f - f_c - f_m)}_{\text{II}} + \underbrace{\frac{j}{2}\delta(f + f_c + f_m)}_{\text{III}} - \underbrace{\frac{j}{2}\delta(f + f_c - f_m)}_{\text{IV}}$$

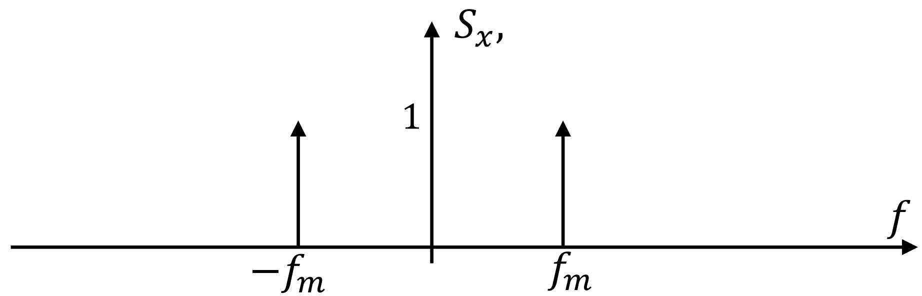
$$\text{I} + \text{IV} = \frac{j}{2}\delta(f - f_c + f_m) - \frac{j}{2}\delta(f + f_c - f_m) \Leftrightarrow -\tfrac{1}{2}\sin(2\pi(f_c - f_m)t)$$

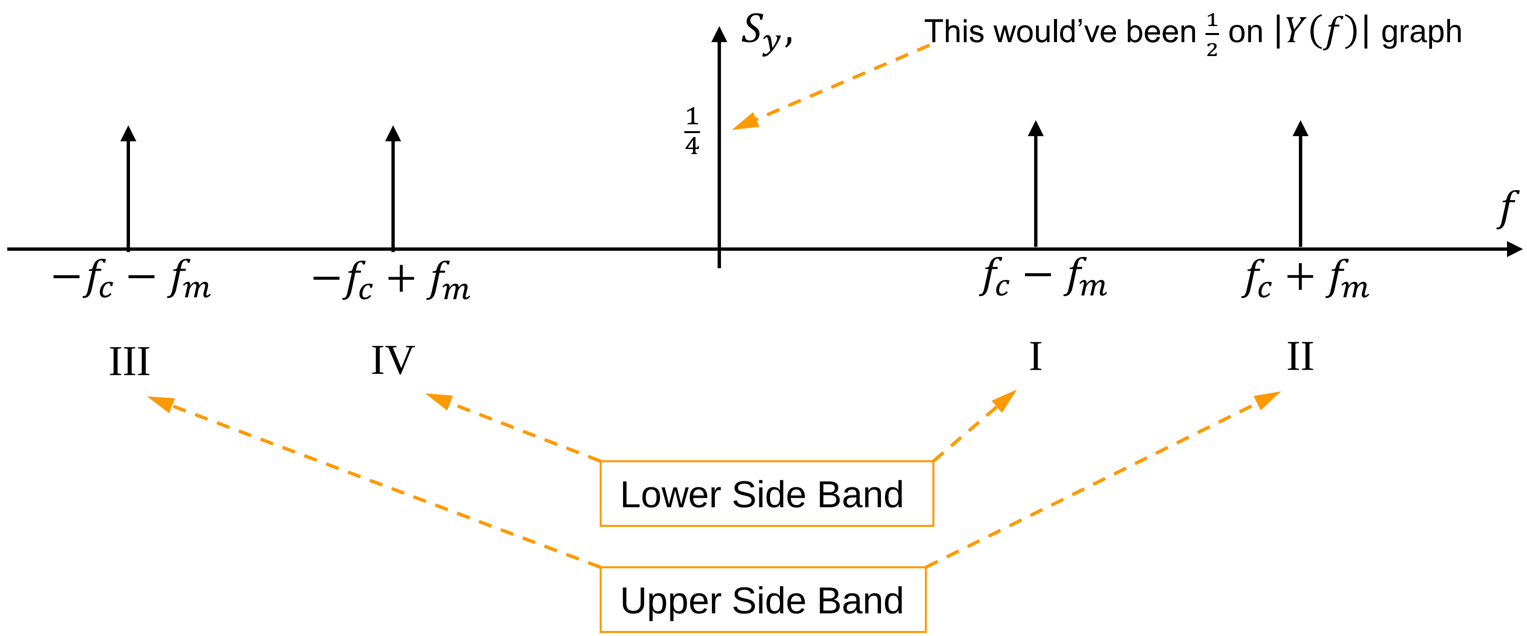
lower frequency components

$$\text{II} + \text{III} = \frac{j}{2}\delta(f + f_c + f_m) - \frac{j}{2}\delta(f - f_c + f_m) \Leftrightarrow \tfrac{1}{2}\sin(2\pi(f_c + f_m)t)$$

upper frequency components

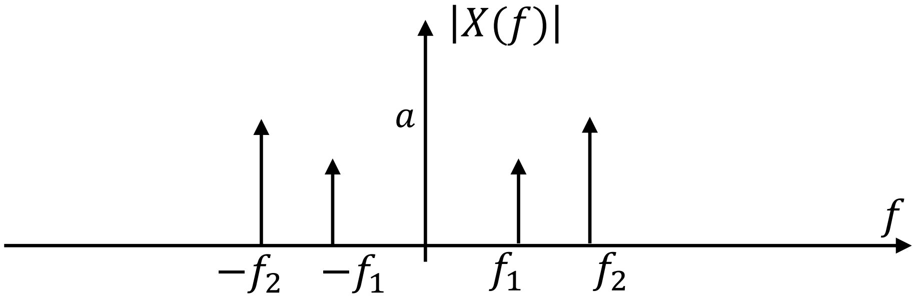
Entire signal $y(t) = \frac{1}{2}\sin(2\pi(f_c + f_m)t) - \frac{1}{2}\sin(2\pi(f_c - f_m)t)$

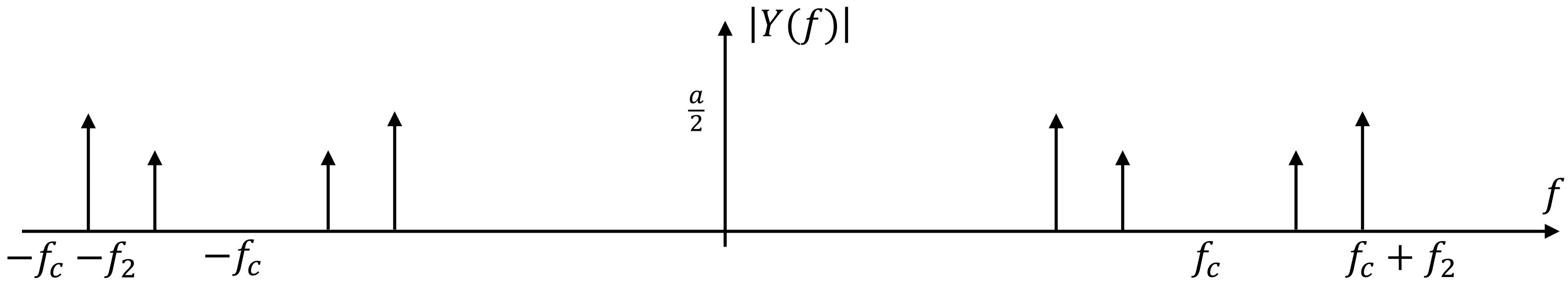




Example : Find the modulated signal $m(t)$ and its Fourier spectrum for

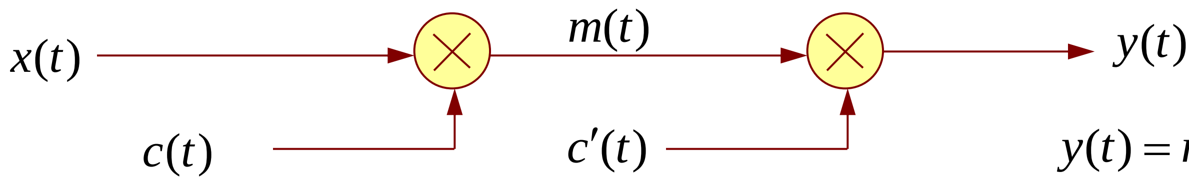
$x(t) = A_1 cos(2\pi f_1 t) + A_2 cos(2\pi f_2 t + \varphi)$ and $c(t) = A_c cos(2\pi f_c t)$





corollary : duplicate the spectrum at f_c and $-f_c$ in order to find the resulting spectrum

Let us apply the same multiplication operation on the modulated signal $m(t)$



$$y(t) = m(t)c'(t)$$

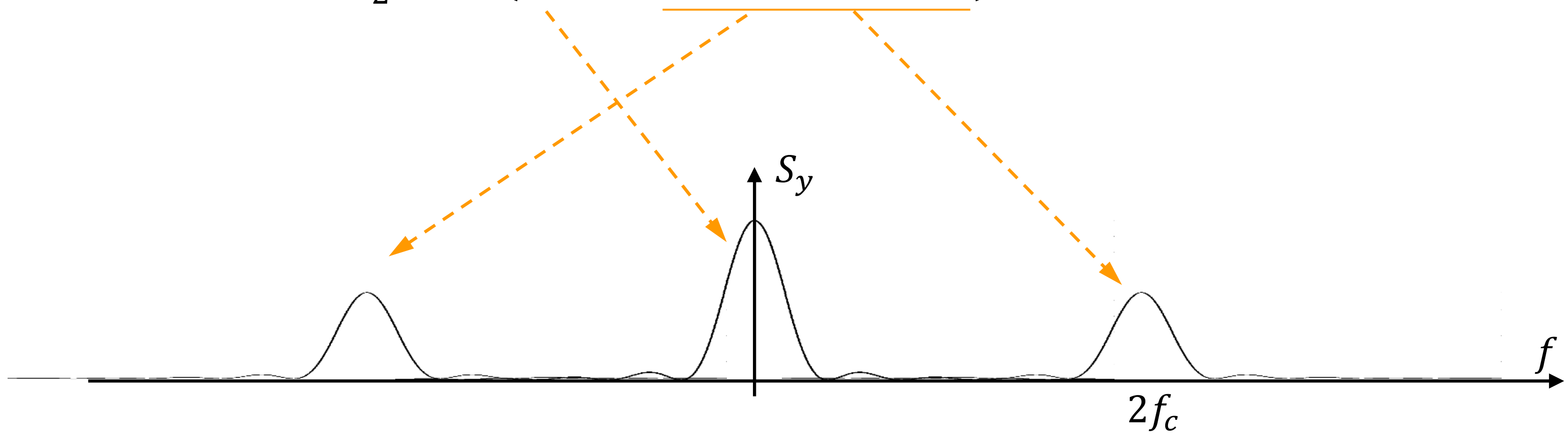
$$m(t) = x(t)c(t)$$

if $c(t) = A_c cos(2\pi f_c t)$ and $\acute{c}(t) = A'_c cos(2\pi f_c t)$

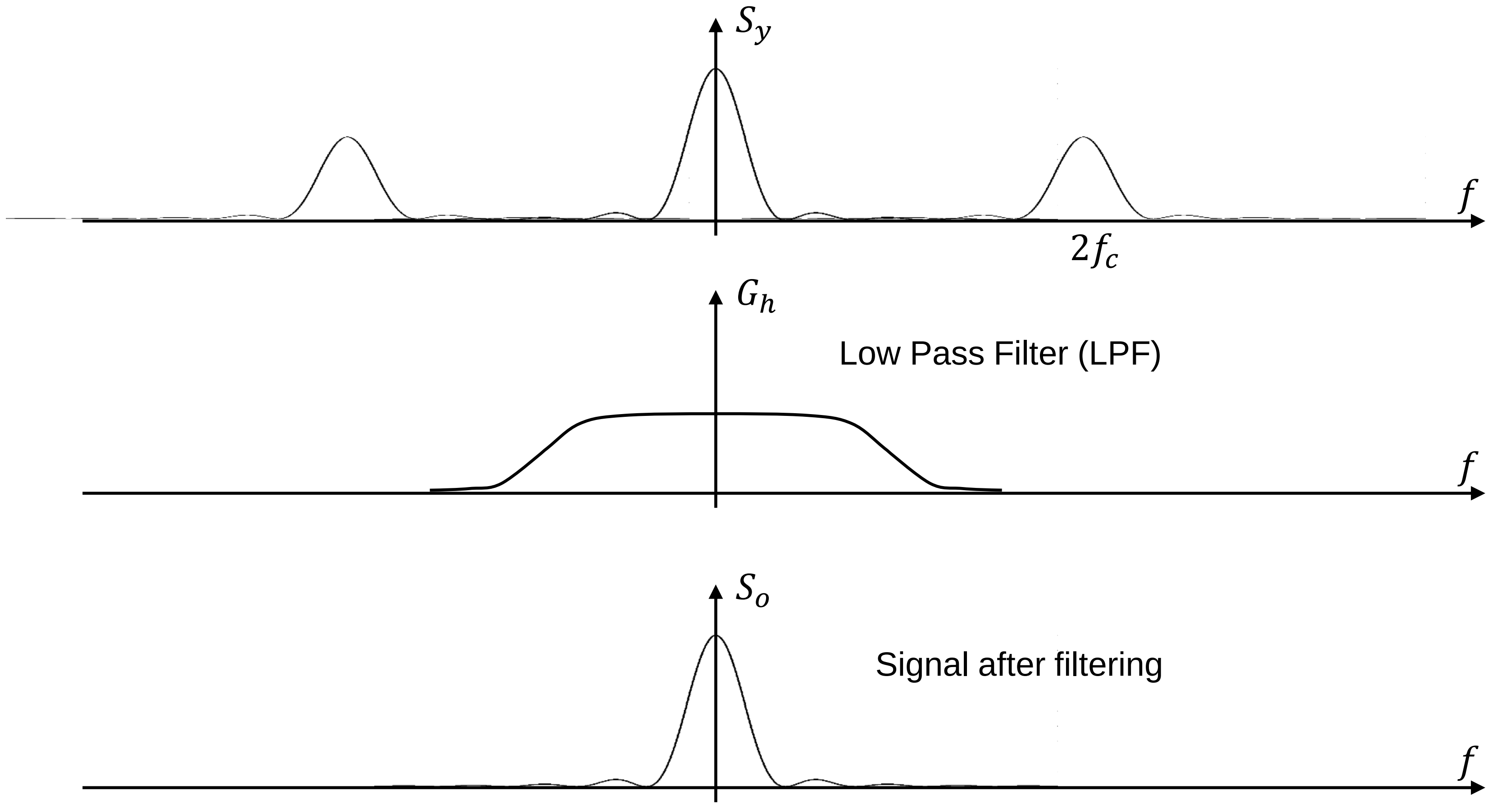
then $y(t) = x(t) A_c A'_c cos^2(2\pi f_c t)$

(using $cos^2(u) = \big(1 + cos(2u)\big)/2$)

$$y(t) = \tfrac{1}{2} A_c A'_c \big(x(t) + x(t)cos^2(2\pi f_c t)\big)$$

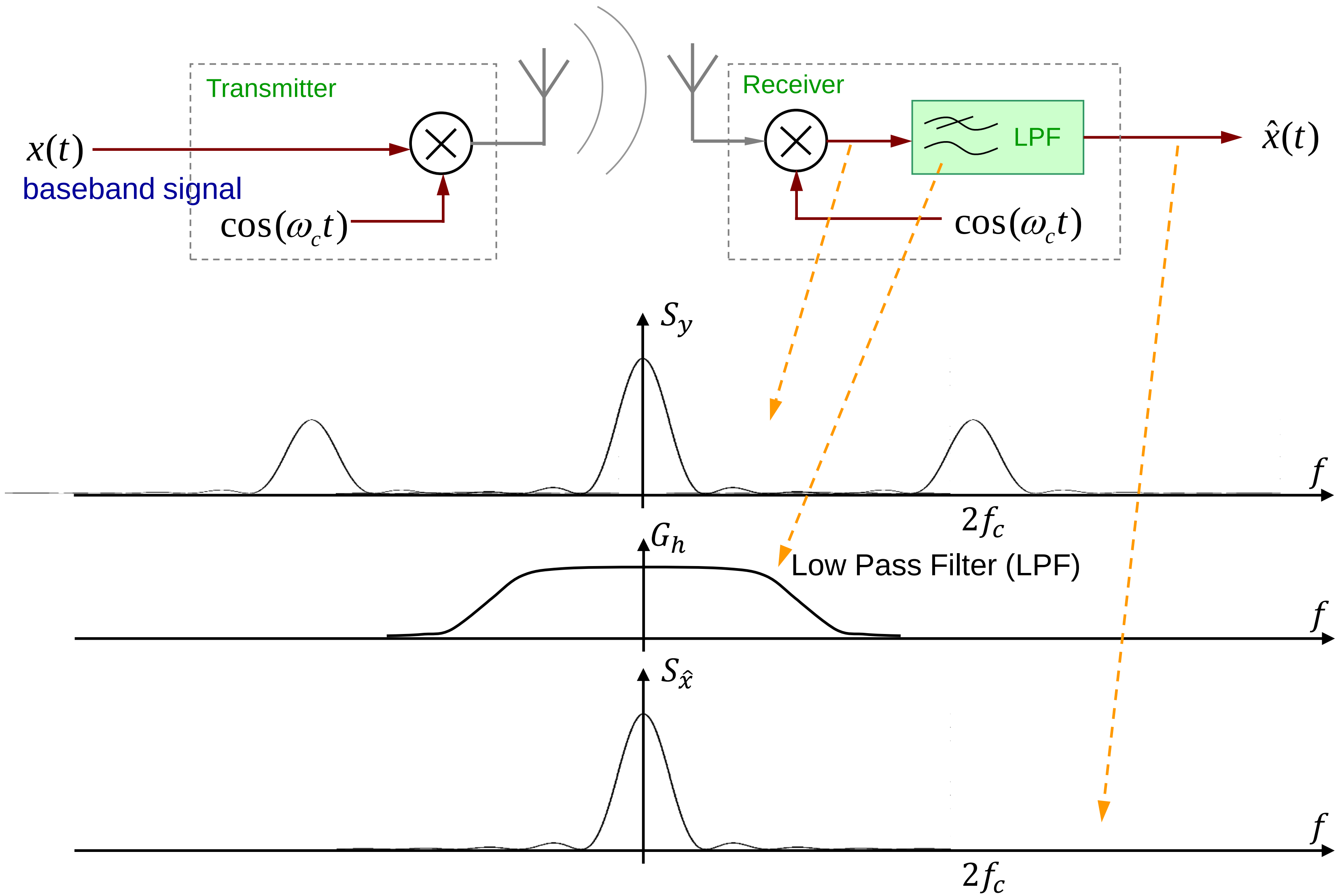


We can now filter/remove unwanted HF components and obtain $x(t)$



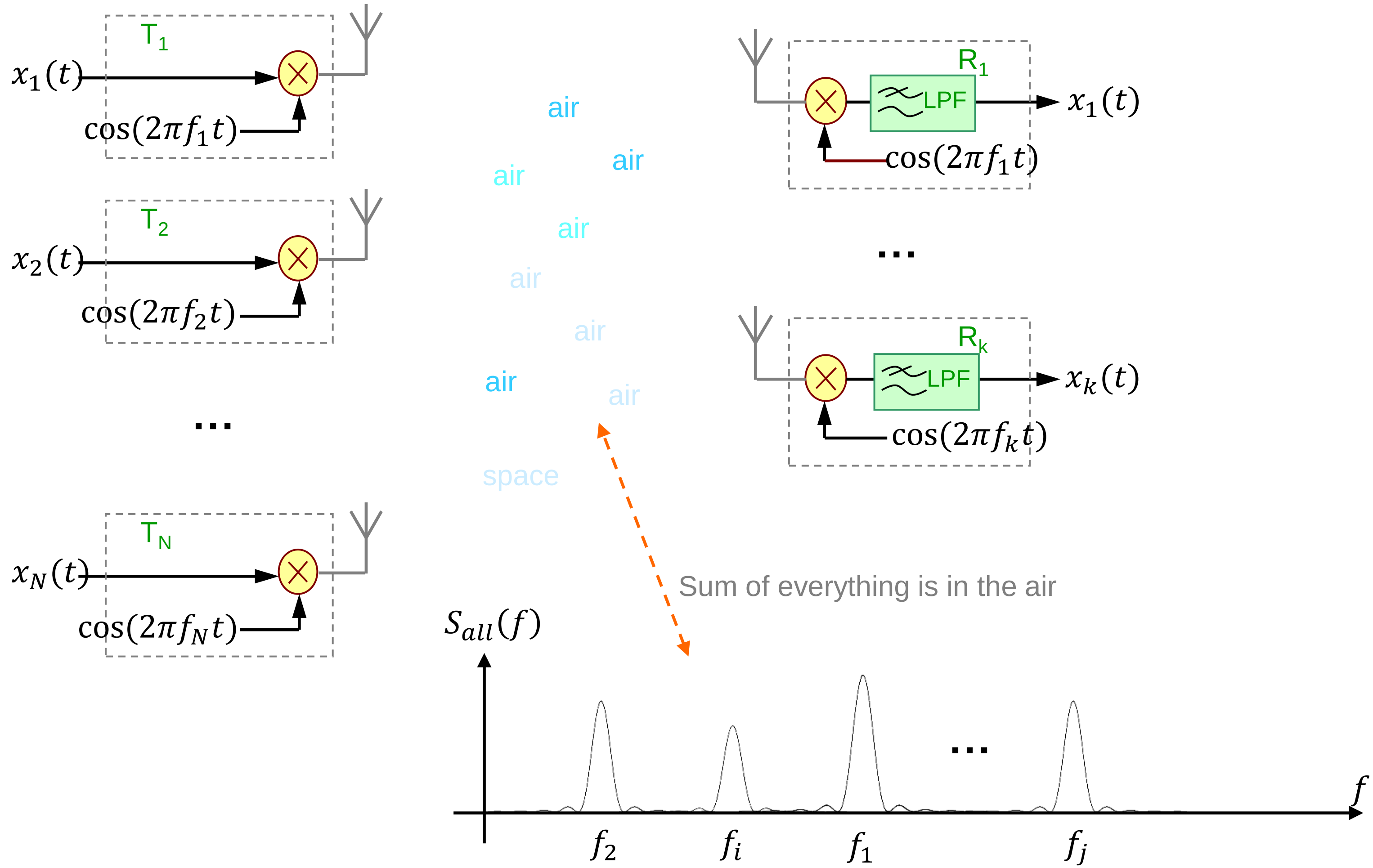
The Problem to Solve Now : How do we obtain the original carrier at the receiver side?

Summary, so far



This is called synchronous demodulation

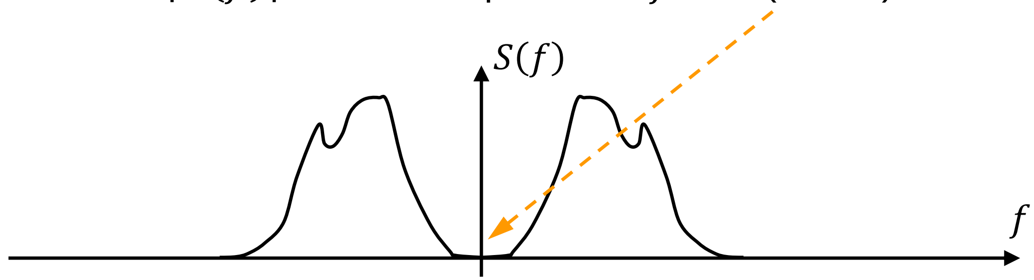
Multiple Transmitters and Receivers



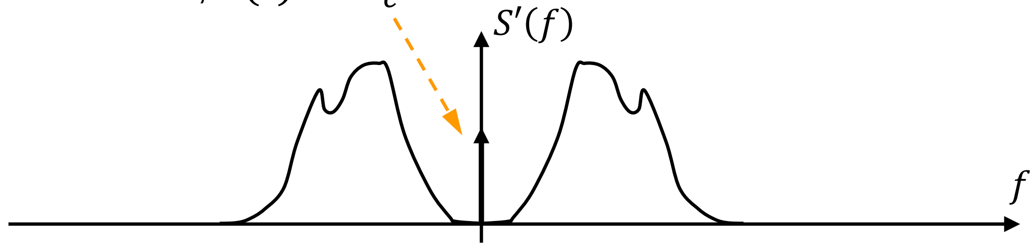
Each receiver can adjust its own center frequency to pick up anyone of the signals

Problem is : how to create $\cos(2\pi f_c t)$ at the receiver in phase with the transmitter oscillator

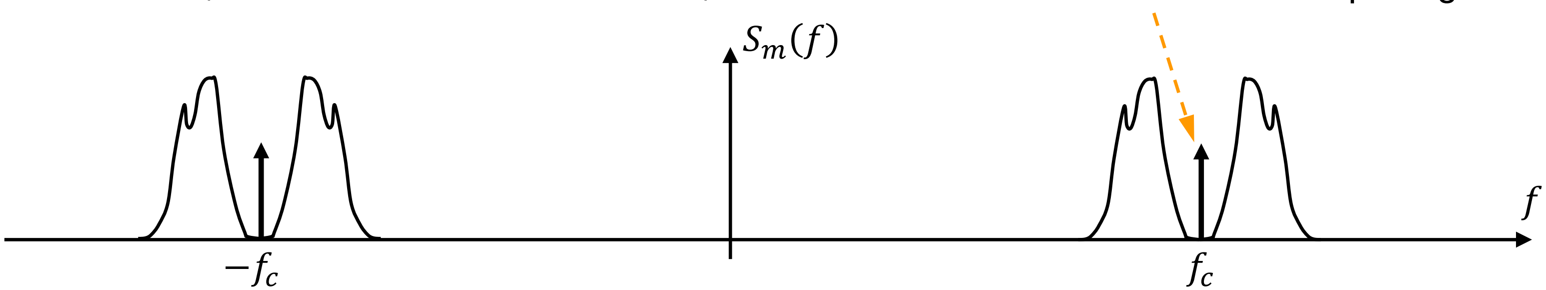
Now, let us assume that $|X(f)|$ has no component at $f = 0$ (no DC) and near (like voice signal)



So that we can add a DC value; $x(t) + m_c$

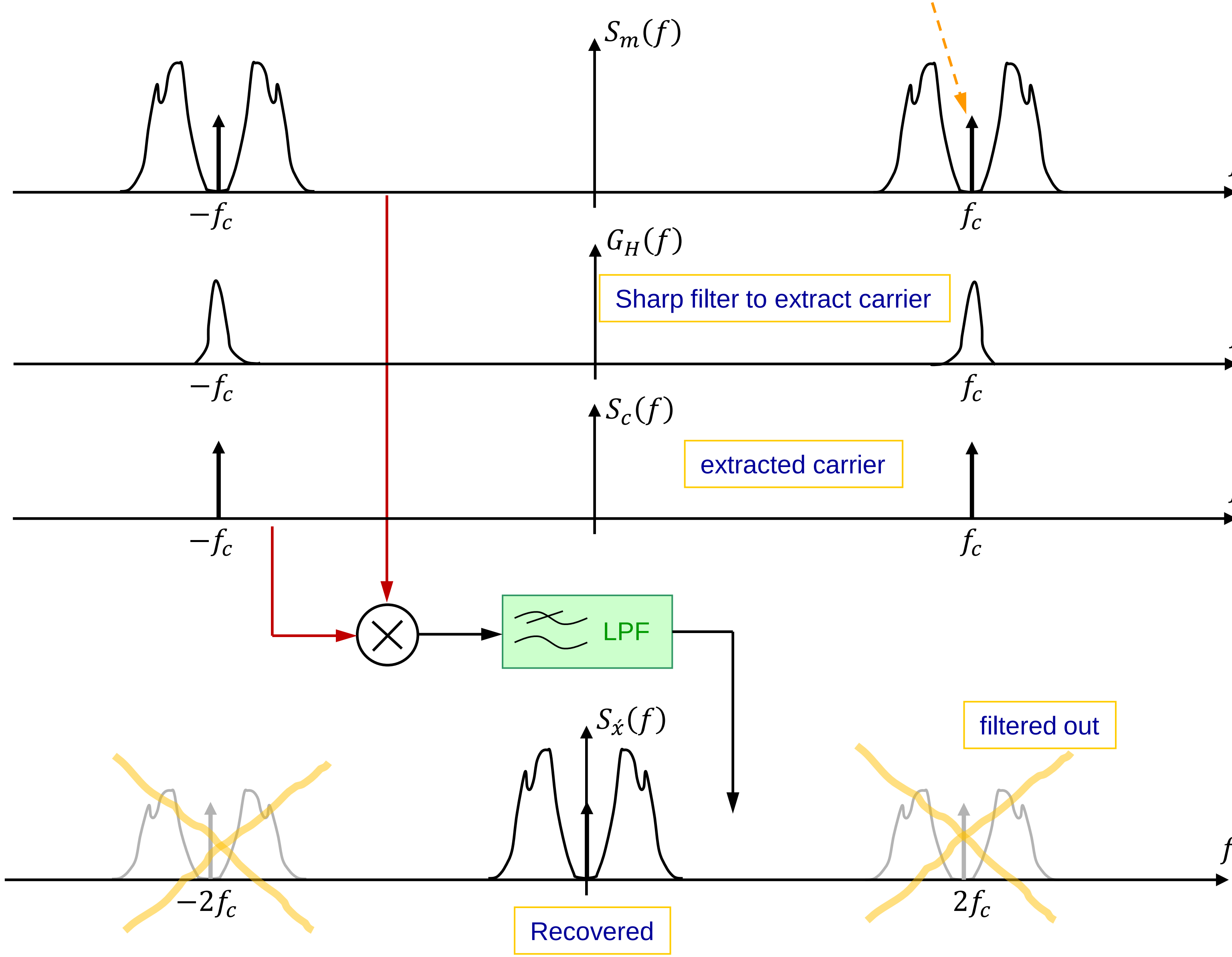


So that, when it modulates the carrier, we would have some carrier in the output signal

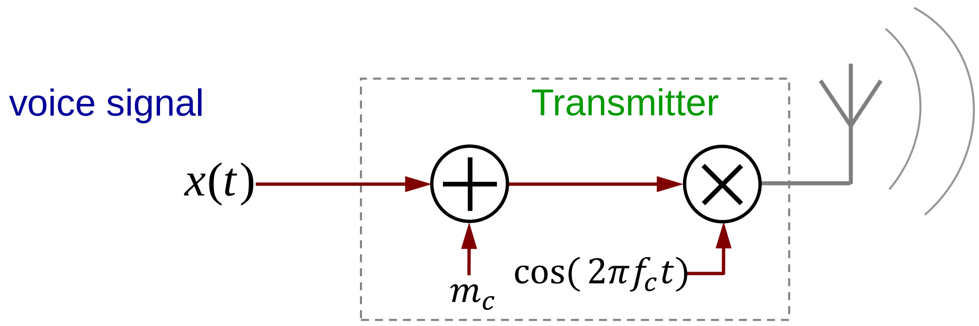


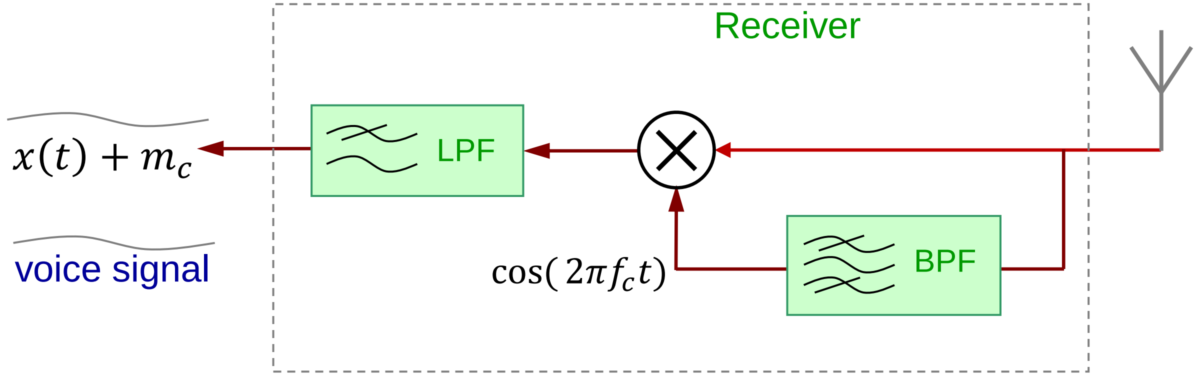
which can be extracted at the receiver to obtain a synchronous carrier

Synchronous demodulation is easier now since we have a carrier signal to extract and use



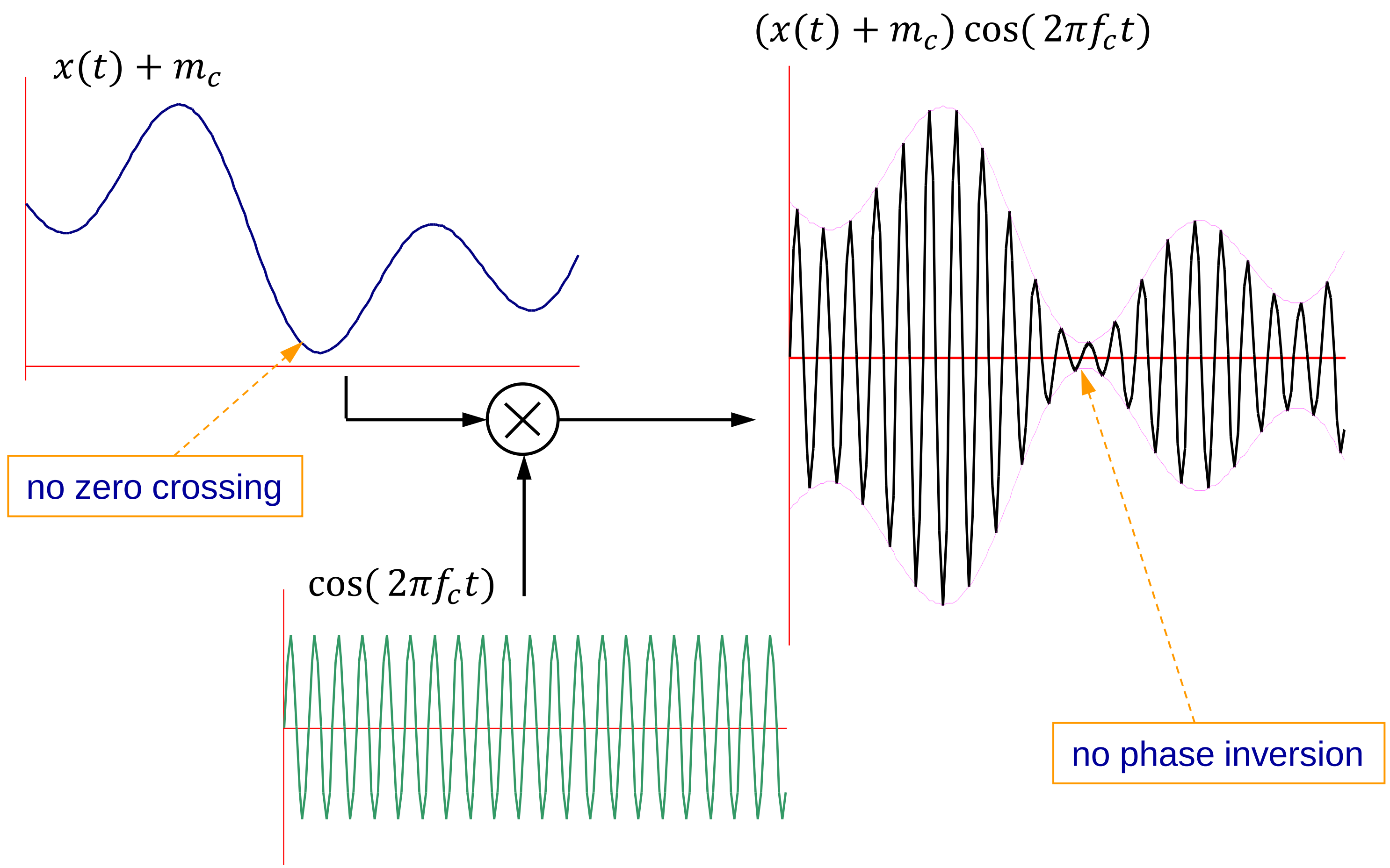
Summary, so far (simplest synchronous radio transmission/reception)





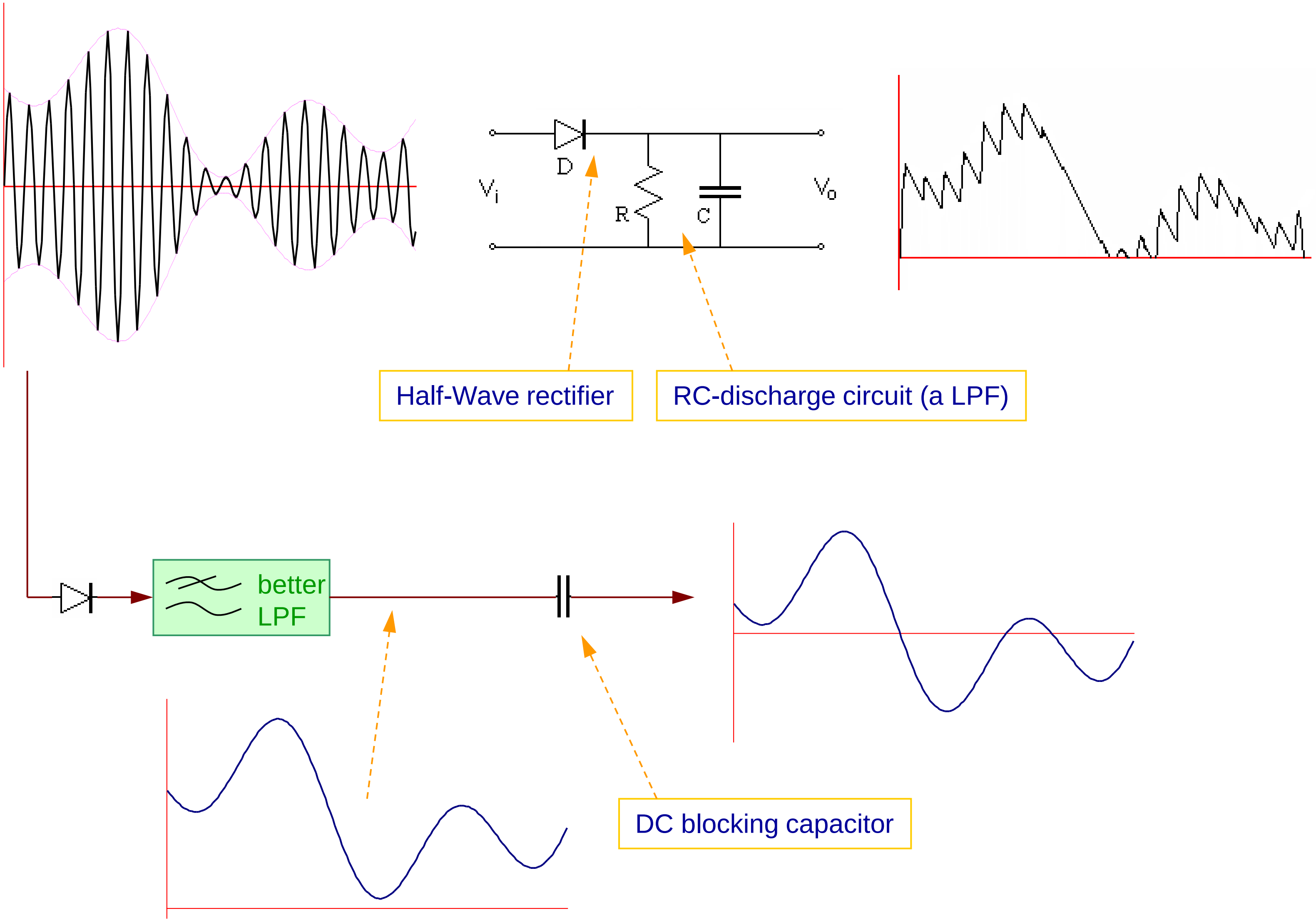
We should remove DC component too, using …

What if we add enough DC so that the sum $x(t) + m_c$ is always positive?



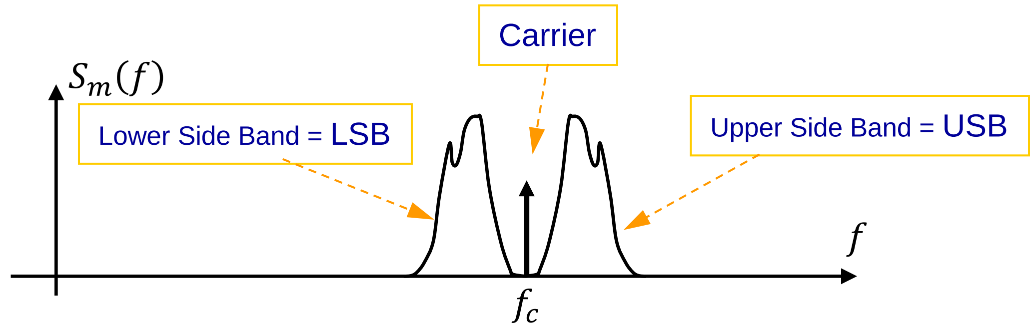
Synchronous demodulation is still possible, but we can do another thing…

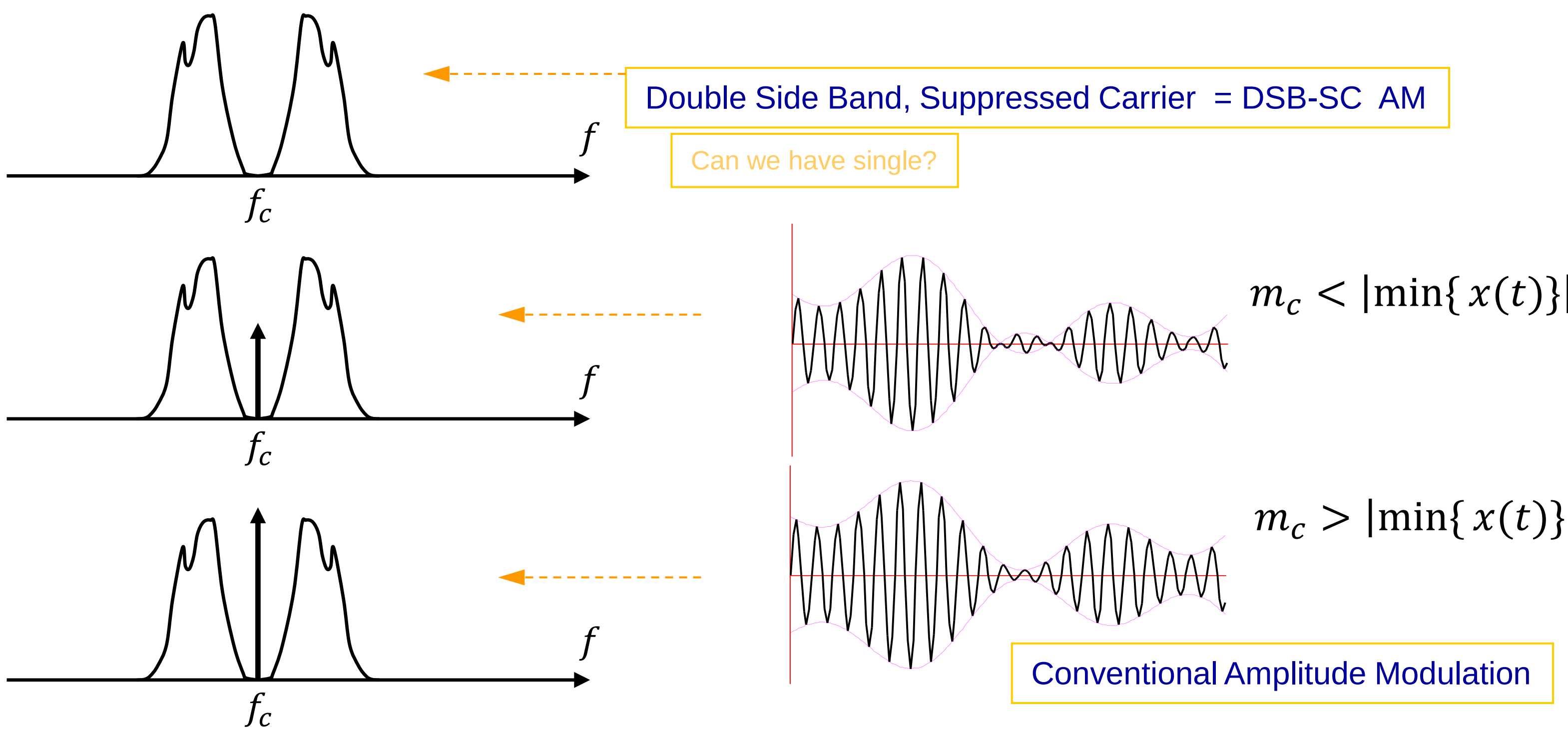
Another Way to Demodulate AM Signal



Note : Better LPF may not be enough. Much higher carrier frequency than illustrated would clearly improve the performance

We notice that normal AM signal carries the same information twice





In general $y(t) = A(1 + a_m x_n(t)) \cos(\omega_c t + \varphi)$

modulation index

normalized signal

$$x_n(t) = \frac{x(t)}{\max|x(t)|}$$ so that $-1 < x_n(t) < 1$

or

$$y(t) = K(m_c + x(t)) \cos(\omega_c t + \varphi)$$

in order for $x(t) + m_c > 0$ $m_c > |\min\{x(t)\}|$

$$a_m = \frac{|\min\{x(t)\}|}{m_c}$$

larger a_m smaller carrier power to signal power ratio

Carrier Power $P_c =$ mean square of $m_c \cos(2\pi f_c t) = \dfrac{m_c^2}{2}$

Sideband Power $P_s =$ mean square of $x(t)\cos(2\pi f_c t) = \frac{1}{2}$ mean square of $x(t) = \dfrac{1}{2}\overline{x^2(t)}$

Power of a single sideband $P_U = P_L = \dfrac{1}{4}\overline{x^2(t)}$

Total Power $P_T = P_s + P_c = \dfrac{1}{2}\left(m_c^2 + \overline{x^2(t)}\right)$

define $\eta = \dfrac{P_s}{P_s + P_c}(\times 100) = \dfrac{\overline{x^2(t)}}{m_c^2 + \overline{x^2(t)}}(\times 100)$ as efficiency

for pure sinusoidal message signal $x(t) = a_m m_c \cos(\omega_m t)$ $\overline{x^2(t)} = \dfrac{(a_m m_c)^2}{2}$

and $\eta = \dfrac{a_m^2}{2 + a_m^2}(\times 100), \quad a_m \leq 1$

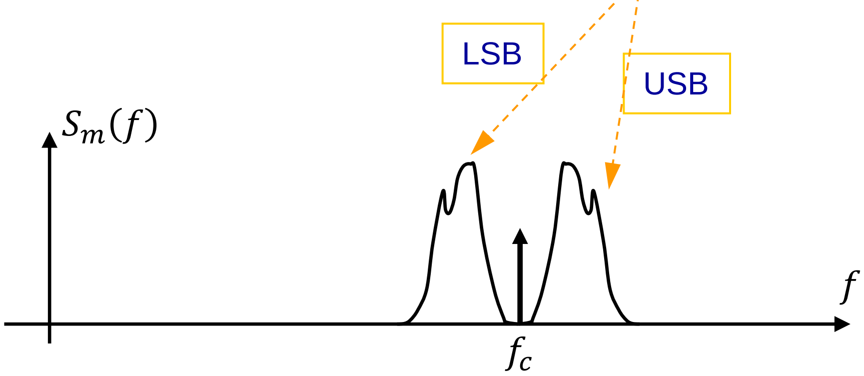
at $a_m = 1$ (best case) $\eta = \eta_{\max}$

For conventional AM two thirds of power is wasted at best. That is, if we want to send 1 then we have to spend additional 2. So, why do we use conventional AM instead of other versions of AM (DSB-SC for example)?

Question : Why do we use conventional AM instead of other versions of AM (DSB-SC for example) even though we know the power disadvantage ?
Simple Answer : Receiver is simpler and cheaper (explain)

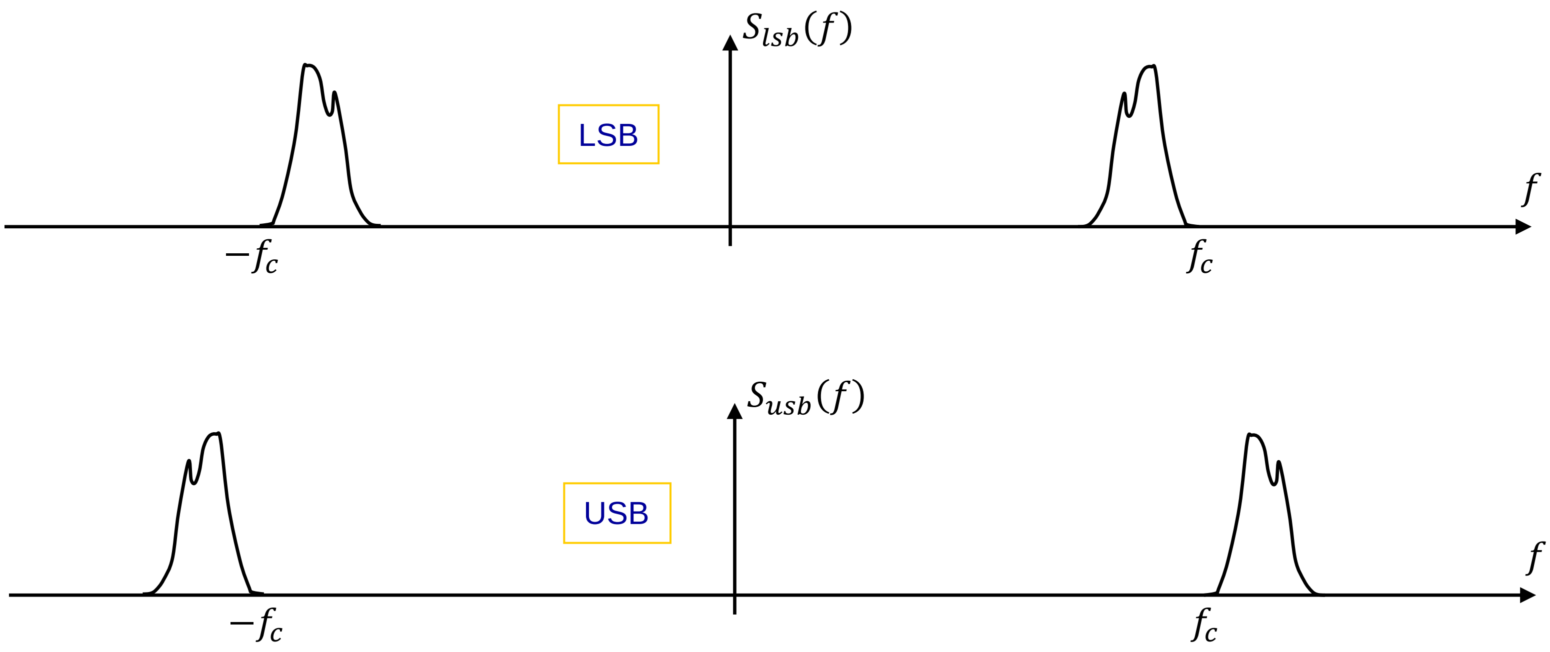
Notice that information within USB and LSB are identical

for real signals



Then, is it enough to transmit only one side to save power (and have the info transmitted of course)?

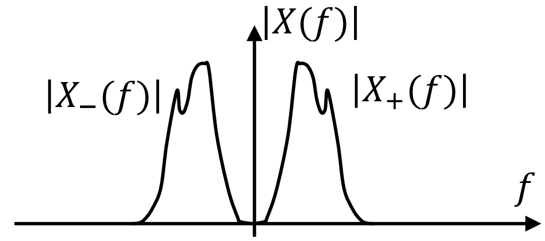
Single Side Band Suppressed Carrier AM



The power advantages are obvious. The question is; **how** do we generate these **SSB-SC AM** signals?

Let us assume that $x(t) = x_-(t) + x_+(t)$

so that $x_-(t) = x_+^*(t)$



It can be written that $x_-(t) = \frac{1}{2}\left[x(t) - jx_h(t)\right]$

and $x_+(t) = \frac{1}{2}\left[x(t) + jx_h(t)\right]$

$$X_+(f) = X(f)U(f)$$

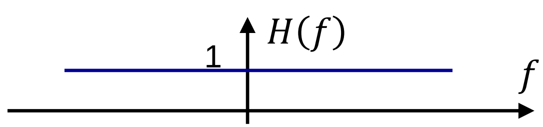
$$X_+(f) = \frac{1}{2}X(f) + \frac{1}{2}X(f)\,\mathrm{sgn}(f)$$

$jx_h(t) \Leftrightarrow X(f)\,\mathrm{sgn}(f)$ or $X_h(f) = -jX(f)\,\mathrm{sgn}(f)$

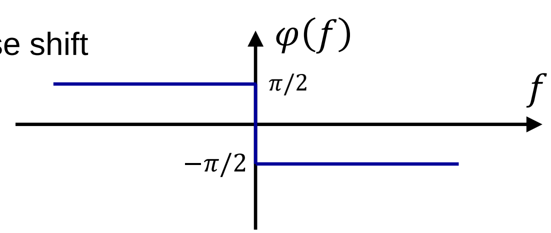
We know that (from tables) $\frac{j}{\pi t} \Leftrightarrow \mathrm{sgn}(f)$ and $\mathcal{F}\{x(t) * y(t)\} = X(f)Y(f)$ (convolution)

Hilbert Transform

$$x_h(t) = \mathrm{F}^{-1}\{-jX(f)\,\mathrm{sgn}(f)\} = \frac{1}{\pi}\int_{-\infty}^{\infty} \frac{x(\alpha)}{t-\alpha}\,d\alpha$$



ideal $\pi/2$ phase shift



$$y_{\mathrm{USB}}(t) = A_c x(t)\cos(\omega_c t) - A_c \hat{x}(t)\sin(\omega_c t)$$

$$y_{\mathrm{LSB}}(t) = A_c x(t)\cos(\omega_c t) + A_c \hat{x}(t)\sin(\omega_c t)$$

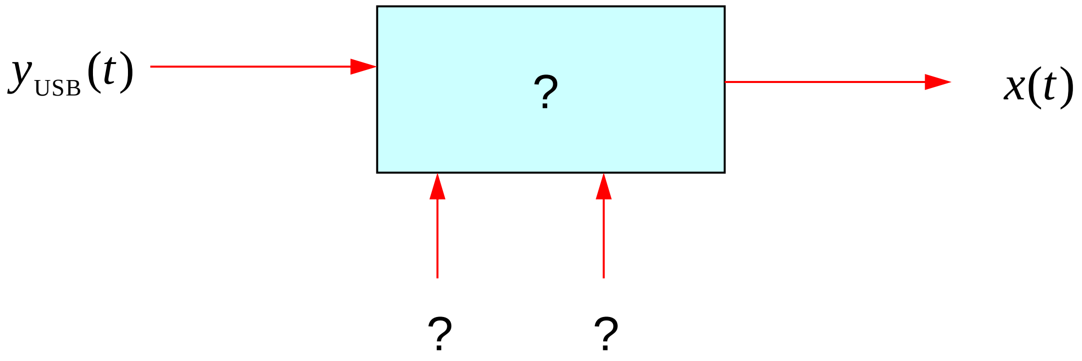
Example : Find $y_{USB}(t)$ for $x(t) = \cos(2\pi f_m t)$ where $f_c >> f_m$

Solution $\hat{x}(t)$ is 90 degrees phase shifted version of $x(t)$ So $\hat{x}(t) = \sin(2\pi f_m t)$

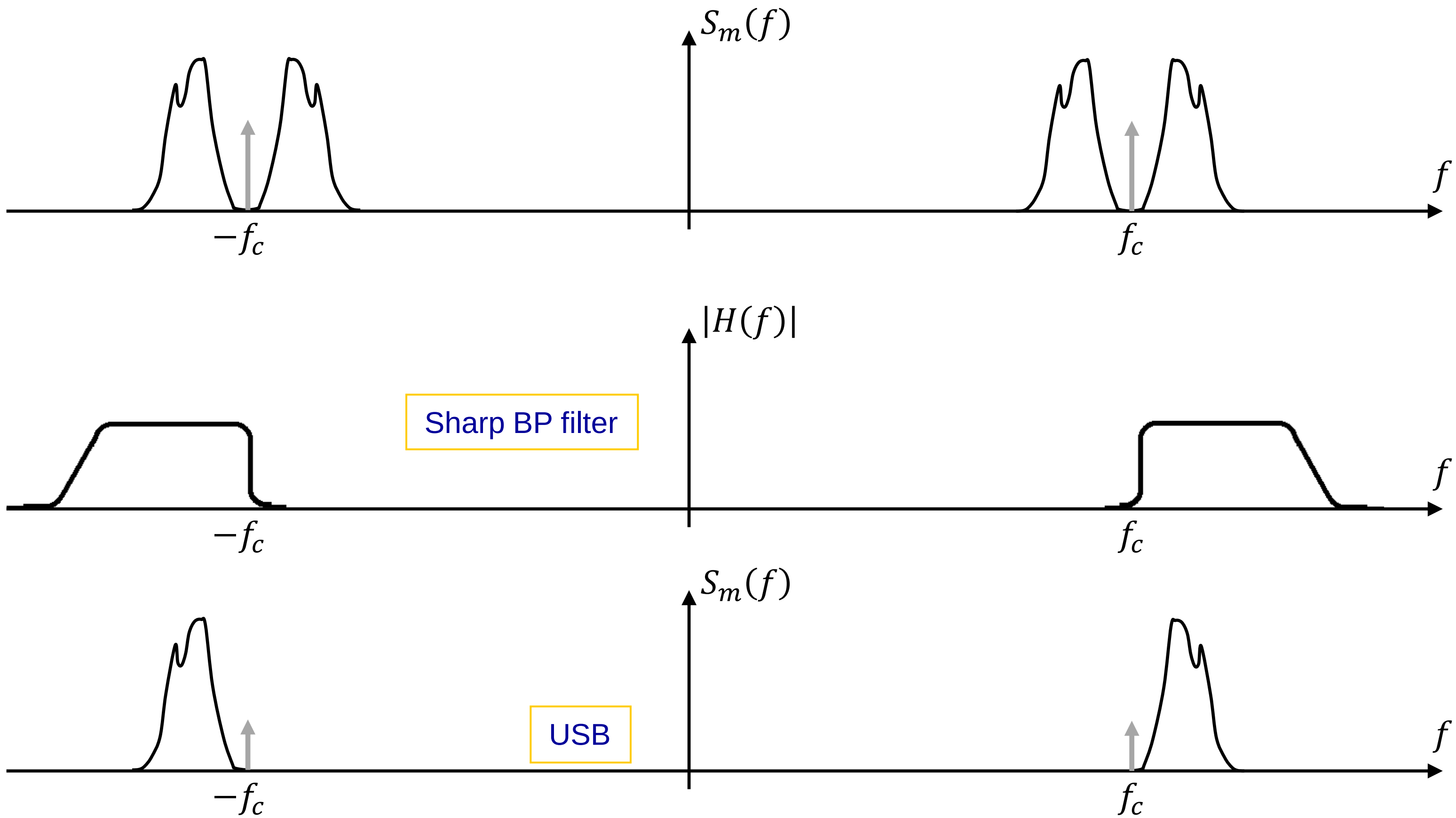
$$y_{USB}(t) = A_c \cos(2\pi f_m t)\cos(2\pi f_c t) - A_c \sin(2\pi f_m t)\sin(2\pi f_c t)$$

$y_{USB}(t) = A_c \cos(2\pi(f_c + f_m)t)$ and $y_{LSB}(t) = A_c \cos(2\pi(f_c - f_m)t)$

Homework How can we demodulate SSB-AM signals?



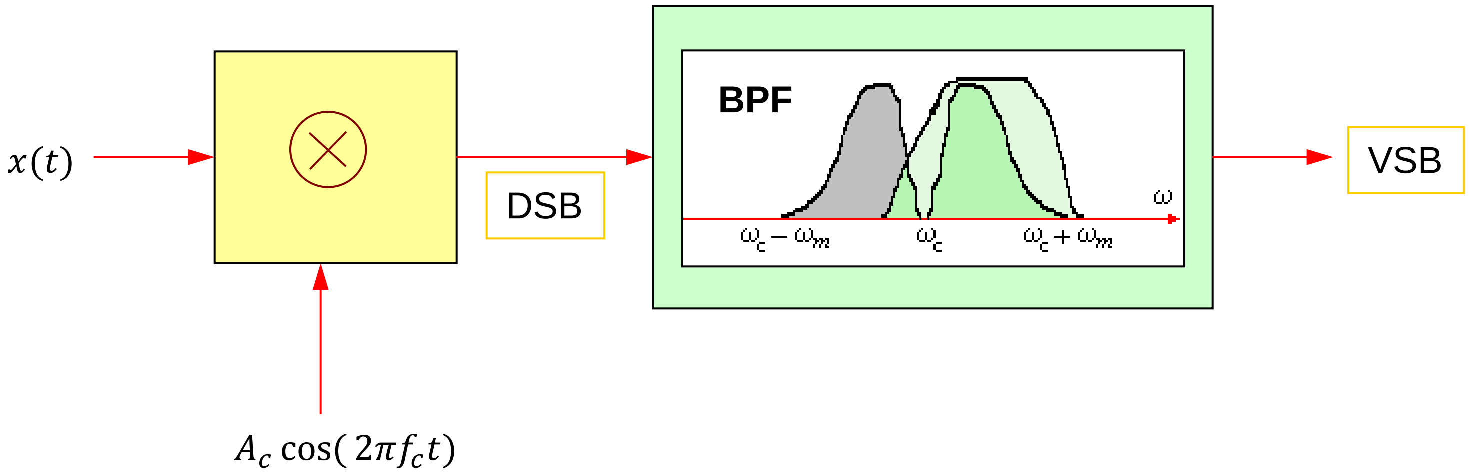
Another way to generate SSB



We need to have very sharp (at least on one cut-off) filters to achieve this.!

VSB : Vestigial Side Band

Instead we can allow a little bit of other sideband to pass; which means a relaxed version of the filter (cheaper)

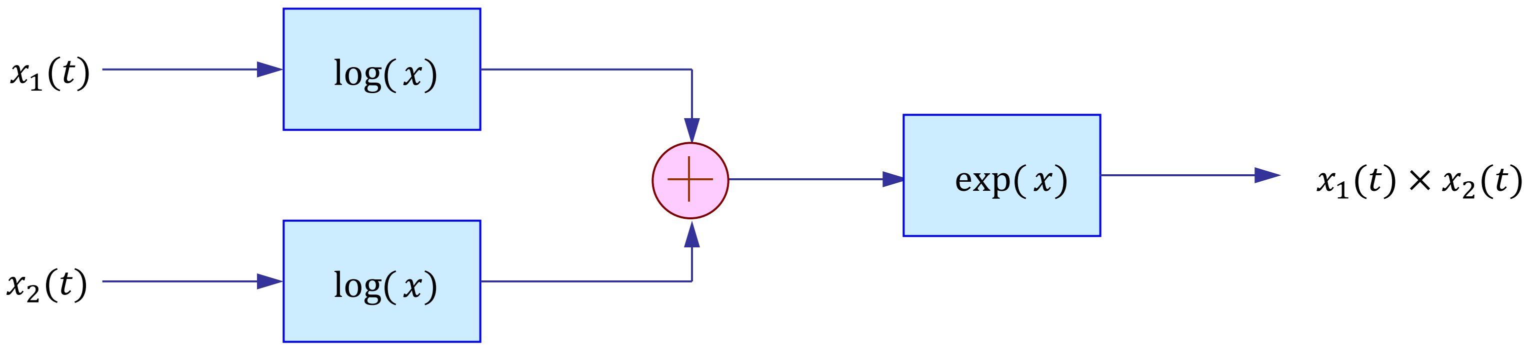


VSB – AM is used in modulation of analog monochrome television picture signals

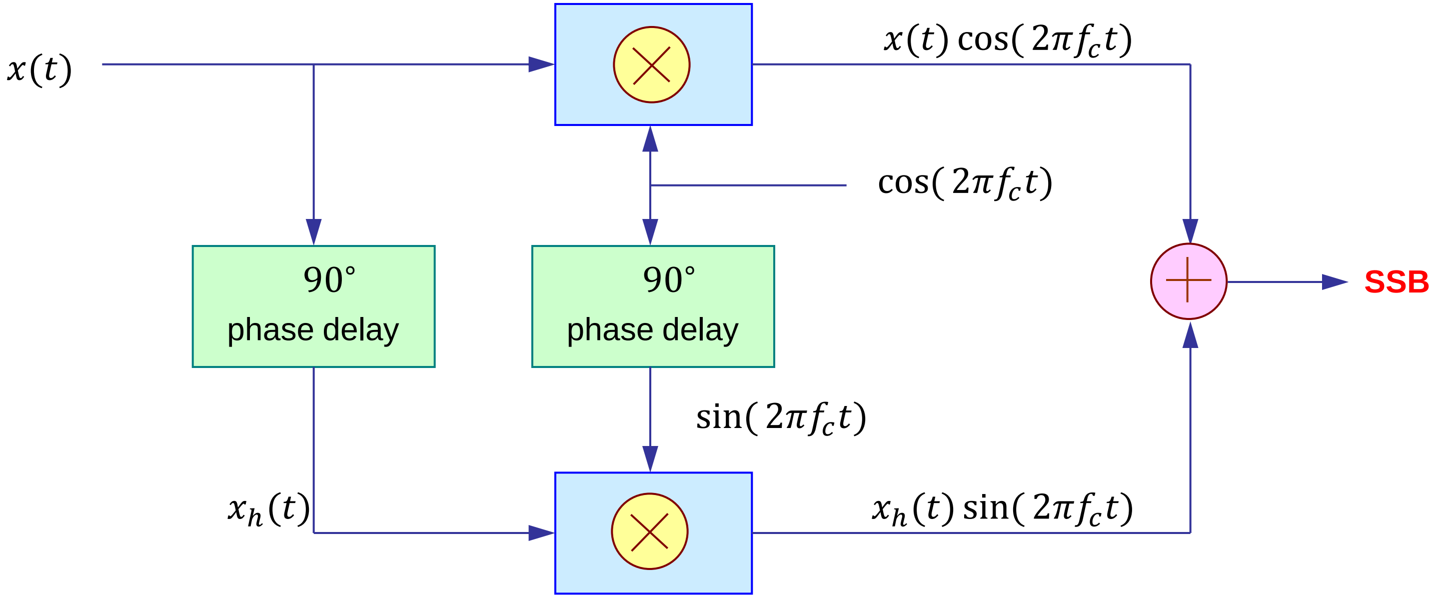
modulation of analog color television picture/color signals is another story

Use of Nonlinear Circuits to Realize Multiplication

(Since Multiplication cannot be realized by Linear Circuits)



Generation of SSB

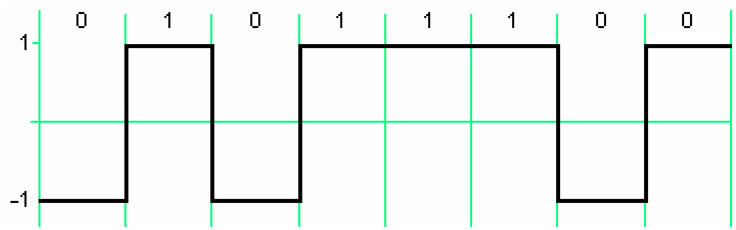


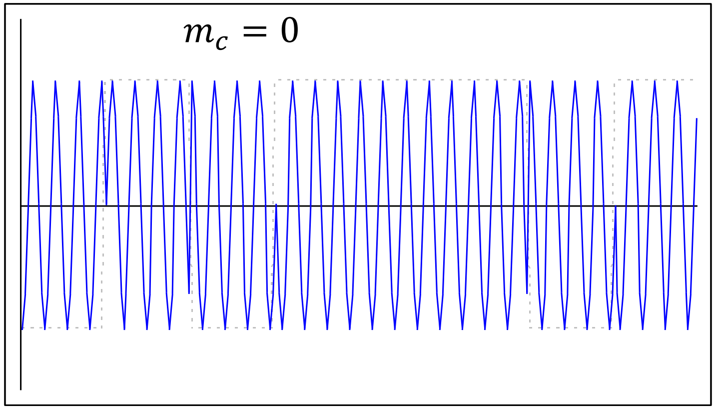
Example : Draw $y(t) = (x(t) + m_c)\cos(2\pi f_c t)$

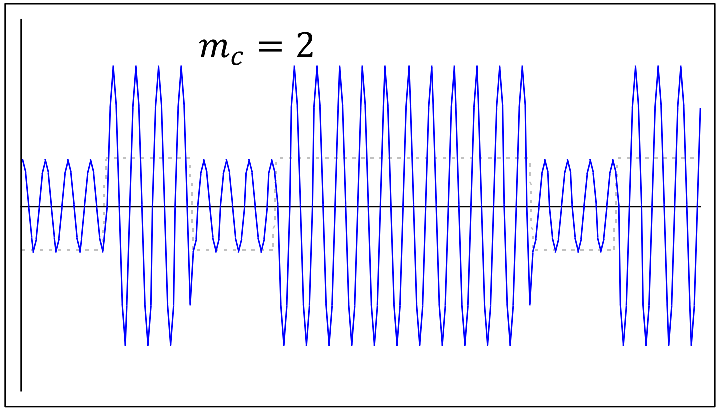
for $m_c = 0$, $m_c = 2$ and $m_c = 1$

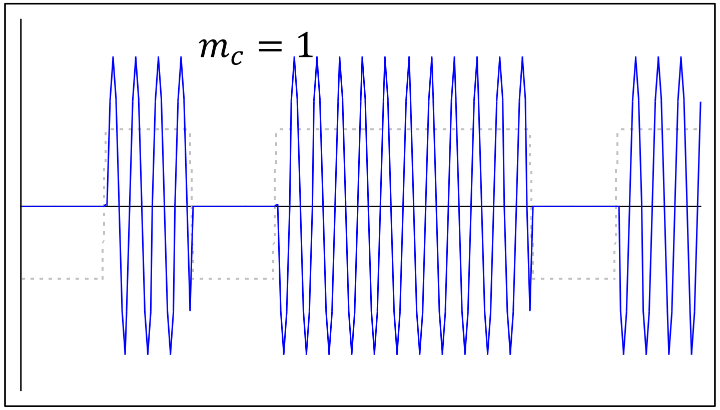
if the binary message signal is given as shown below.

Assume that carrier has high enough frequency that at least 3 cycles fit into a binary period.









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