Examples

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For the course "Communications"

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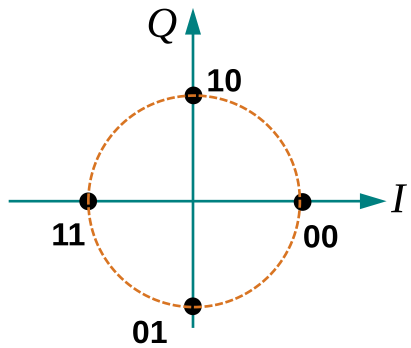
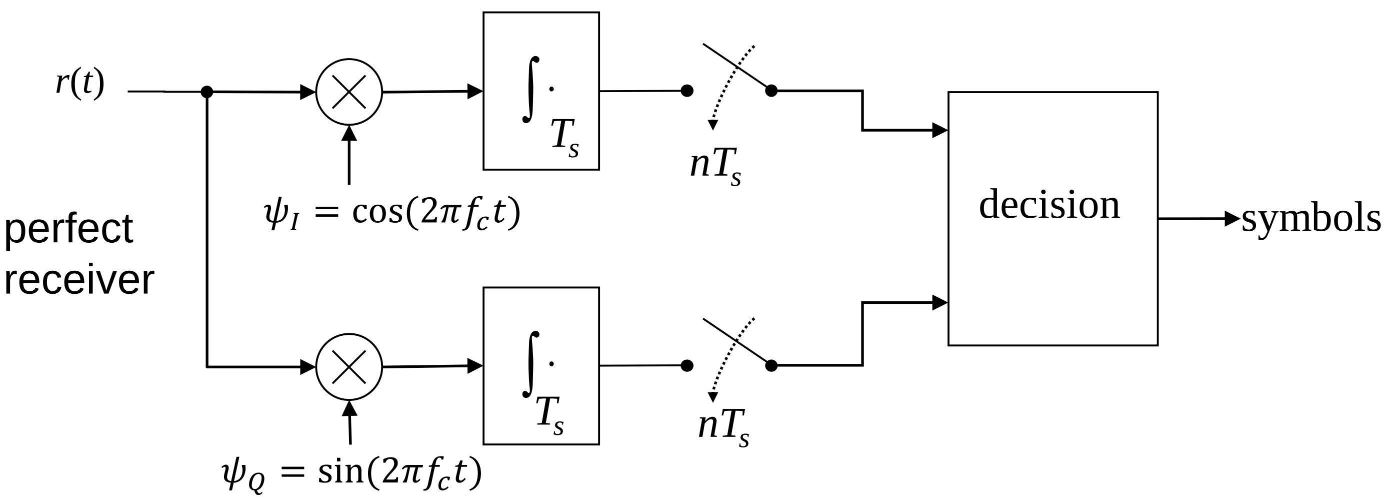
Question: The following constellation diagram is used in a transmitter. Symbol rate is 10 Msps and the carrier frequency is 20 MHz. Determine the output signal for the binary stream 0011010110. Determine probability of detection error for a perfect receiver assuming that probabilities of sending all symbols are equal.

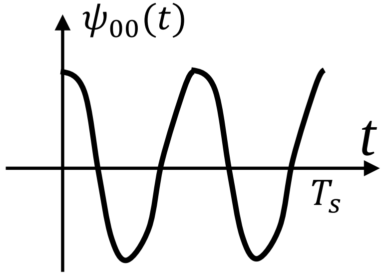


$f_c = 20\ MHz$

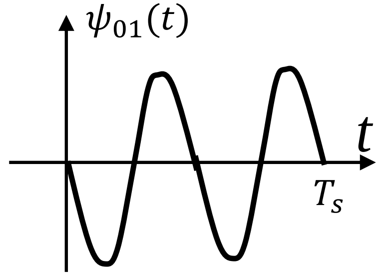
$R_s = 10\ Msps$



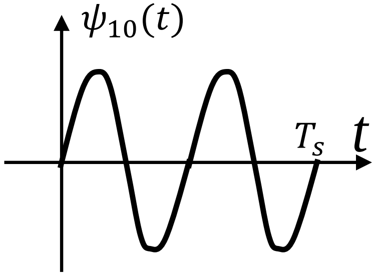
We have 4 waveforms representing 4 symbols



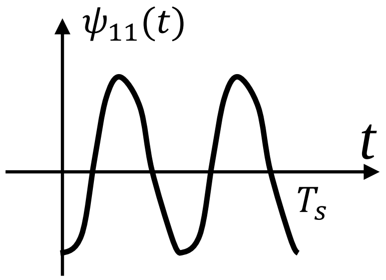
$\psi_{00} = \cos(2\pi f_c t)$



$\psi_{01} = -\sin(2\pi f_c t)$



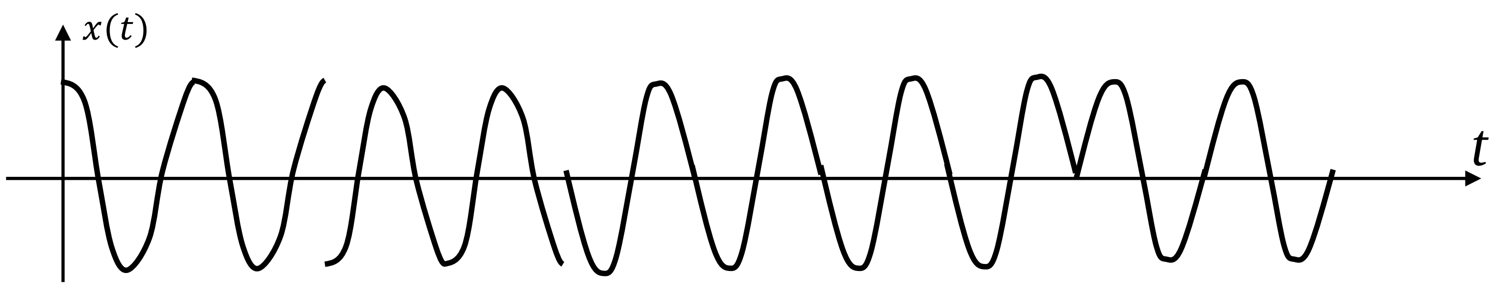
$\psi_{10} = \sin(2\pi f_c t)$



$\psi_{00} = -\cos(2\pi f_c t)$

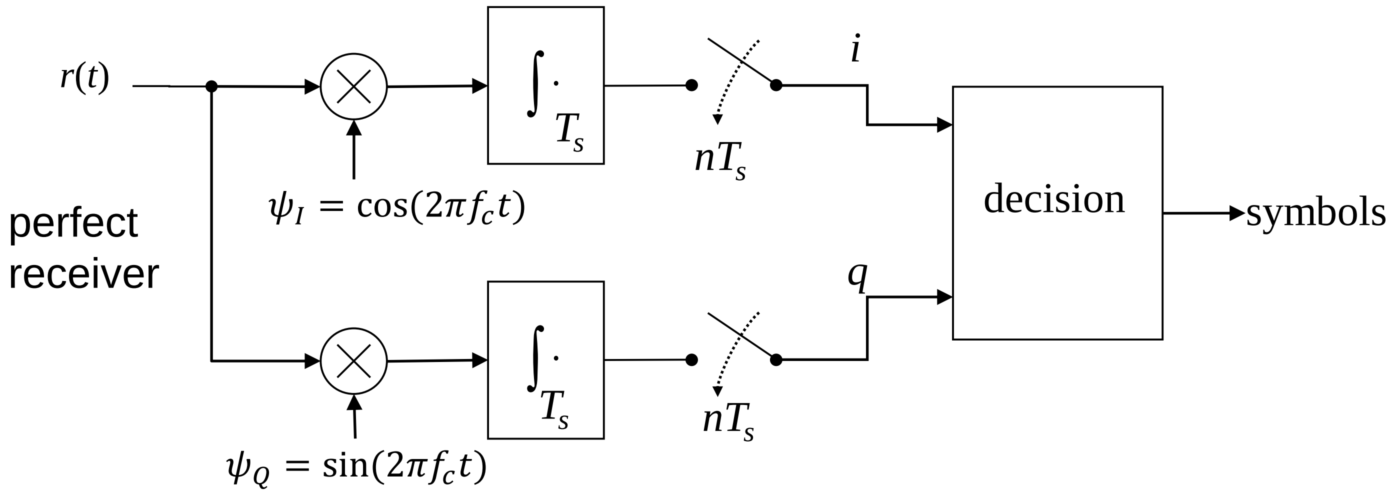
Each waveform lasts two periods of carrier, because carrier frequency is twice the symbol rate.

The generated signal for the symbol stream 00-11-01-01-10 will then be



How will the correlator receiver work?

At the end of each symbol period, i and q will be available at the output of the correlators. The decision circuit will compare them to the expected (i,q) pairs and output the symbol whose expected (i,q) is closest to the received pair.



The expected (i,q) pairs are;

$$i_{00} = \int_0^{T_s} [\cos(2\pi f_c t)]^2 dt = \frac{T_s}{2} = 0.5 \times 10^{-7} \qquad q_{00} = 0$$

$$i_{01} = 0 \qquad q_{01} = -0.5 \times 10^{-7}$$

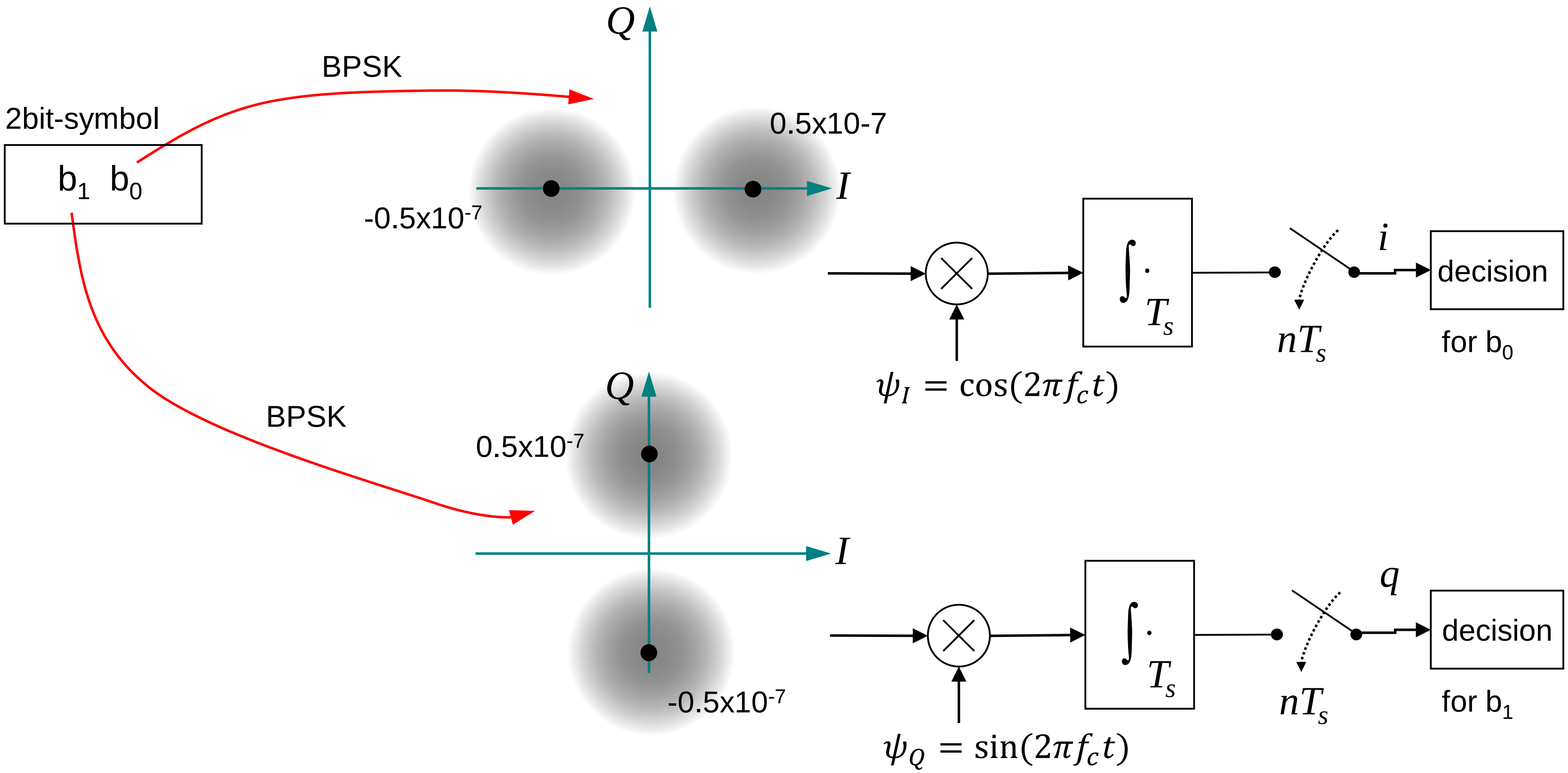
$$i_{10} = 0 \qquad q_{10} = 0.5 \times 10^{-7}$$

$$i_{11} = -0.5 \times 10^{-7} \qquad q_{11} = 0$$

Look-up table

sym	i	q
00	0.5×10^{-7}	0
01	0	-0.5×10^{-7}
10	0	0.5×10^{-7}
11	-0.5×10^{-7}	0

Let us now assume that QPSK is the summation of two BPSKs having carriers that are orthogonal (90° phase difference) to each other.

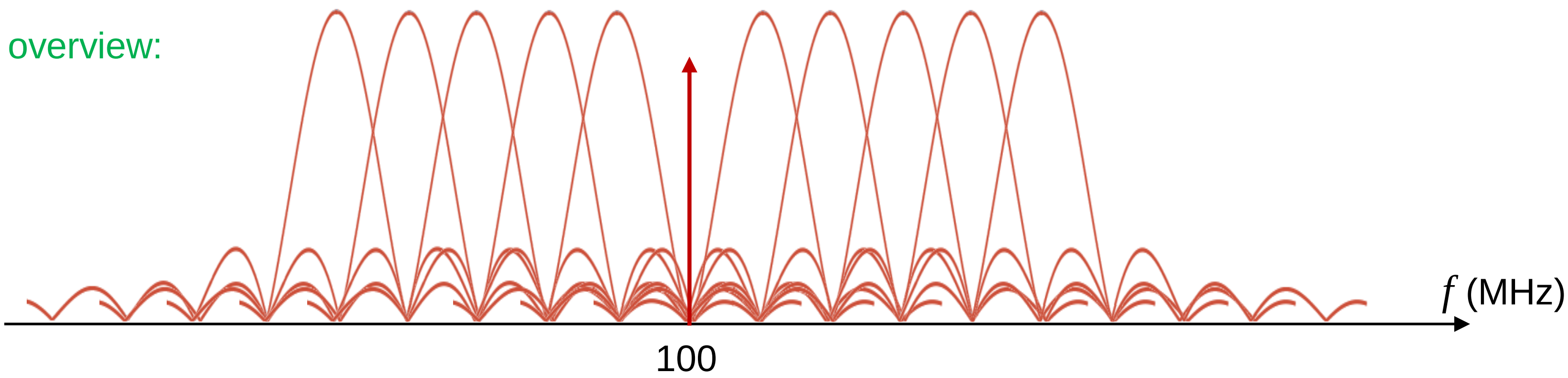


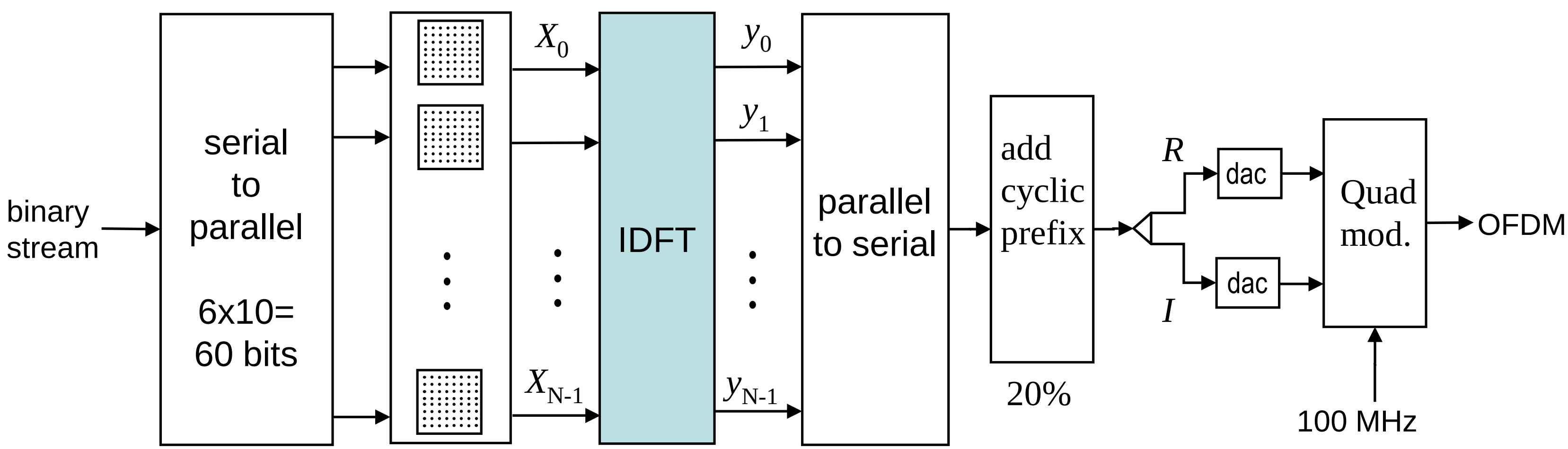
Assuming that the symbol probabilities are equal, than $p_{e-QPSK} = p_{e-BPSK}$

p_e: probability of bit error

Question: An OFDM system uses 11 carriers one of which is a pilot carrier at 100 MHz. Null-to-null BW of each sub-channel is 1 MHz and all employs 64-QAM. A $0.2T_s$ cyclic prefix is used. What is the physical bit rate?

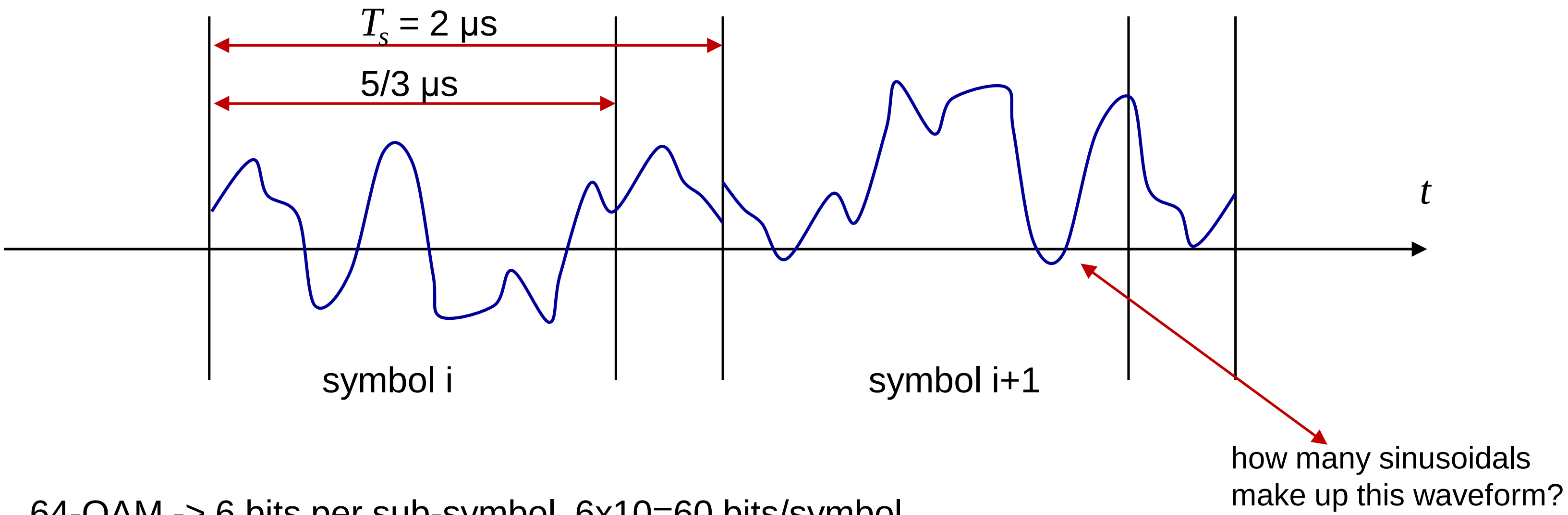
overview:





Solution:

Sub channel half BW is 500 kHz -> sub-channel symbol rate is 1/(500 kHz) = 2 μs.



64-QAM -> 6 bits per sub-symbol. 6x10=60 bits/symbol.

(60 bits) / (2 μs) = 30 Mb/s

In case we do not use cyclic prefix;
The bit rate would be (60 bits) / (5/3 μs) = 36 Mb/s
That is, cyclic prefix clearly reduces channel efficiency. So, why do we use it?
answer: to reduce ISI

Question: A 20 Mbps binary stream will be transmitted. Design an example OFDM system with all the bells and whistles (that we are aware of already) and describe it?

Solution: we need to decide on several parameters since this can be done in several different ways.

Let N=32 (easier DFT/IDFT)

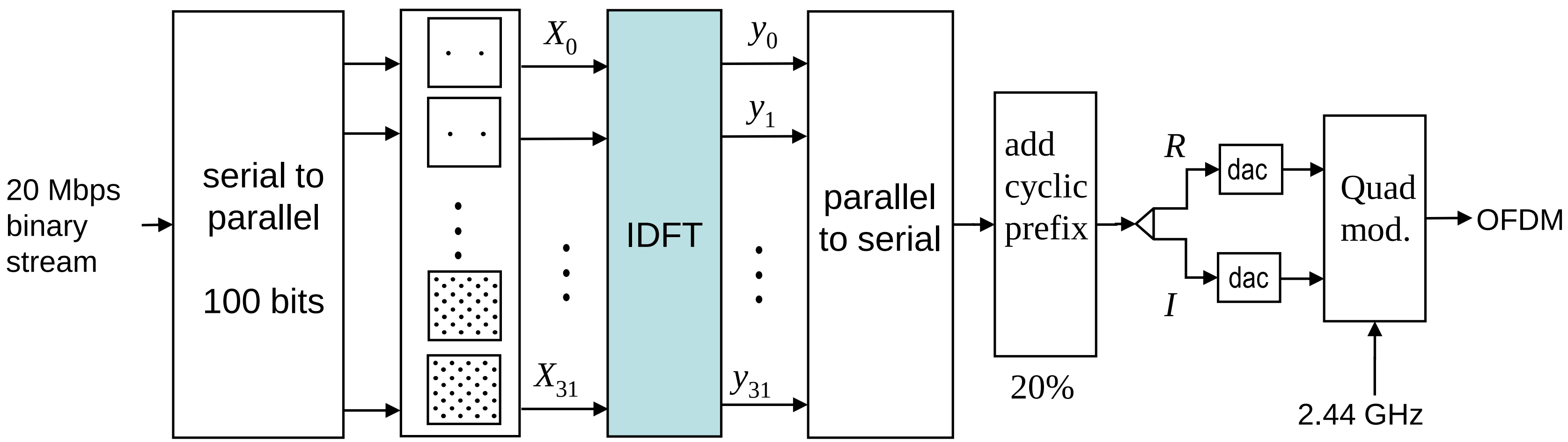
Let DC component is used as a pilot carrier

Let symbols are 100 bits and each channel uses 1 to 5 bits (just to see what happens)

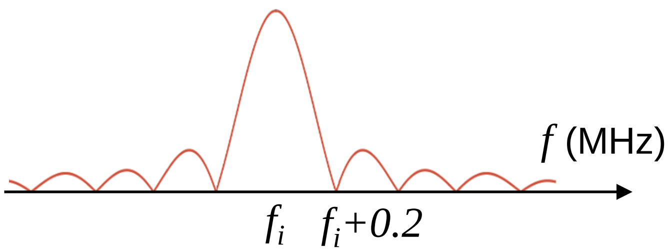
Let the system uses 2.4 GHz ISM band (choice?)

Let the cyclic prefix is 20%+ of the total symbol duration.

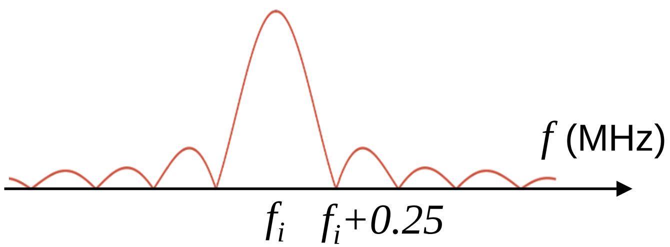
Initial design block diagram:



20 Mbps / 100 bpsym = 200 ksymbols per second.
Therefore, without cp, spectrum of each channel would be like



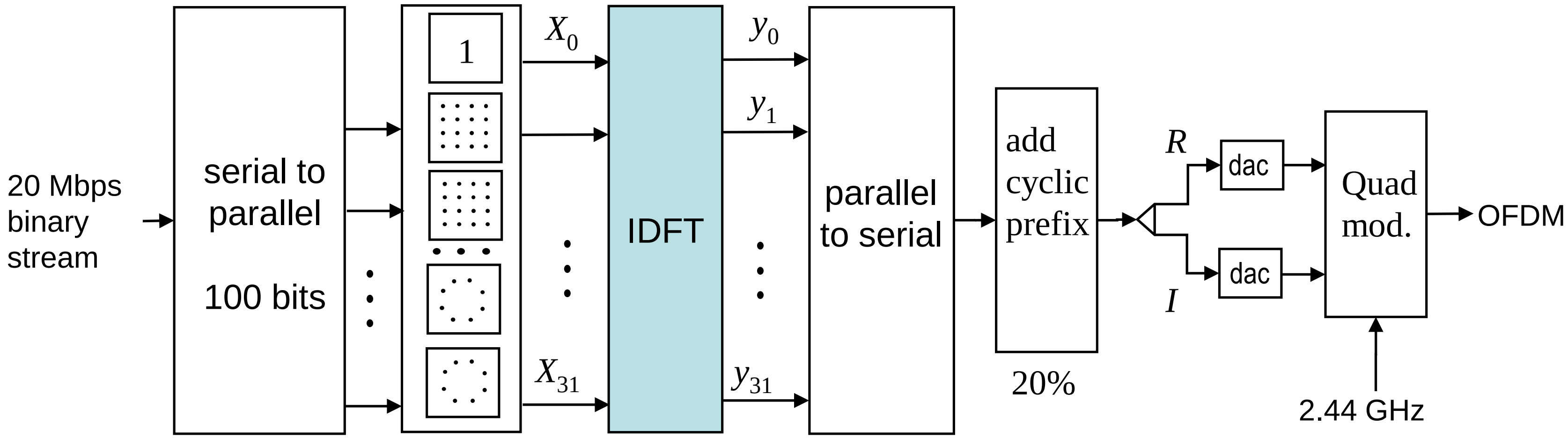
However, with 20% cp, it will be like



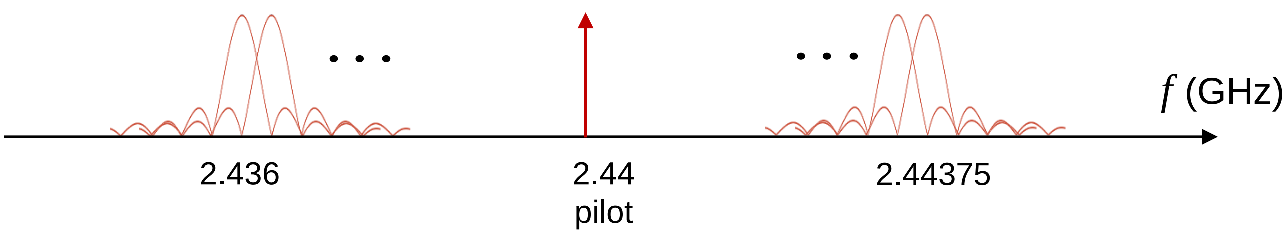
so that, (200 kHz)/(250 kHz)=0.8 and/or ((250 kHz)-(200 kHz))/(250 kHz) =20%

Then, what will be the modulation plan for each channel?
Here, we need to find N_x that satisfies
$N_{BPSK} \times 1 + N_{QPSK} \times 2 + N_{8PSK} \times 3 + N_{16QAM} \times 4 + N_{32QAM} \times 5 = 100$
where N_x is the number of channels that use modulation schema x.
We also need to satisfy $N_{BPSK} + N_{QPSK} + N_{8PSK} + N_{16QAM} + N_{32QAM} = 31$ (plus 1 for pilot carrier)

It seems that $10 \times 4 + 20 \times 3 = 100$, that is $N_{16QAM} = 10$ and $N_{8PSK} = 20$.
Nyquist carrier will be null.



10 16QAM, 20 8PSK and 1 pilot channels

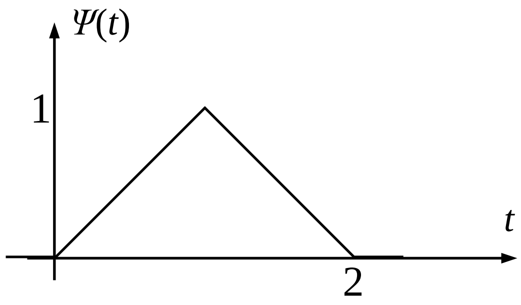


However, it is no good to use Nyquist frequency carrier for data transmission
Therefore, one might choose to set 16th carrier to null and redistribute the modulation types accross the channels.

It is also safer to leave a few edge carrier locations for null carriers to prevent interference between neighboring transmissions.

Question:

An antipodal binary communication system uses triangular pulses as shown



$T_b=2$

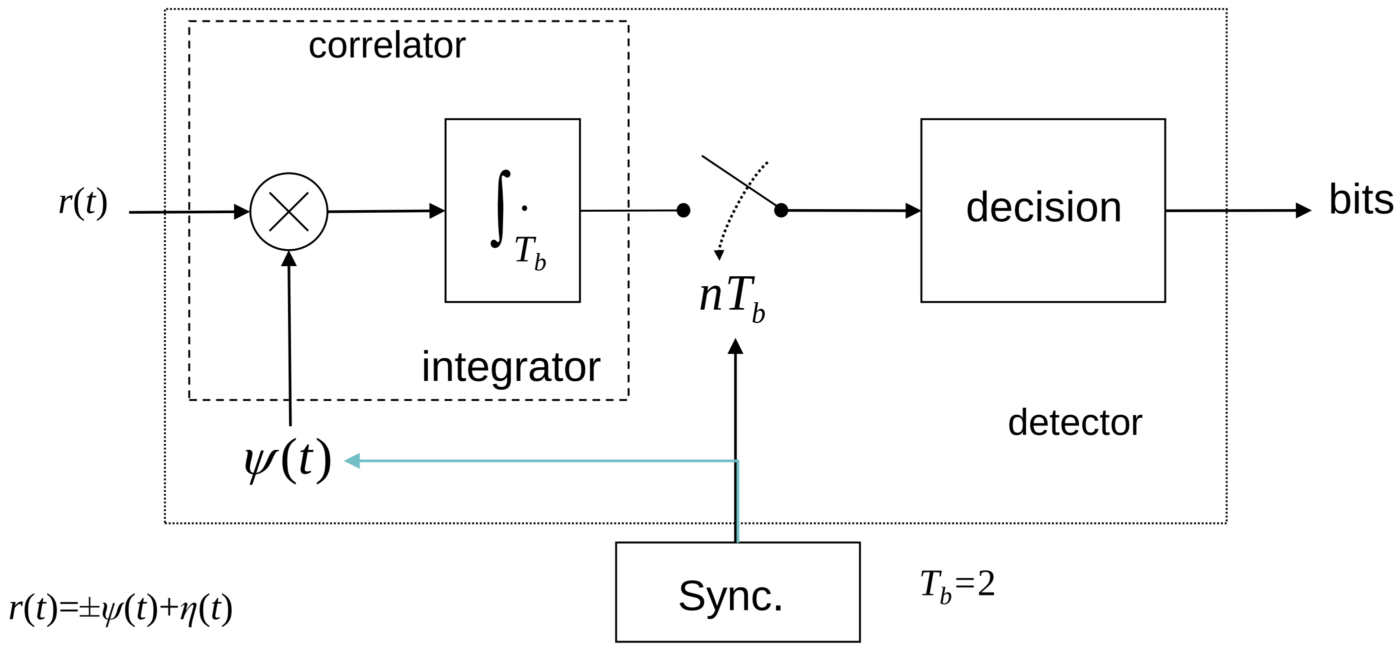
$$\psi(t) = \begin{cases} t, & 0 < t < 1 \\ 2 - t & 1 \leqq t < 2 \\ 0 & otherwise \end{cases}$$

and $-\Psi(t)$, of course.

Channel noise is AWGN with the spectral density of N_0 = 0.16 W/Hz.

this is a lot of noise actually

Q : What is the probability of decision error p_e in the optimal receiver.



Instead of a correlator, one might use a matched-filter as the optimal receiver. Their performances are identical.

We could re-run through all the math we had in the Baseband slides.

But it is easier to just use the results we had there.

$$p_e = Q\left(\sqrt{\frac{2E_b}{N_0}}\right)$$

So, we need to calculate E_b first and then p_e.

$$E_b = \int_{-\infty}^{\infty} \psi^2(t)dt \qquad = 2\int_0^1 t^2 dt$$

we used symmetry here

$E_b = \frac{2}{3}$ Joules

putting it all together, we get

$$p_e = Q\left(\sqrt{\frac{2\times 2}{3\times 1.6\times 10^{-1}}}\right) \qquad = Q(2.887) \qquad = 4.46\text{x}10^{-5}$$

I used *erfc* function in Matlab

Let us have an inverse problem for the triangular waveform now.

Let the AWGN channel noise has the spectral density of N_0 = 1x10^{-9} W/Hz.

Q : What is the maximum bit rate for p_e<0.0001 ?

Do it as a homework.!

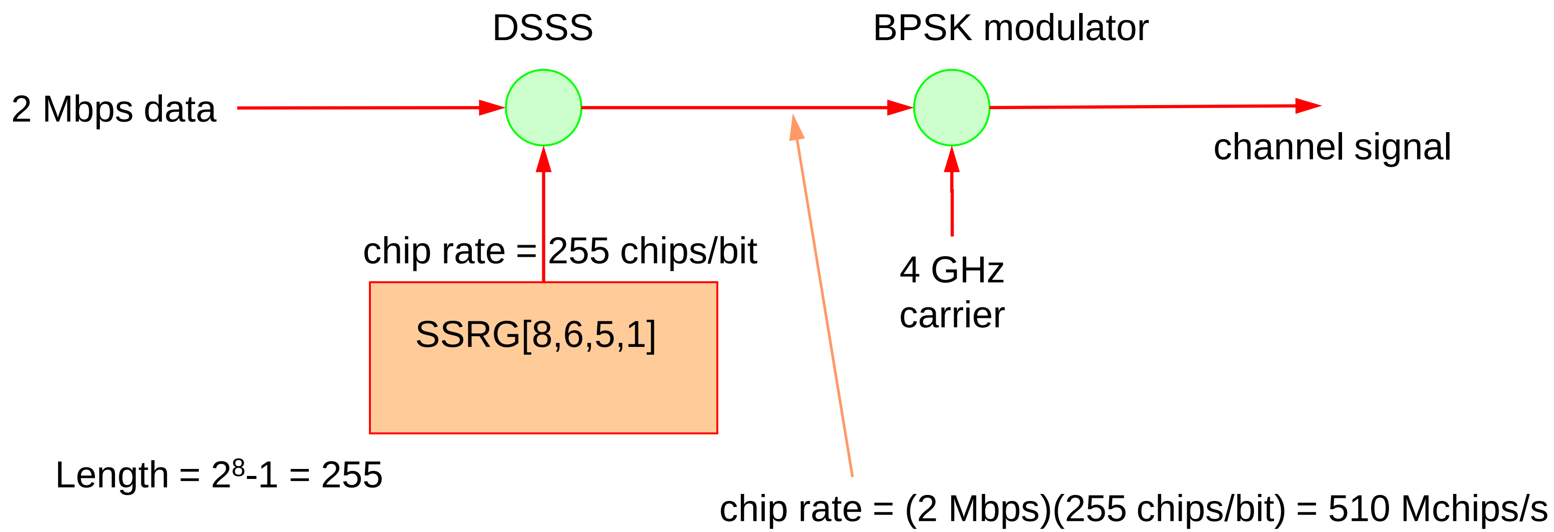
Hint :

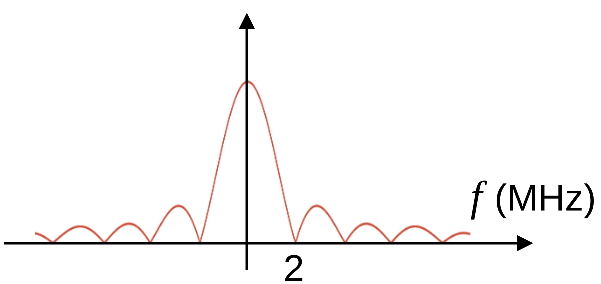
1. Find the argument of $Q(x)$ function first
2. Find E_b
3. Find T_b. Bit rate is the inverse of it.

$$p_e = Q\left(\sqrt{\frac{2E_b}{N_0}}\right)$$

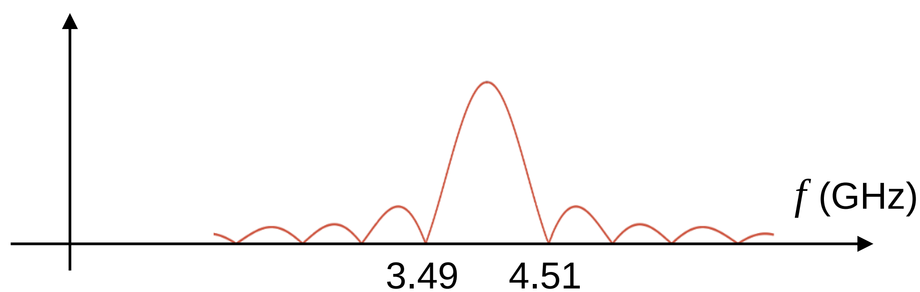
Have fun.!

Question: A 2 Mbps binary stream is first spread using pn-code generated by SSRG[8,6,5,1] with DSSS method (1 pn period per bit) and then shifted to 4 GHz using BPSK.
What are the spectral properties of the resulting signal?





Baseband signal spectrum



channel signal spectrum

END