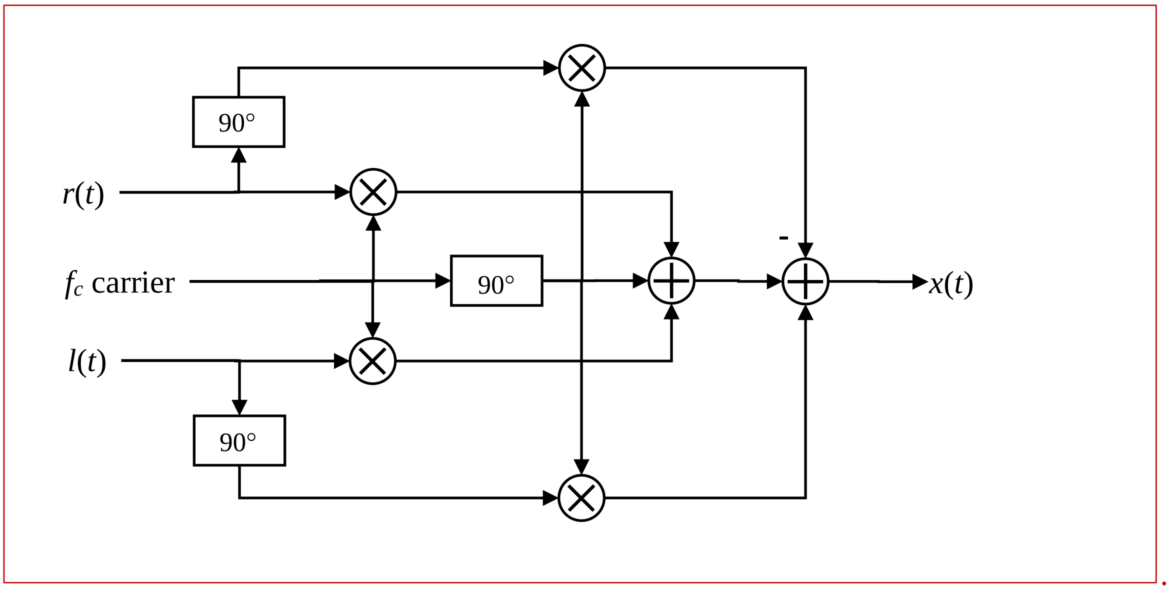
A1.

It is obvious that L channel is LSB-AM and R channel is USB-AM. We need to obtain USB for $r(t)$ and LSB for $l(t)$ and then sum them up along with the carrier.



We need 2 multipliers, three 90° phase-shifters and one 5 input adder, in order to generate the signal.

A2.

Output noise power is

$$P_\eta = \int_{18\times10^3}^{24\times10^3} 10^{-6}|H(f)|^2 df = 10^{-6}\left[\int_0^{4\times10^3} \left|\frac{1}{4\times10^3}f\right|^2 df + \int_0^{2\times10^3} \left|\frac{1}{2\times10^3}f\right|^2 df\right]$$

where the first term is for $f = (18k, 22k)$ and the second term is for $f = (22k, 24k)$. Note that we moved the curves to origin so that the calculation will be easier. We can do this as long as we also move the signal (noise). In our case the noise is constant over the spectrum, so no spectral shift for noise is necessary.

$$P_{\eta1} = \frac{10^{-6}}{16\times10^6}\int_0^{4\times10^3} f^2 df = \frac{10^{-12}}{16\times3}f^3\Big]_0^{4\times10^3} = \frac{10^{-12}\times(4\times10^3)^3}{16\times3}$$

$$= \frac{10^{-12}\times64\times10^9}{16\times3} \cong 1.33\times10^{-3} = 1.33\ mW$$

$$P_{\eta2} = \frac{10^{-6}}{4\times10^6}\int_0^{2\times10^3} f^2 df = \frac{10^{-12}}{4\times3}f^3\Big]_0^{2\times10^3} = \frac{10^{-12}\times(2\times10^3)^3}{4\times3}$$

$$= \frac{10^{-12}\times8\times10^9}{4\times3} \cong 0.66\times10^{-3} = 0.66\ mW$$

We did not need to calculate the second term actually, it was obvious that the bw is halved. So the power is also halved. The total noise power is then,

$P_\eta = P_{\eta 1} + P_{\eta 2} \cong 2\, mW$

Amplitude of the signal portion at 20 kHz is half of the input (from the graph). That is,

$y_s(t) = \frac{1}{2}\cos(40\pi t + \varphi)$ (with irrelevant phase shift φ). Power of this sinusoidal signal at the output is then

$$P_s = \frac{1}{T}\int_0^T \left|\frac{1}{2}\cos(2\pi t/T)\right|^2 dt = \frac{1}{8}\, W$$

As we know, frequency is also irrelevant, it will cancel out in calculations, hence T is ok.

We could calculate the signal power using $P_s = V_p^2/2$ too, since it is a zero mean sinusoidal.

Signal to Noise ratio is then,

$$SNR = \frac{P_s}{P_\eta} = \frac{1/8}{2\times 10^{-3}} \cong 62.5 \simeq 17.9\, dB$$

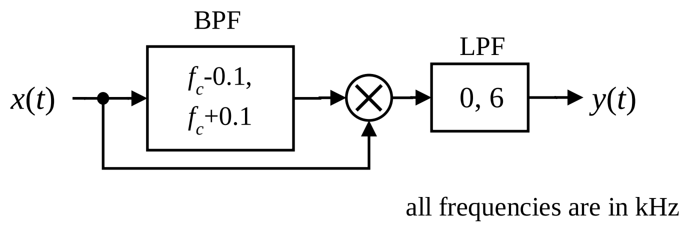
A3.
Code table for non-extended source is found to be
A={a,b,c},z={0.6,0.3,0.1} => C_1={0,10,11}, L={1,2,2}
$L_{avg1} = \sum_i z_i l_i$=1.4, and the compression ratio R_{c1} =1.4/2 = 0.7
Code table for 2nd extension is found to be
A={aa,ab,ba,ac,...,cc}, u={0.36,0.18,0.18,0.09,0.06,0.06,0.03,0.03,0.01} =>
C_2={0,100,101,1100,1110,1111,11010,110110,110111}, L_2={1,3,3,4,4,4,5,6,6}.
$L_{avg2} = \sum_i u_i l_i$=2.67, and the compression ratio R_{c2} =2.67/4 = 0.6675

A4.
$H(f) = (1 + j2\pi RCf)^{-1}$, $|H(f)| = (1 + (2\pi RCf)^2)^{-1/2}$
At 4 kHz $\langle H(4000) = \langle(1 + j2\pi \times 2 \times 10^3 \times 50 \times 10^{-9} \times 4000)^{-1}$
$= \langle(1 + j0.8\pi)^{-1} \simeq \langle(1 + j2.5133)^{-1} = \tan^{-1}(-2.5133) \simeq -68.3°$.
Since the sinusoidal input is advanced by $\frac{\pi}{4} = 45°$, resulting phase of the sinusoidal is $-23.3°$.
Amplitude of the sinusoidal at the output is $|H(4000)| \simeq 0.37$. The power of the sinusoidal is, then, $P_s = \frac{4\times 0.37^2}{2} \simeq 273.8\, mW$.
The noise power at the output, on the other hand, is
$P_\eta = \int_0^\infty N_0 |H(f)|^2 df = \int_0^\infty 10^{-6}(1 + (2\pi RCf)^2)^{-1} df$
$= \int_0^\infty 10^{-6}(1 + (j2\pi \times 2 \times 10^3 \times 50 \times 10^{-9} \times f)^2)^{-1} df$
$= \int_0^\infty 10^{-6} \frac{1}{1+4\pi^2 \times 10^{-8} f^2} df = 5\pi 10^{-3} \tan^{-1}(2\pi 10^{-4} f)|_0^\infty \cong 2.47 mW$

SNR at the output of the filter is then SNR_o=273.8/2.47 ≈ 110.85 ≈ 40.9dB.

A5.

We need to extract the carrier and multiply it with the incoming signal to obtain a baseband and $2f_c$ centered components. We use a LPF to extract the baseband components afterwards.



It is no different than the synchronous detection of AM signal.

A6.

The noise power at the output of the filter, assuming that the noise power spectral density at the input is N_0, is

$P_o = \int_0^{\infty} N_0|H(f)|^2 df = N_0 \int_0^{10000} 1^2 df + N_0 \int_0^{4000} (f/4000)^2 df$

For the second part, we took the advantage of having same power for shifted/flipped characteristics.

$P_o = N_0 f]_0^{10000} + \frac{N_0}{4000^2 \times 3} f^3\Big]_0^{4000} = N_0 10000 + \frac{N_0}{4000^2 \times 3} 4000^3 = N_0 11333.33$ [W]

If it were an ideal BPF, then

$\acute{P}_o = \int_0^{\infty} N_0|H_i(f)|^2 df = \int_0^{B_{neq}} N_0 1^2 df = N_0 B_{neq}$ [W]

Since we need $P_o = \acute{P}_o$, it turns out that

$B_{neq} = 11333.33$ [Hz]

A7.

We need to find the pairwise correlation coefficients. If $R_{x,0} > R_{x,1}$ then we will conclude that $\psi_x(t)$ is more similar to $\psi_0(t)$ than to $\psi_1(t)$.

$R_{x,0} = \int_0^1 \psi_x(t)\psi_0(t)dt = \int_0^1 1.1 \sin(\pi t)\, dt = \frac{-1.1}{\pi}\cos(\pi t)\Big]_0^1 = \frac{1.1}{\pi}(1+1) = \frac{2.2}{\pi} \cong 0.7$

$R_{x,1} = \int_0^1 \psi_x(t)\psi_1(t)dt = \int_0^{0.5} 1.1 \sin(\pi t)\, 2.4t dt + \int_{0.5}^1 1.1 \sin(\pi t)\, 2.4(1-t)dt$

Because of the symmetry, we can calculate the first term and double it.

$R_{x,1} = 2\int_0^{0.5} 1.1 \sin(\pi t)\, 2.4t dt = 5.28 \int_0^{0.5} t \sin(\pi t)\, dt = 5.28 \left[\frac{1}{\pi^2}\sin(\pi t) - \frac{t}{\pi}\cos(\pi t)\right]_0^{0.5}$

$R_{x,1} = 5.28 \left[\frac{1}{\pi^2}\right] \cong 0.527$

Since 0.7>0.527, we conclude that $\psi_x(t)$ is more similar to $\psi_0(t)$ than to $\psi_1(t)$.

A8.

$p_e = Q\left(\sqrt{2SNR}\right)$

$10^{-3} = Q\left(\sqrt{2SNR_1}\right)$ and $10^{-5} = Q\left(\sqrt{2SNR_2}\right)$.

$\sqrt{2SNR_1} \cong 3.1$, $\sqrt{2SNR_2} \cong 4.25$ (found approximately from tables)

$SNR_1 = 4.805$, $SNR_2 = 9.03$

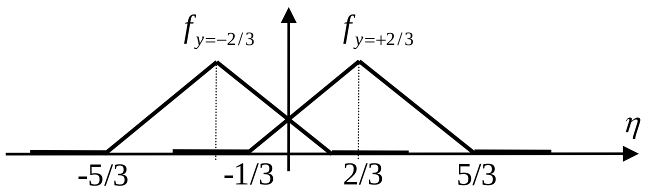
$\frac{SNR_2}{SNR_1} = 1.88$.

Assuming that the noise stays the same, this number is the power gain necessary to achieve BER=10^{-5}.

A9.

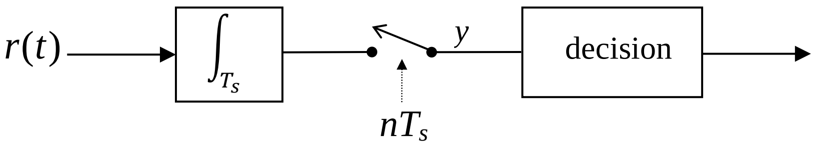
In full synchronization $R[nT_b] = \pm\int_0^{T_b} \psi_1^2(t)dt = \pm 2\int_0^1 t^2 dt = \pm \left.\frac{2t^3}{3}\right|_0^1 = \pm\frac{2}{3}$

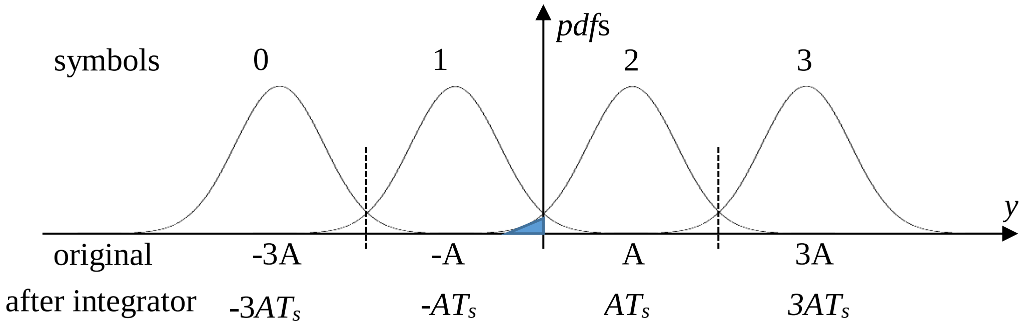
At the decision instant, pdf's for the two output possibilities will be as shown



Assuming that the decision threshold is at 0, $p_e = \int_{-1/3}^{0}\left(u+\frac{1}{3}\right)du = \int_0^{1/3} u du = \frac{1}{18}$

A10.
Since, for rectangular pulses, it will not make any difference in detection, we can just have an integrator instead of 4 correlators at the receiver.





In that case, signal part of the correlator output will be

$$I_s = \int_0^{T_s} S dt = \begin{cases} -3AT_b, & S = -3A \\ -AT_b, & S = -A \\ AT_b, & S = A \\ 3AT_b, & S = 3A \end{cases}$$

This situation is shown in the figure.
Since the integration is linear operation, we can assume that the noise part will be Gaussian, but we do not know its variance, yet.
Detection thresholds should be at the halfway between symbols (ML criteria); $-2AT_s$, 0 and $2AT_s$.
Let us assume that the noise variance is small enough so that we can also assume p(A|-3A)=p(3A|-3A) =p(3A|-A) =p(-3A|A)=p(-3A|3A)=p(-A|3A)=0.
The remaining areas (one is shown gray) are all the same;
p=p(-A|-3A)=p(-3A|-A)=p(A|-A)=p(-A|A)=p(3A|A)=p(A|3A).
The noise part of the integration result is $I_\eta = \int_0^{Ts} \eta(t)dt$. We cannot calculate it but we can calculate its variance. The variance of the output noise is, by definition

$$\sigma_\eta^2 = ExpectedValue(I_\eta^2)$$

$$\sigma_\eta^2 = \frac{1}{T_s}\int_0^{T_s}\left[\int_0^{T_s}\eta(t)dt\right]^2 d\tau$$

Since we assumed that the noise is uncorrelated, we can directly write

$$\left[\int_0^{T_s}\eta(t)dt\right]^2 = \int_0^{T_s}\eta^2(t)dt = T_sN_0/2$$

Hence, $\sigma_\eta^2 = \frac{1}{T_s}\int_0^{T_s} T_sN_0/2\, d\tau = \frac{T_sN_0}{2}$ and $\sigma_\eta = \sqrt{\frac{T_sN_0}{2}}$ (standard deviation of noise part).

The grayed area is then

$$p = \frac{1}{\sqrt{\frac{T_s N_0}{2}}\sqrt{2\pi}} \int_{u_1 = AT_s}^{\infty} e^{-u^2/T_s N_0} du$$

Let us now try to make this integral look like

$$Q(x) = \frac{1}{\sqrt{2\pi}} \int_{t_1 = x}^{\infty} e^{-t^2/2} dt$$

by changing variables $u^2/T_s N_0 = t^2/2$.

(Why are we doing it?: Because we cannot calculate the integral by analytical calculus means. We need to refer to the tables or do numerical integration. Tables are given for $Q(x)$)

We have $u^2 = T_s N_0 t^2/2$ and $u = \sqrt{\frac{T_s N_0}{2}} t$ $t = \sqrt{\frac{2}{T_s N_0}} u$ and $du = \sqrt{\frac{T_s N_0}{2}} dt$

For $u_1 = AT_s$ we have $t_1 = \sqrt{\frac{2}{T_s N_0}} AT_s = \sqrt{\frac{2A^2 T_s}{N_0}}$

Putting these into p equation, we have

$$p = \frac{1}{\sqrt{2\pi}} \int_{t_1 = \sqrt{\frac{2A^2 T_s}{N_0}}}^{\infty} e^{-t^2/2} dt = Q\left(\sqrt{\frac{2A^2 T_s}{N_0}}\right)$$

The total average detection error is then

$$p_{se} = 0.25p(1|0) + 0.25[p(0|1) + p(2|1)] + 0.25[p(1|2) + p(3|2)] + 0.25p(2|3)$$

Since all symbol error probabilities are the same, we have

$p_{se} = 0.25p + 0.25[p + p] + 0.25[p + p] + 0.25p.$

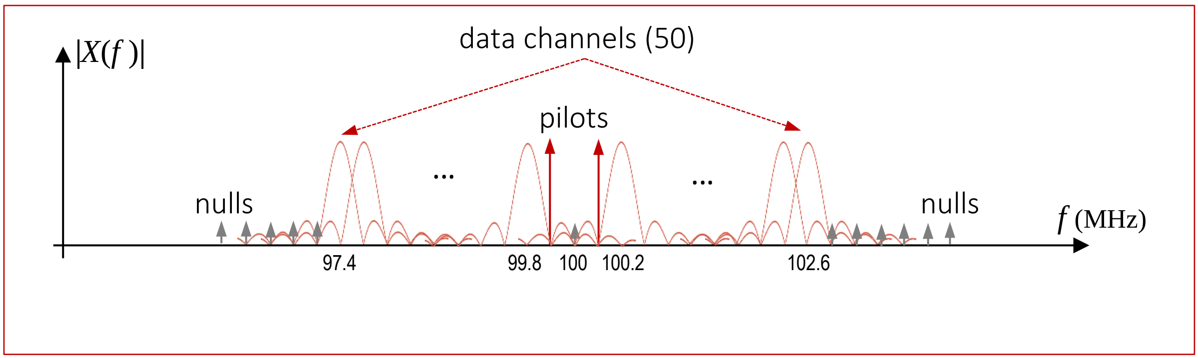
Hence,

probability of symbol error is $p_{se} = \frac{3}{2} Q\left(\sqrt{\frac{2A^2 T_s}{N_0}}\right)$

With the assumption that the neighboring symbols have one bit difference, we can also assume that bit error probability is half of the symbol error probability.

probability of bit error is $p_e = \frac{3}{4} Q\left(\sqrt{\frac{2A^2 T_s}{N_0}}\right)$.

A11.
Out of 64 carriers 1+2+5+6=14 are not used for data transmission. Remaining 50 channels employ BPSK, QPSK or 16-QAM.



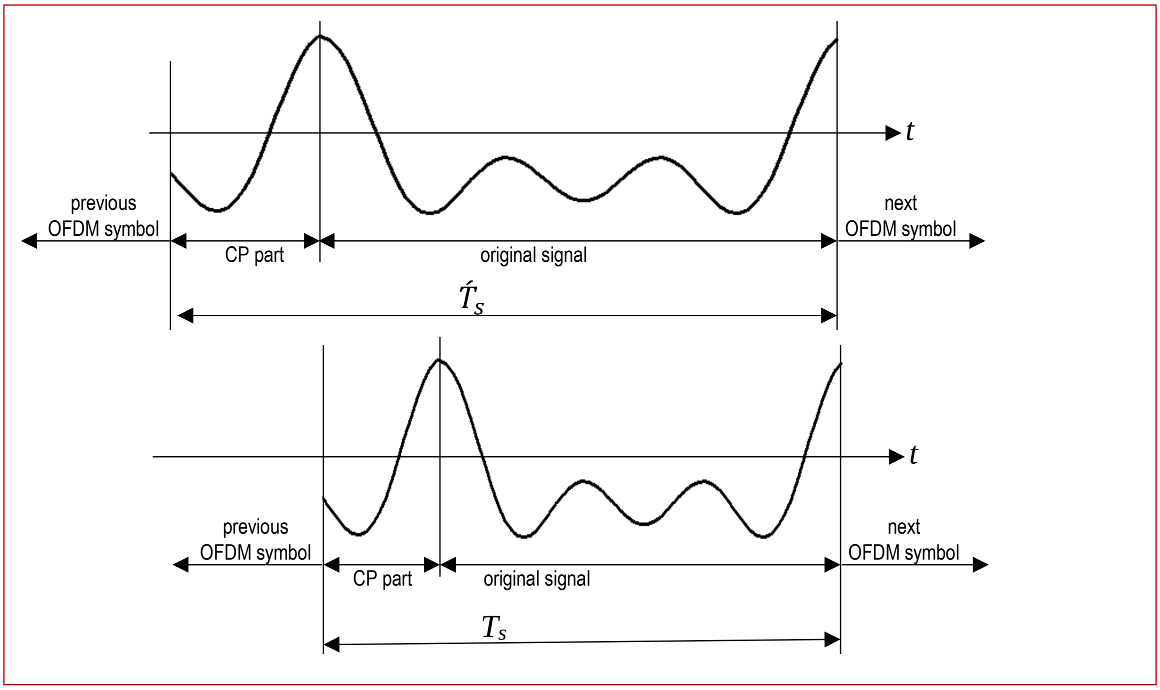
a) Null-to-null bw of each channel is 2x100ksps=200 kHz. Pilot carriers are at 99.9 and 100.1 MHz. 26th channel position is the highest data channel carrier frequency. Since the carriers are positioned 100 kHz apart, it corresponds to 102.6 MHz. Similarly lowest data channel carrier frequency is 97.4 MHz.

b) Maximum bit rate is obtained when all channels employ 16-QAM (4 bits/sym). In that case, 50x4=200 bits per OFDM symbol is obtained.

Then, total bit rate is 200 bits/sym x 100 ksym/s = 20 Mbps. However, 25% of it will be lost on CP. We will have a raw bit rate of 15 Mbps.

Some explanation on CP:

It seems that we have two choices on how prefixed signal is generated in reality. Let us explain them with the following two baseband OFDM signal (real part only).



Let us keep in mind that the part shown as "original signal" is the output of IFFT, which is composed of several sinusoidals that are orthogonal within T_s.

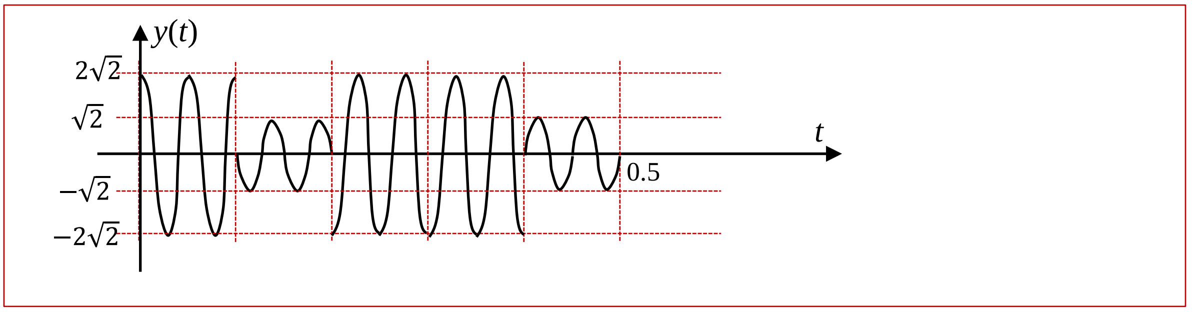
The first option increases the symbol duration, therefore decreases the symbol rate by 25%. As a result, bit rate is reduced by 25%.

In the second option (the signal is squeezed in time) the symbol duration is kept the same but all frequencies are increased by 25%. Possibly, IFFT needs to be calculated 25% faster than the first option. Occupied bandwidth is also increased by 25%.

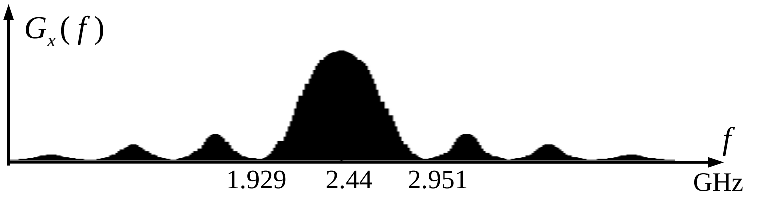
Individual components of both time tomain signals may or may not be orthogonal when multiplied with a carrier unless the carrier frequency is an integer multiple of the lowest baseband frequency (other than DC).

Standards usually limit the bandwidth and possible carrier frequency locations too. Therefore, when implementing a standard, one needs to take these into consideration. That is, it is not logical to leisurely select neither extension nor squeezing the time domain signal. Both OFDM symbol duration and bandwidth are usually given.

A12.

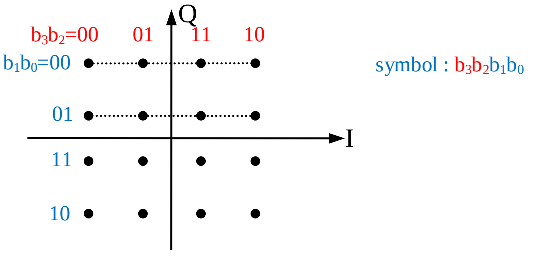


A13.
The BW of the resulting signal when $x(t)$ is spread with a code of 511 chips will be 511 MHz. BW will be 2x511=1022 MHz when multiplied with the carrier.



A14.

a) Any placement of 16 symbols on the constellation would be counted as 16-QAM as long as there are amplitude and phase differences between them. But the generally accepted/used/optimal placement is shown below.



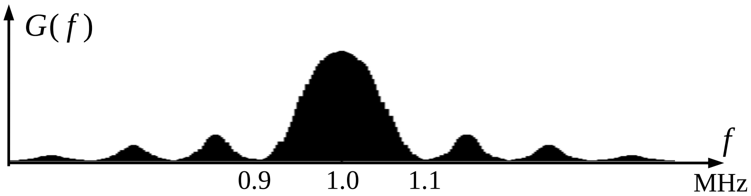
Keep in mind that 16-PSK is not counted as 16-QAM since there is no amplitude difference between symbols/waveforms in 16-PSK.

b) There are four levels on both I and Q direction. Let them be ±1 and ±3 in order to have uniform distances between constellation symbols. In that case, I and Q components (cos and sin multipliers, respectively) of the 16 symbols will be (from top-left to bottom-right)
{(-3,3),(-1,3),(1,3),(3,3),(-3,1),(-1,1),(1,1),(3,1),(-3,-1),(-1,-1),(1,-1),(3,-1),(-3,-3),(-1,-3),(1,-3),(3,-3)}.
In table form, it will be

symbol	I	Q
0000	-3	3
0001	-3	1
0010	-3	-3
0011	-3	-1
0100	-1	3

0101	-1	1
0110	-1	-1
0111	-1	-3
1000	3	3
1001	3	1
1010	3	-3
1011	3	-1
1100	1	3
1101	1	1
1110	1	-3
1111	1	-1

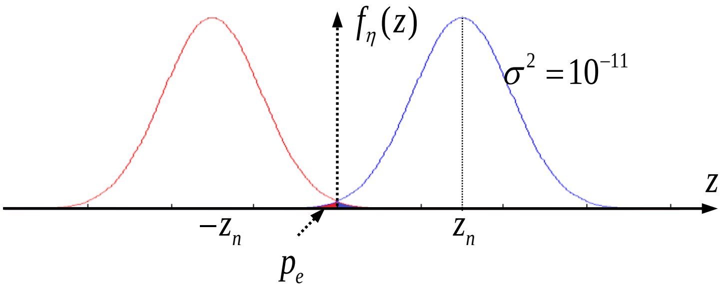
A15.



Correlator output signal part at decision instant

$$z_n = \pm\int_0^{T_b} \cos^2(2\pi f_c t)dt = \pm\int_0^{T_b} [\tfrac{1}{2} + \tfrac{1}{2}\cos(4\pi f_c t)]dt = \pm\frac{T_b}{2} \pm \frac{1}{8\pi f_0}\sin(4\pi f_c T_b)$$

$$= \pm\frac{T_b}{2} \pm \frac{1}{8\pi f_0}\sin(40\pi) = \pm\frac{T_b}{2} = \pm 5\times 10^{-6}$$



$$p_e = \frac{1}{\sqrt{2\pi}\sigma}\int_{-\infty}^{0} e^{-\frac{(z-z_n)^2}{2\sigma^2}} dz = \frac{1}{\sqrt{2\pi}\sigma}\int_{z_n}^{\infty} e^{-\frac{z^2}{2\sigma^2}} dz$$

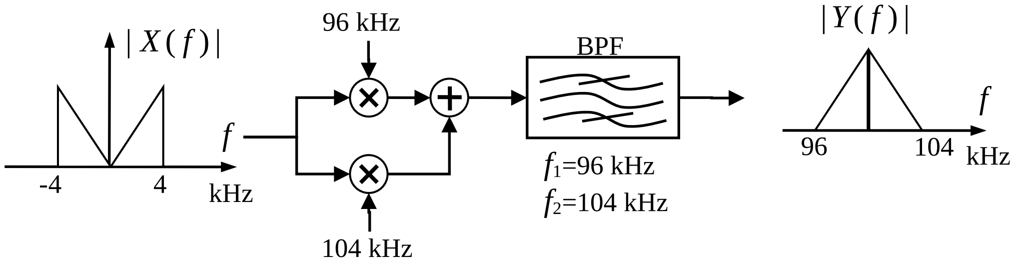
Letting $\frac{z^2}{2\sigma^2} = \frac{u^2}{2}$, $z = \sigma u$ and $dz = \sigma du$, and for the lower boundary

$\frac{z_n}{\sigma} = u_n = \frac{5\times 10^{-6}}{\sqrt{10^{-11}}} \cong 1.58$ we get $p_e = \frac{1}{\sqrt{2\pi}}\int_{u_n=1.58}^{\infty} e^{-\frac{u^2}{2}} du = Q(1.58)$.

From tables of $Q(x)$, we find $Q(1.58) \cong 0.056$, that is $p_e \cong 0.056$.

A16.

Among the several solutions, one with two DSB modulators and a BPF is shown below.

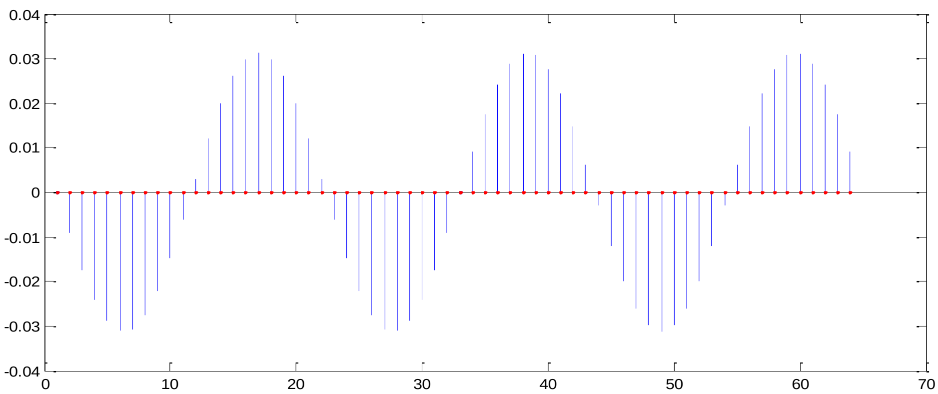


A17.

$X(3)$ and $X(61)$ corresponds to 3rd harmonic, that is 3 times the frequency of the fundamental frequency. We will have 64 samples of exactly 3 periods of a real sinusoid. Imaginary part will be zero since the complex numbers given are conjugate of each other. +j in $X(3)$ and -j in $X(62)$ corresponds to a cosine signal shifted by +90°, meaning that it is a negative sine. Therefore, the output of IFFT is 64 samples of real -sine signal.

Experimenting in Matlab/Octave : Since Matlab indexes start from 1 instead of 0, we should increment the indexes to get the Matlab array indexes. That is, the array indexes will be $X(4)$ and $X(62)$ in frequency domain.

```
x=zeros(64,1);x(4)=1i;x(62)=-1i; y=ifft(x);
figure;stem(real(y),'marker','none');
hold on;stem(imag(y),'marker','none','color','red');
```



When this samples are played out 1 sample per μs, a period is completed in 64/3 μs. So the frequency of the sinusoidal when converted to analog will be

3/64 [1/μs] = 3/64 MHz ≈ 47 kHz. (this answer is for $y(t)$ in a). it should be multiplied by 2 for b)).