Resampling

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For the course "`Data Compression`"

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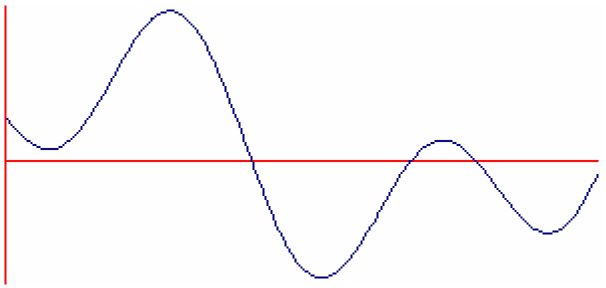
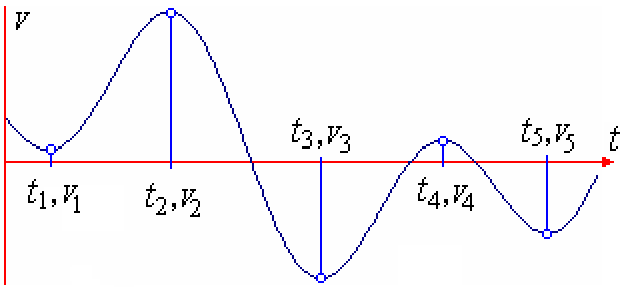
Sampling

Problem : We have a continuous waveform which we would like to replicate at another location, but we are only allowed to describe it.

Is it possible? : Someone looks at the signal and tells about the shape of it to someone at the other end (eg. over the phone) and tries to make him construct the waveform. ☺



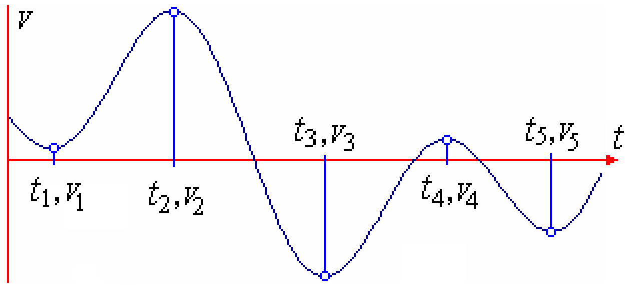
Example : Tell some values at some critical points. How about min-max points?



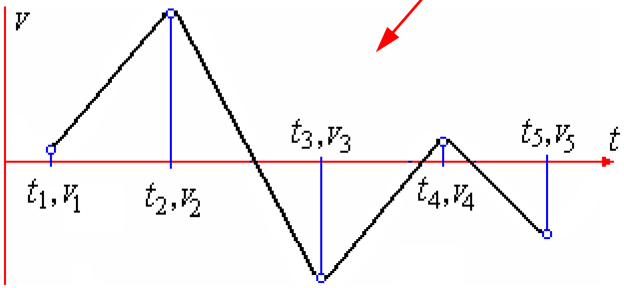
…and transmit time and value

$$(t_1,V_1)(t_2,V_2)(t_3,V_3)(t_4,V_4)(t_5,V_5)\ldots$$

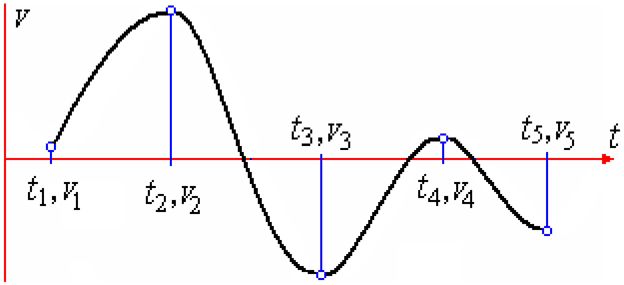
…and reconstruct the signal from its irregular samples.



reconstruct from samples

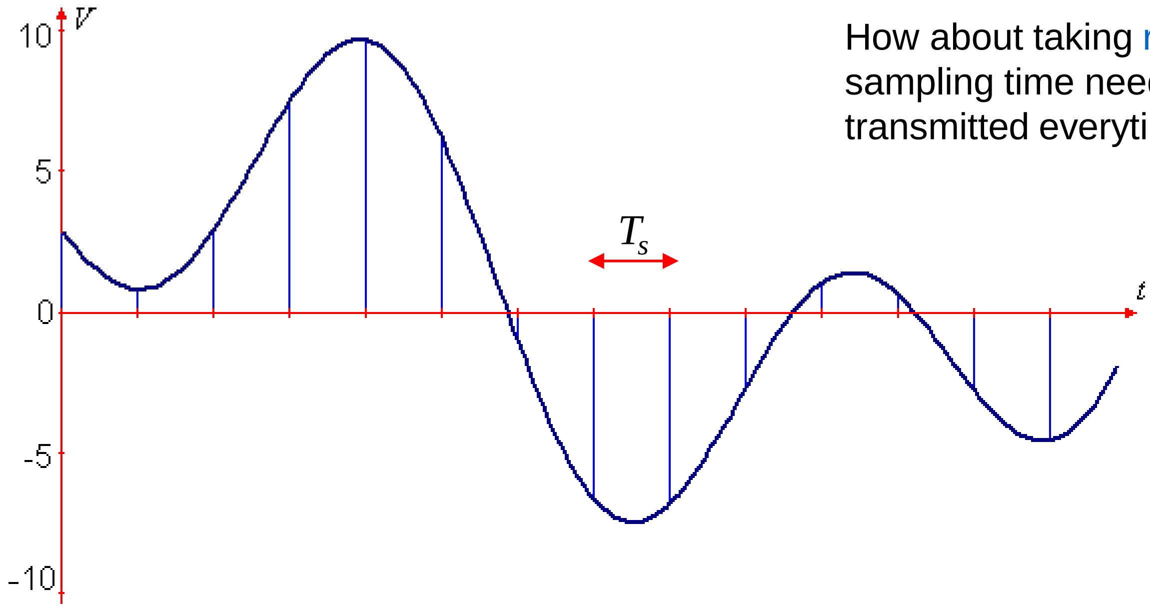


linear interpolation

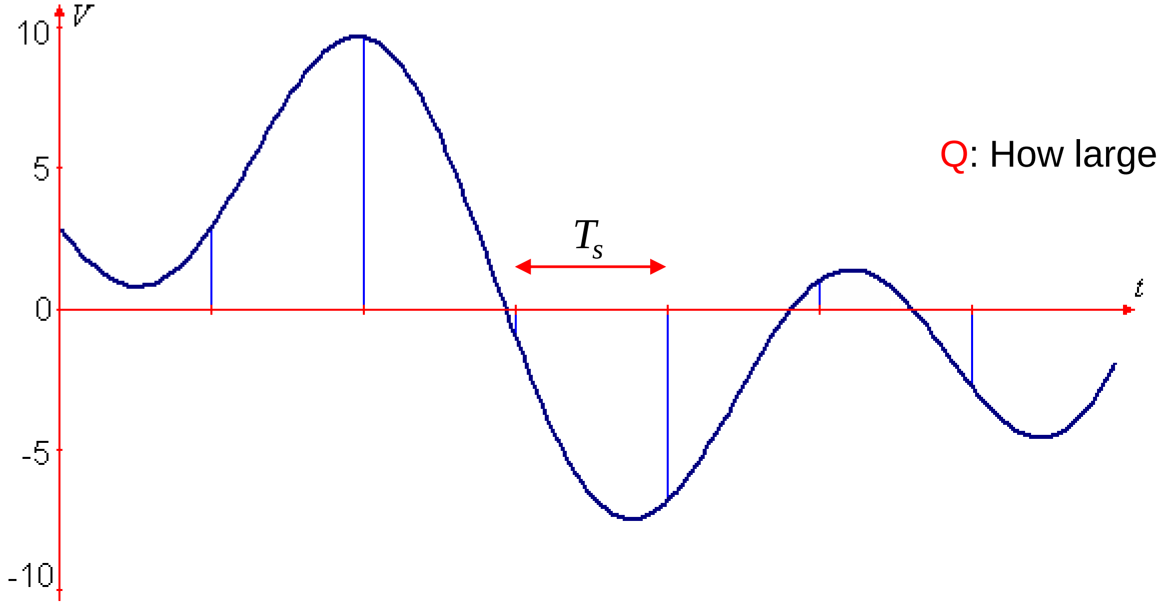


Higher order curve fit techniques

Any difficulty ? ☹



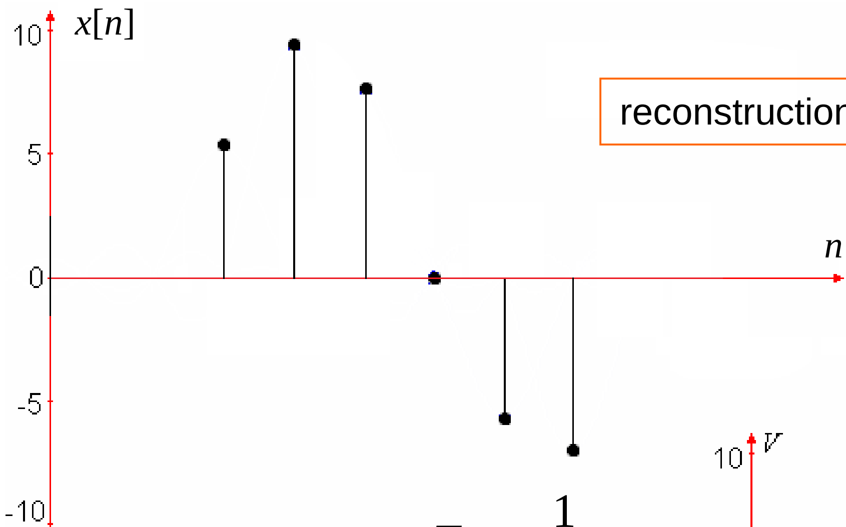
How about taking regular samples in which sampling time need not be measured and transmitted everytime a sample is taken?



Q: How large the intervals can be?

Nyquist's sampling criterion : A baseband signal with the highest frequency component at f_m can be reconstructed from its samples if the interval between samples is less than $1/2f_m$

That is; sampling frequency must be higher than twice the highest frequency of the signal

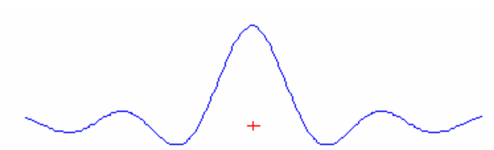


reconstruction formula

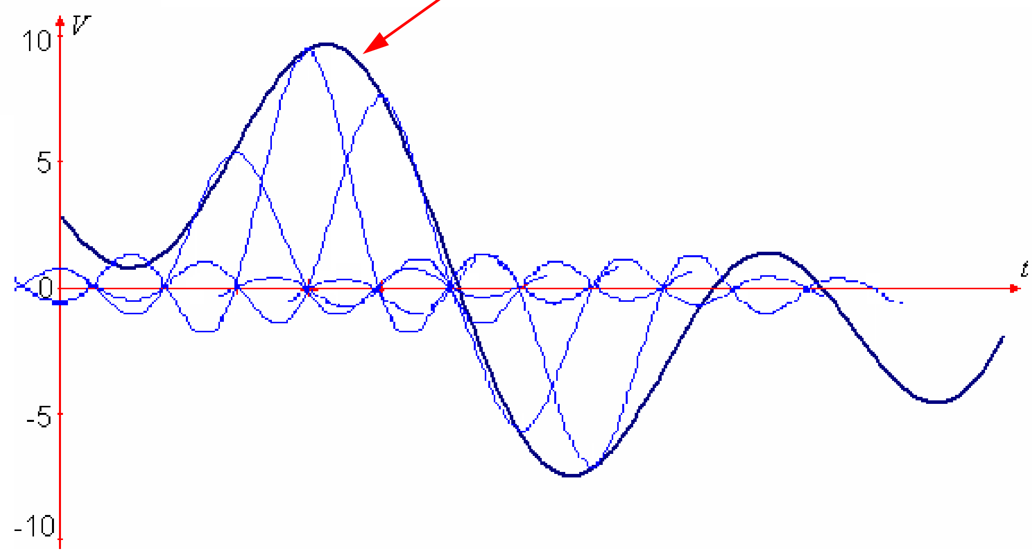
$$x(t) = \sum_{n=-\infty}^{n=\infty} x(nT_s)\,\mathrm{sinc}(2f_m(t-nT_s))$$

The sum of all **sinc**s

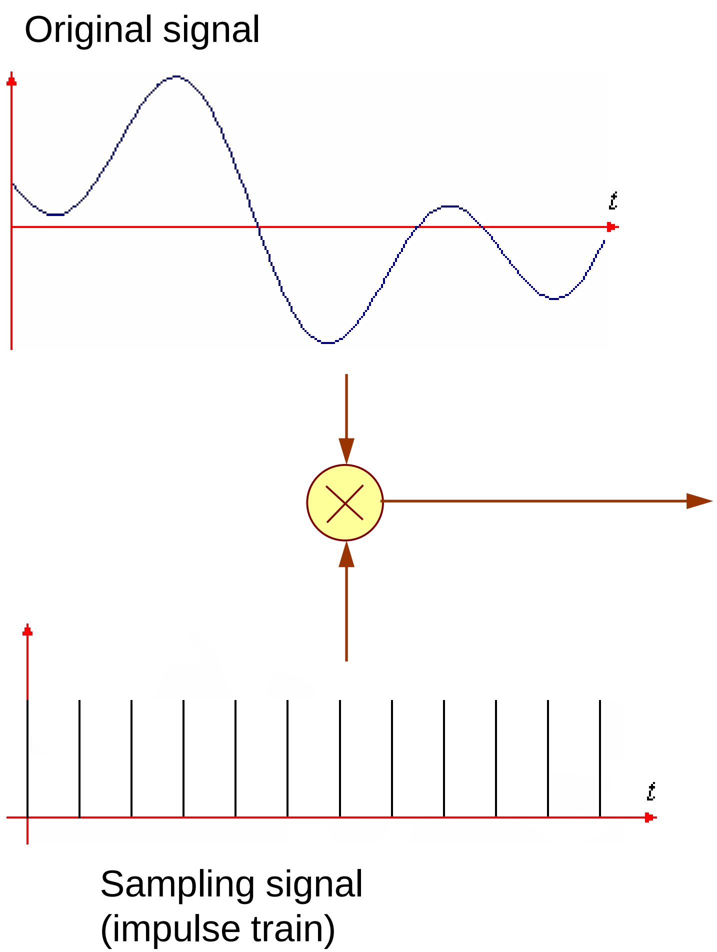
a special case when $T_s = \frac{1}{2f_m}$

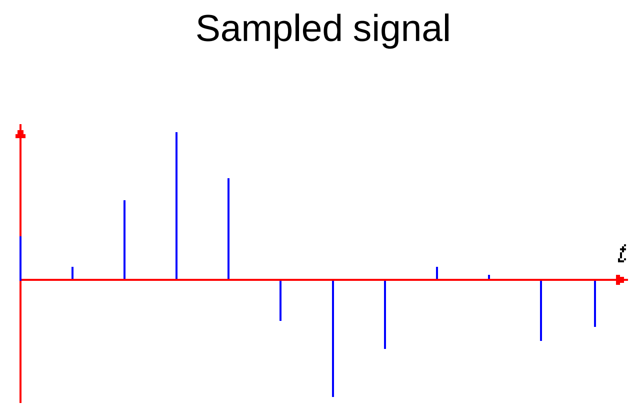


truncated sinc function



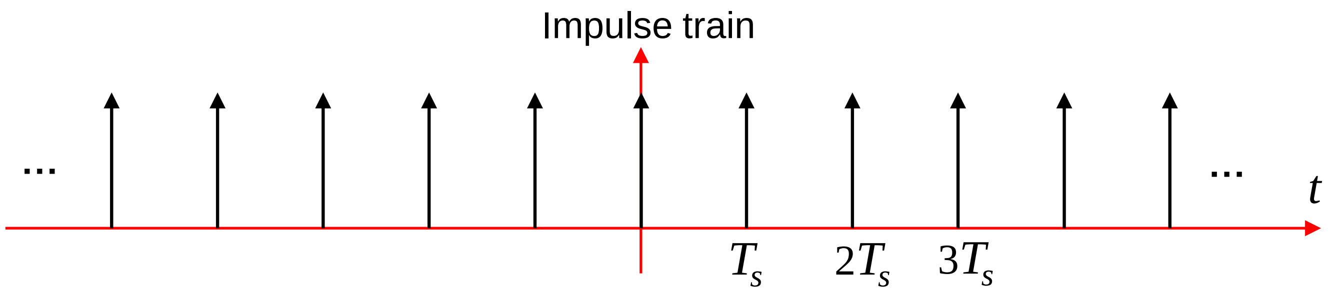
What happens in frequency domain

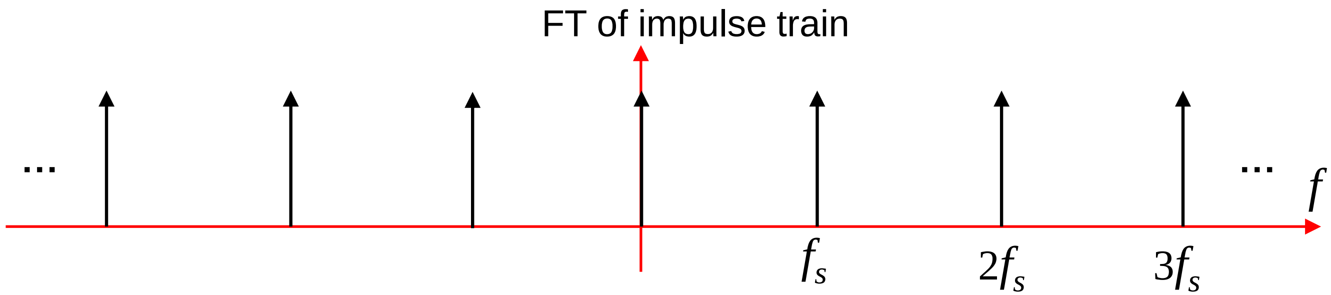




Q: How is the spectrum of the sampled signal?

Spectrum of the sampling train





FT property $\mathcal{F}\{x(at)\} = \frac{1}{|a|} X\left(\frac{\omega}{a}\right) \longrightarrow f_s = 1/T_s$

$$s(t) = \sum_{i=-\infty}^{\infty} \delta(t - iT_s) \quad \Leftrightarrow \quad S(f) = f_s \sum_{i=-\infty}^{\infty} \delta(f - if_s)$$

FT Property (convolution)

Convolution in time domain corresponds to Multiplication in frequency domain

$$\mathcal{F}\{x(t) * y(t)\} = \mathcal{F}\{x(t)\}\mathcal{F}\{y(t)\} = X(f)Y(f)$$

convolution is defined as $$x(t) * y(t) = \int_{-\infty}^{\infty} x(\tau)y^*(t-\tau)d\tau$$

Multiplication in time domain corresponds to Convolution in frequency domain

$$\mathcal{F}\{x(t)y(t)\} = \mathcal{F}\{x(t)\} * \mathcal{F}\{y(t)\} = X(f) * Y(f)$$

or, shortly

$$x(t)y(t) \Leftrightarrow X(f) * Y(f)$$

$$x(t) * y(t) \Leftrightarrow X(f)Y(f)$$

Sifting Property of Impulse

convolution with impulse

$$\int_{-\infty}^{\infty} \delta(t - T)x(t)dt = x(T)$$

same in F domain

$$\int_{-\infty}^{\infty} \delta(f - T)X(f)df = X(T)$$

in general $\int_{a}^{b} \delta(t - T)x(t)dt = \begin{cases} x(T) & a < T < b \\ 0 & otherwise \end{cases}$

therefore

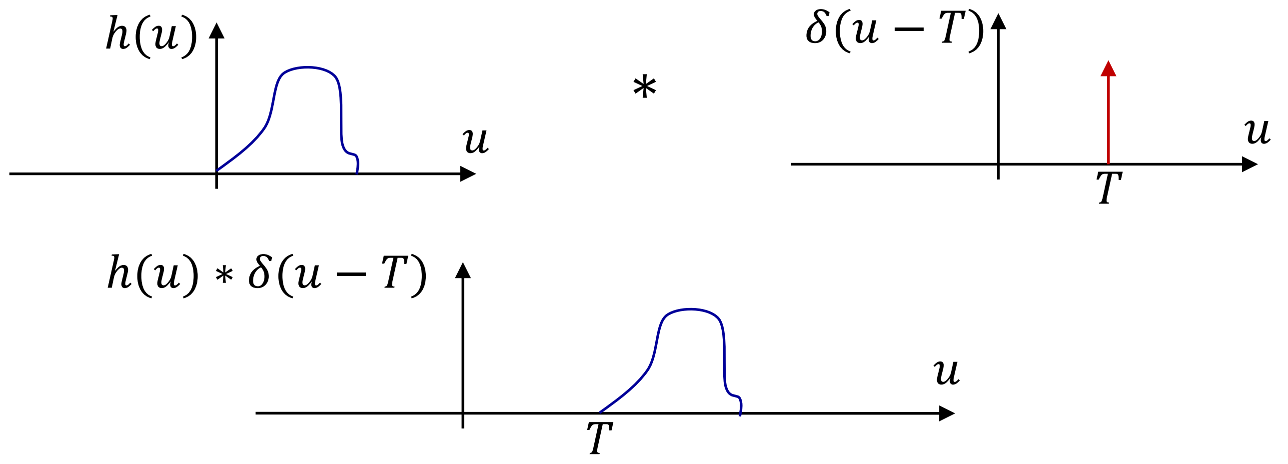
$$\int_{-\infty}^{\infty} \delta(\tau - t)x(t)dt = \int_{-\infty}^{\infty} \delta(t)x(\tau - t)dt = x(\tau)$$

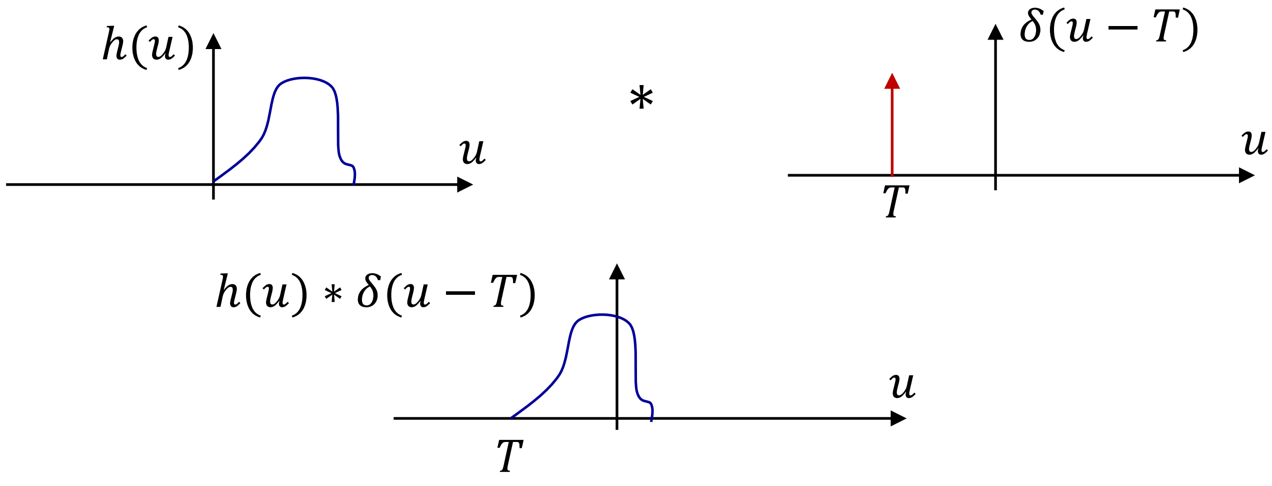
which means $\delta(t) * x(t) = x(t)$

similarly $\delta(t - T) * x(t) = x(t - T)$

Sifting Property of Impulse

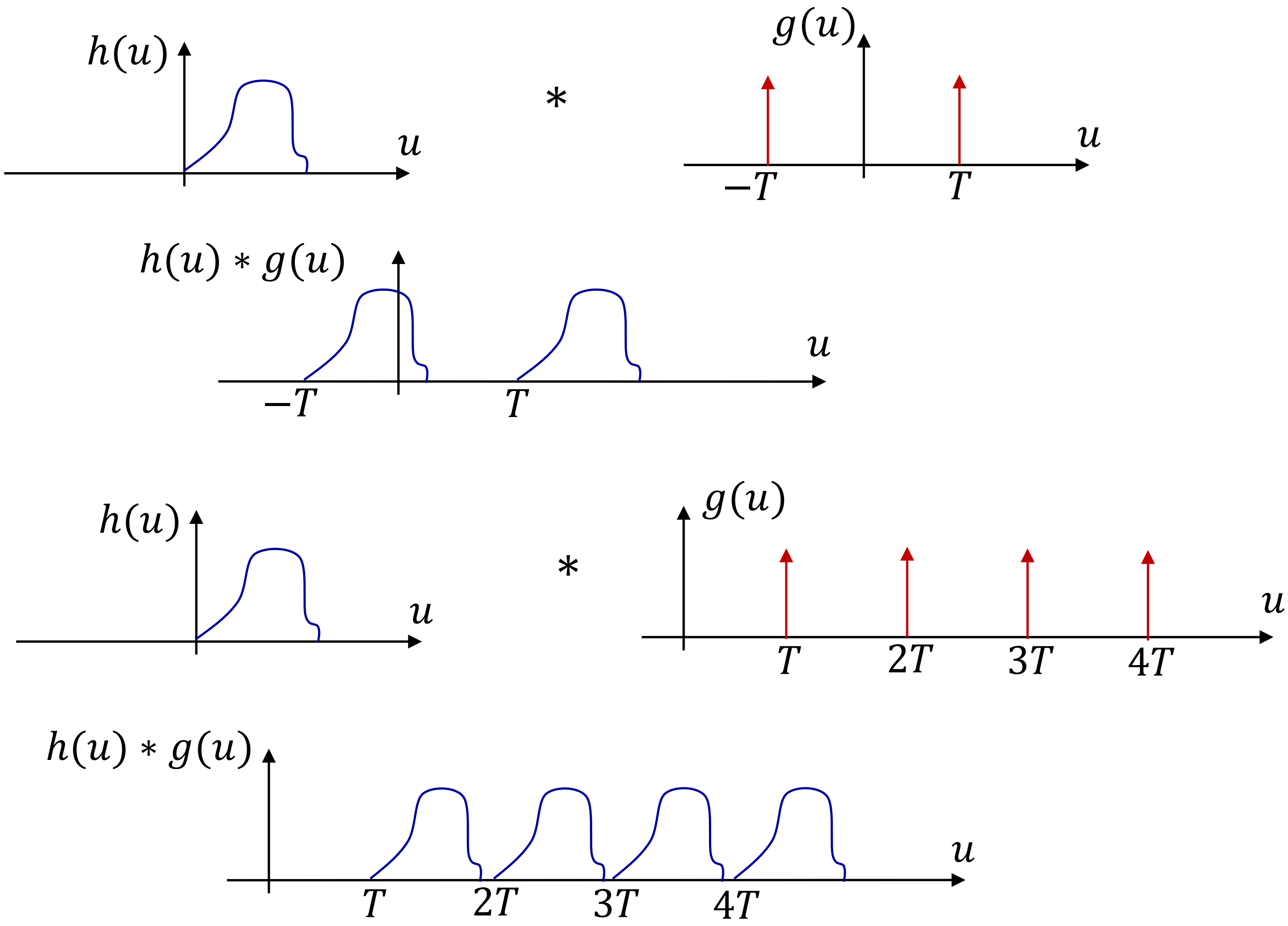
convolution with impulse



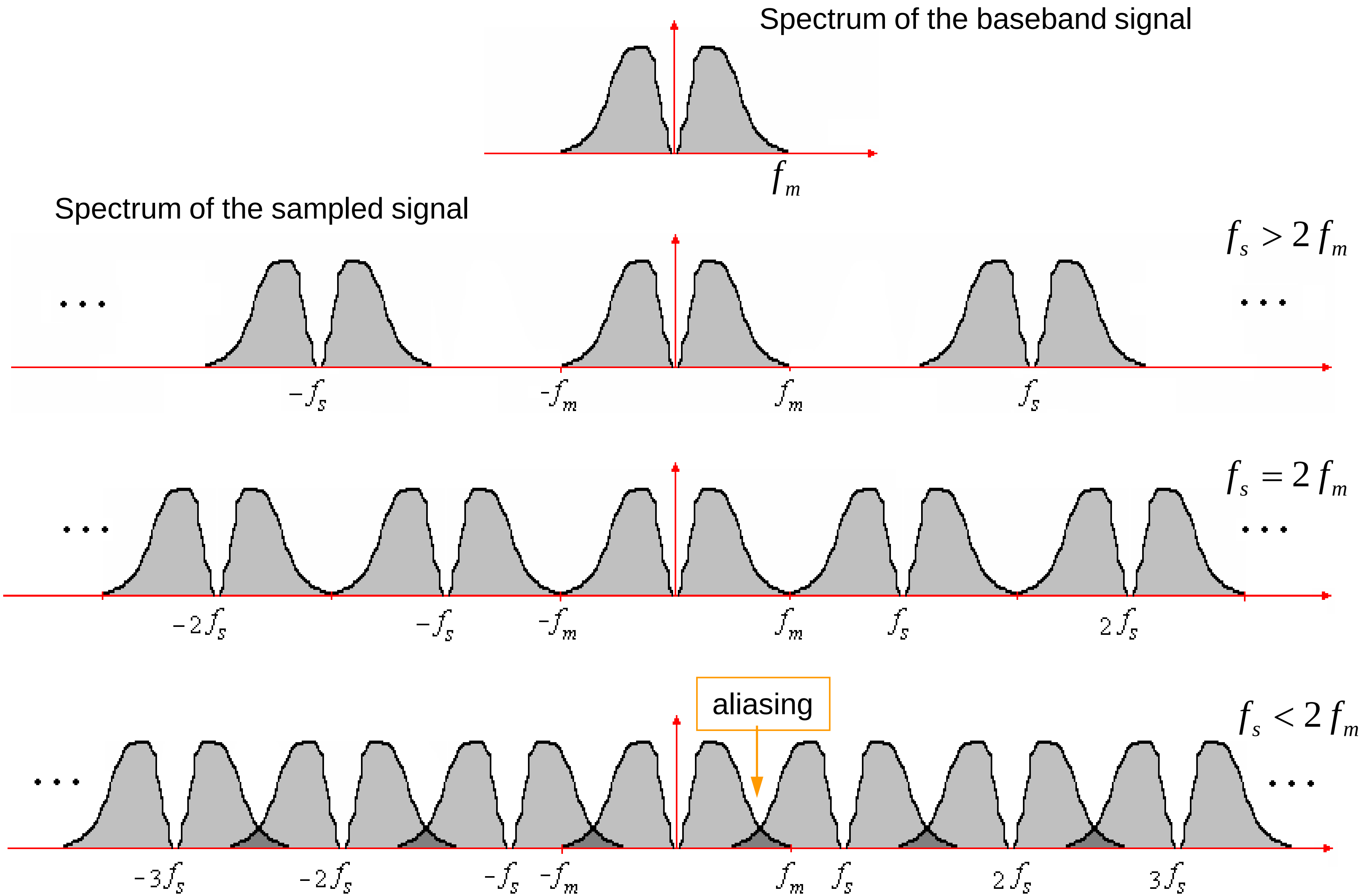


Sifting Property of Impulse

convolution with impulse

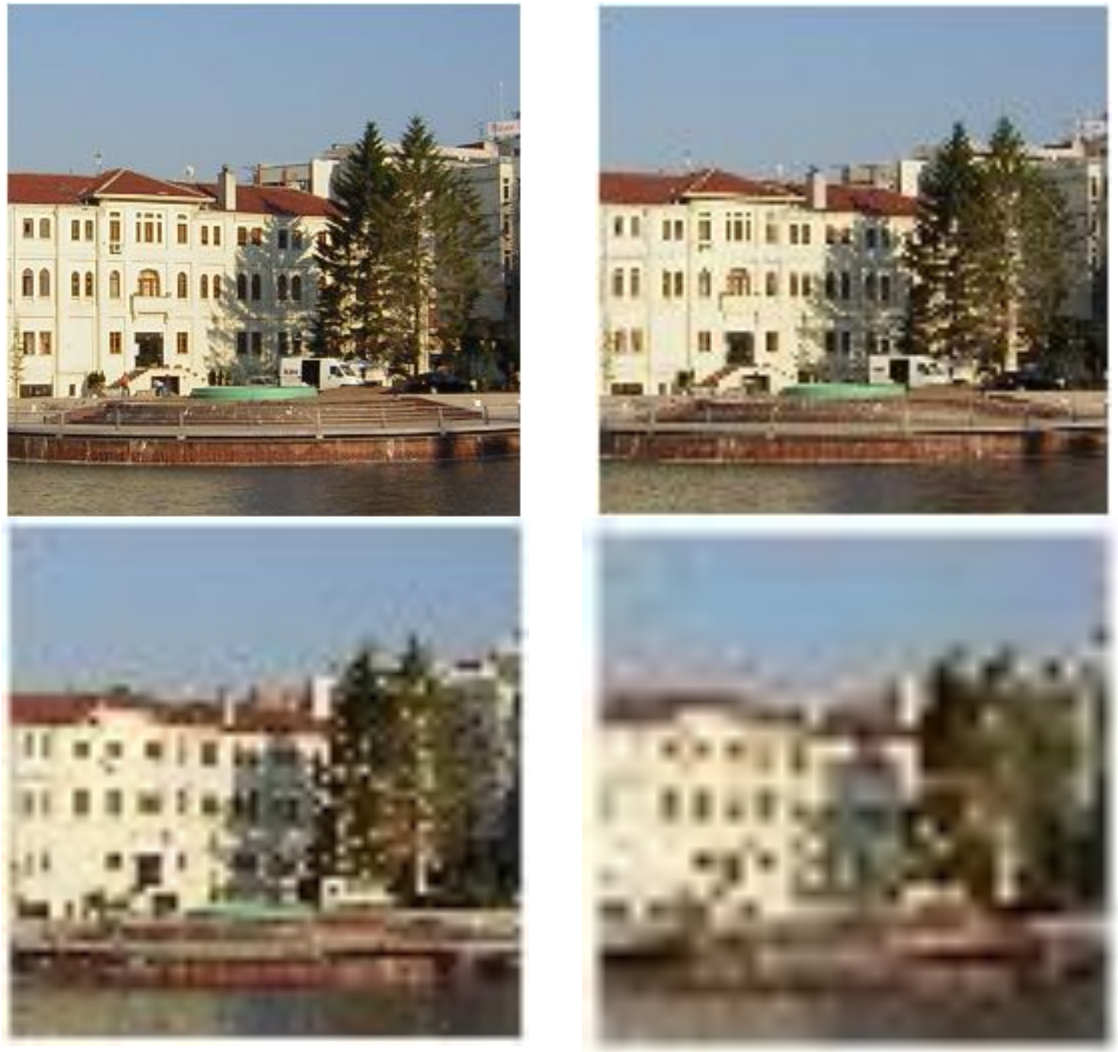


Spectrum of the sampled signal



now we see why Nyquist has such a criteria

Re-Sampling



high frequency components are lost

depending on how re-sampling and interpolation is done, information loss is inevitable

note: pictures are resized with interpolation

Re-Sampling

original

upper row : downsampled in image domain

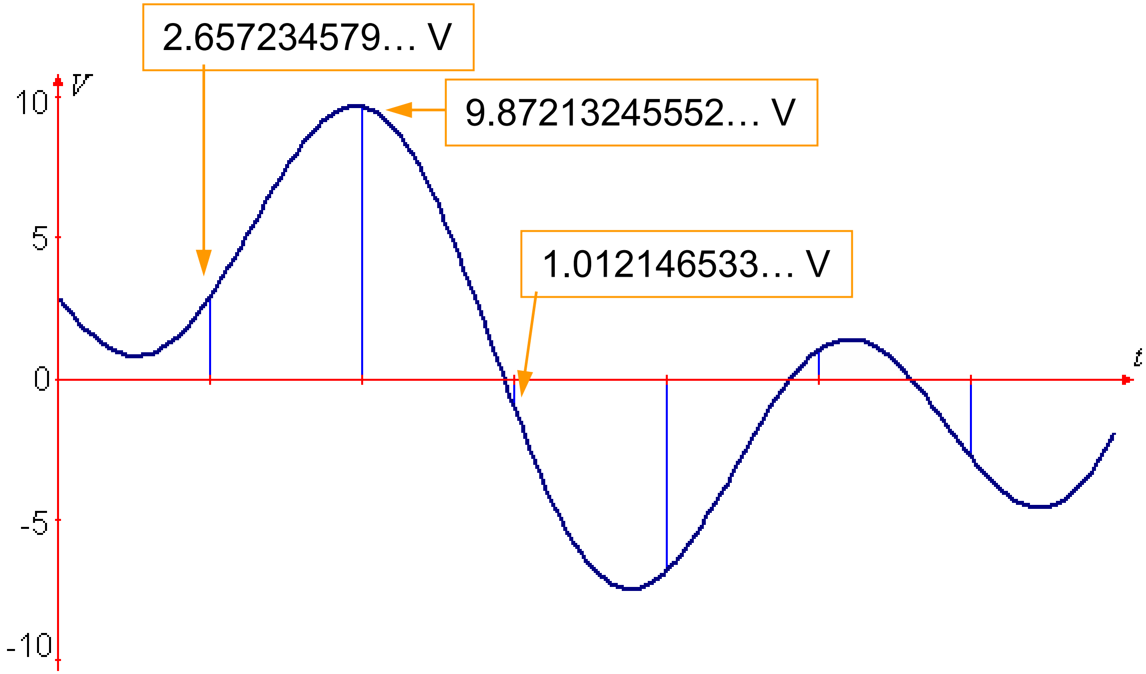
lower row : downsampled in spatial frequency domain

1/2 of the samples are deleted

3/4 of the samples are deleted

It seems that downsampling in frequency domain preserves more detail (no quantization involved yet)

Quantization

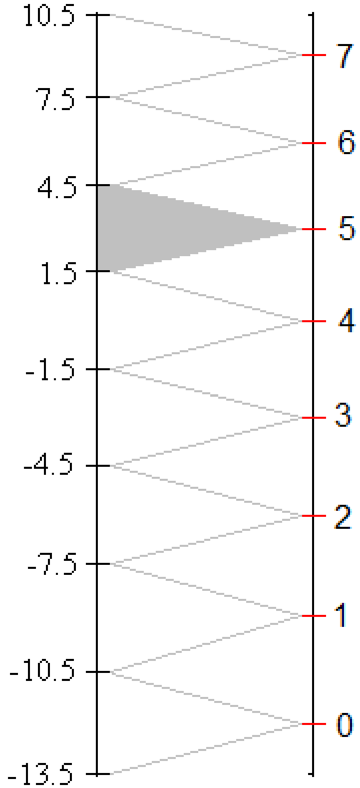


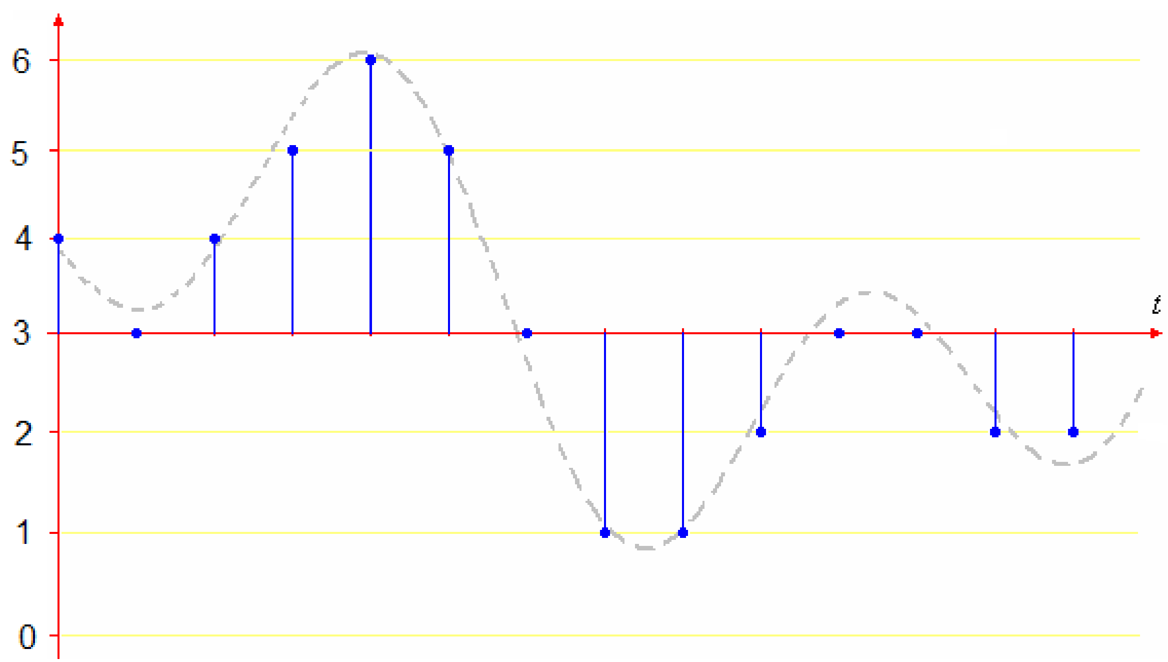
We can not keep the sample values like 2.657234579… V 9.87213245552… V …?

We need to reduce the number of digits like 2.6… V 9.8… V etc. for obvious reasons

The truncation done is called **quantization**.

It is a mapping from ranges to integer numbers



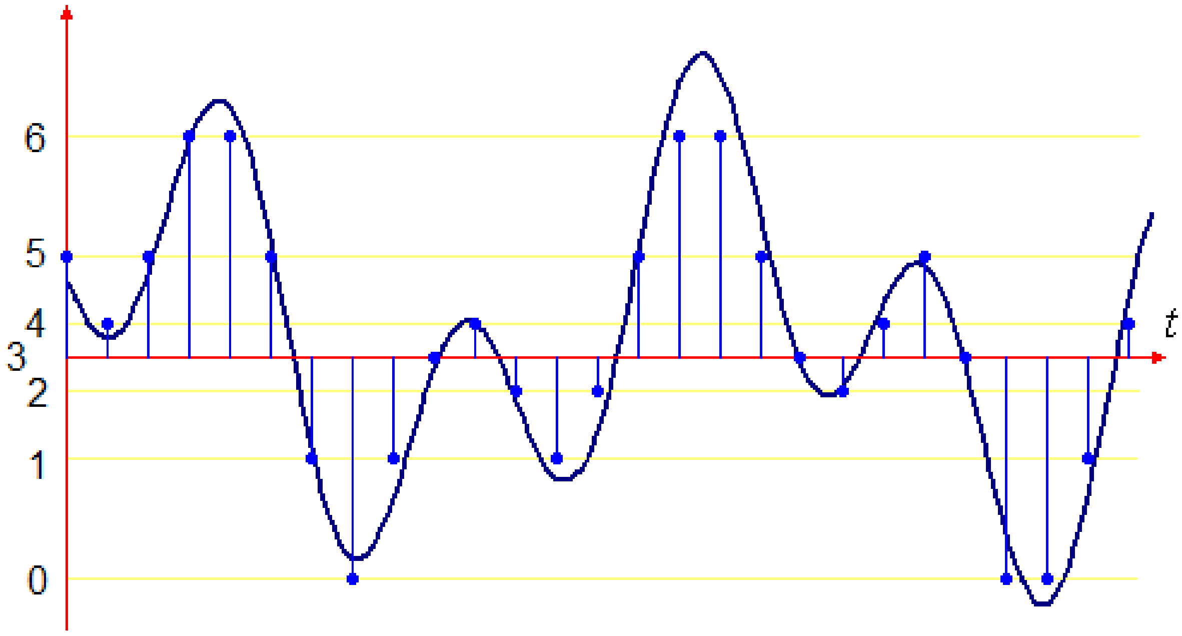


obviously there are some differences what the sample values are and what they are going to be after the truncation/quantization. Because of this differences, the original value can never be recovered.

Q : Why are quantization levels uniform?

A : There is no reason other than practical issues. One can use non-uniform levels for best representation, where *best* depends on the application.

Here are non-uniform levels for a signal that mostly roams around zero



Keys for compression of statistically dependent data:

(statistically dependent = high correlation with itself,
meaning that it is not a series of random numbers)

1- Avoid/eliminate redundancy/oversampling
2- Determine a quantization schema optimized for the application
3- Have a reasonable balance between loss and
   - a) Re-sampling ratio
   - b) Number of quantization levels

4- Have a reasonable balance between compression ratio and algorithmic complexity

The compression schemas JPEG and MPEG try to achieve these goals for applications in which images and motion picture sequences are intended for human visual sense.

END