

No : Answer **Name : Solution**
Eskişehir Osmangazi University, Faculty of Engineering and Architecture
Department of Electrical Engineering & Electronics, “Communications”

Name : Solution

Eskişehir Osmangazi University, Faculty of Engineering and Architecture

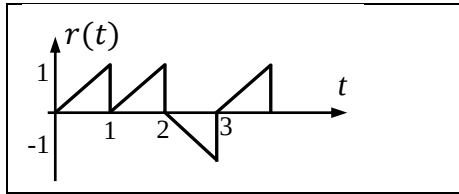
Department of Electrical Engineering & Electronics, “Communications” Final

1.07.2026

I handed over all smart devices (mobile phone, smartwatch, earphones, etc.) to the exam supervisor before the exam. I take full responsibility if any are found on me.
Exam duration: 90 minutes. No books, no notes, no communication.

Sign here

1. $r(t)$ input to a correlator is given as shown in the figure. Correlation period is $T = 1$ and its other input is $\psi(t) = 1$. What is the correlator output value at $t = 2$?



- a) -0.5 d) 0.5
b) 1 e) -0.25
c) -1 f) -1/3

$$R(2) = \int_2^3 r(t)\psi(t)dt = \int_0^1 tdt = \left.\frac{t^2}{2}\right|_0^1 = 0.5$$

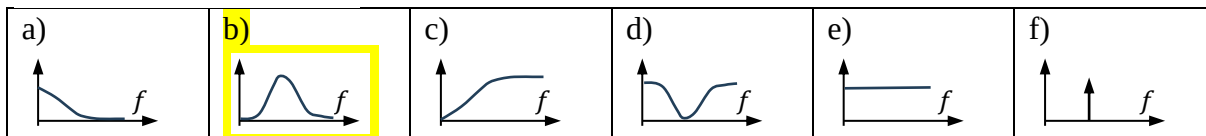
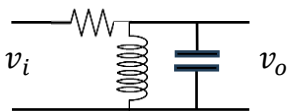
2. What is the noise-equivalent bandwidth of a LPF defined as $|H(f)| = \begin{cases} 1 & |f| < 1 \\ 2 - |f| & 1 < |f| < 2 \\ 0 & \text{otherwise} \end{cases}$?

- | | | | | | |
|---------|----------------|---------|---------|--------|---------|
| a) 2.27 | b) 1.33 | c) 0.71 | d) 0.53 | e) 3.5 | f) 1.99 |
|---------|----------------|---------|---------|--------|---------|

$$P_{neq} = 2 \int_0^1 1^2 df + 2 \int_1^2 |2 - f|^2 df = 2 + 2 \int_0^1 f^2 df = 2 + 2 \left. \frac{f^3}{3} \right|_0^1 = \frac{8}{3} \cong 2.67 \text{ (two sided)}$$

$$2B_{neq} \times 1^2 = 2.67 \Rightarrow B_{neq} = \frac{2.67}{2} = 1.33 \text{ Hz.}$$

3. Input to a RLC-filter is White Gaussian Noise with spectral density $N_o = 1 \mu\text{W/Hz}$. What would the shape of output noise spectral density be like when R, L and C have reasonable values?



4. The symbol probabilities of a memoryless source are given as $z = \{0.3, 0.3, 0.2, 0.1, 0.1\}$. What is the average codeword length of the optimal Huffman code?

- | | | | | | |
|----------|--------|--------|----------|--------|---------|
| a) 1.625 | b) 3.0 | c) 2.0 | d) 1.875 | e) 2.2 | f) 1.65 |
|----------|--------|--------|----------|--------|---------|

Finding Huffman tree;

$p_5=0.1+0.1=0.2$. $p_6=0.2+0.2=0.4$. $p_7=0.3+0.3=0.6$. $p_8=0.6+0.4=1.0$ (check). Code lengths are then $L=\{2,2,2,3,3\}$ which makes the average to be

$$L_{avg} = \sum_{i=0}^4 p_i l_i = 0.3 \times 2 + 0.3 \times 2 + 0.2 \times 2 + 0.1 \times 3 + 0.1 \times 3 = 2.2 \text{ bits/sym}$$

5. What is the output value at $t = 2$ of the matched filter designed for the waveform

3. What is the output value at $t = 2$ of the matched filter designed for the waveform $\psi(t) = \begin{cases} 1 & 0 < t < 1 \\ -1 & 1 < t < 2 \\ 0 & \text{elsewhere} \end{cases}$ when the same waveform is applied to its input?

- | | | | | | |
|------|------|-------|-------|------|------|
| a) 2 | b) 1 | c) -2 | d) -1 | e) 4 | f) 0 |
|------|------|-------|-------|------|------|

We get matched filter by flipping $\psi(t)$ in time axis and sliding it towards $+t$ axis in order to make it

causal. When done, it will be $h_\psi(t) = \begin{cases} -1 & 0 < t < 1 \\ 1 & 1 < t < 2 \\ 0 & \text{elsewhere} \end{cases}$. Response is the convolution of $\psi(t)$ and $h_\psi(t)$,

[illegible]

which is the same as autocorrelation function of $\psi(t)$ (shifted to the right to make it causal response). However, since the value at $t = 2$ is asked, the answer is E_ψ . Hence,

$$E_\psi R_\psi(\tau = 2) = \int_0^2 \psi^2(t) dt = 2.$$

6. A communication system uses binary antipodal pulses of $\psi_i(t) = \pm \begin{cases} 1 & , 0 < t < 1 \\ -0.5 & , 1 < t < 3 \\ 0 & , \text{elsewhere} \end{cases}$ with symbol period $T_b = 3$. What is the average power?

a) 3.2	b) 4	c) 1	d) 1.5	e) 0.5	f) 1.6
--------	------	------	--------	--------	--------

$$P = \frac{1}{T_b} \int_0^{T_b} |\psi(t)|^2 dt = \frac{1}{3} \left(\int_0^1 1^2 dt + \int_1^3 0.5^2 dt \right) = \frac{1}{3} (1 + [0.25t]_1^3) = \frac{1}{3} (1 + 0.5) = 0.5.$$

7. A communication channel contains low frequency hum noise centered around 50 Hz whose two-sided psd is approximated as $G_\eta(f) = e^{-||f|-50|}$ W/Hz. What should be the lowest cutoff frequency of the ideal high-pass filter at the input of the receiver for the filter output noise power be less than 50 μ W?

a) 69.4 Hz	b) 60 μ W	c) 51 Hz	d) 101.2 Hz	e) 1.2 kHz	f) 60.6 Hz
------------	---------------	----------	-------------	------------	------------

$$P_\eta = 2 \int_{f_c}^{\infty} G_\eta(f) df = 2 \int_{f_c}^{\infty} e^{-(f-50)} df = 2e^{-(f_c-50)} \text{ and we require } P_\eta < 50 \mu\text{W} \text{ or explicitly } 2e^{-(f_c-50)} = 50 \mu\text{W} \text{ for minimum } f_c \text{ value. From here, we find that } f_c \geq 60.6 \text{ Hz.}$$

8. Which of the following modulation schemes have a constant envelope?

A. 8-PSK

B. Binary FSK

C. 16-QAM

D. FM

E. VSB-AM

F. 4-ASK

a) D,E,F	b) A,B,D	c) A,C,E	d) B,D,E	e) A,E,F	f) C,D,F
----------	----------	----------	----------	----------	----------

9. The QPSK signal with carrier frequency of 10 kHz received from a noisy communication channel has peak amplitude of 10. Assume that the symbol rate is low enough that almost all of the signal power is contained within the frequency range 8–12 kHz. Noise has two-sided power spectral density of $N(f) = |f|$ nW/Hz up to 30 kHz and flat afterwards. The receiver uses an ideal low-pass filter with $f_{cutoff}=20$ kHz at the input. What is the SNR at the output of the filter?

a) 20	b) 0.4	c) 50 dB	d) 21 dB	e) 50	f) 125 dB
-------	--------	----------	----------	-------	-----------

Since the signal frequency (10 kHz) is within passband of the filter, signal power is $P_s = \frac{V_P^2}{2} = 50$. (Note: we assumed an ideal filter. Even in that case, the signal should be well within passband of the filter as filters remove outband components of the signal. Think of sinc). Noise power is $P_\eta = 2 \int_0^{20k} f 10^{-9} df = 10^{-9} f^2 |_0^{20k} = 10^{-9} \times 4 \times 10^8 = 0.4$.

$$\text{SNR is then, } \text{SNR} = \frac{P_s}{P_\eta} = \frac{50}{0.4} = 125 \cong 21 \text{ dB.}$$

10. A correlator receiver is used for receiving 4-ASK $r(t) = A \cos(20\pi t)$ under AWGN where $A = \{-3, -1, 1, 3\}$ and $T_s=1$ s. Symbols have equal probability. Assuming that the receiver is in sync with the received signal and that the local oscillator generates $\psi(t) = \cos(20\pi t)$, what are the ML decision thresholds for the decision circuitry that follows the integrator output?

a) $\{-2, 0, 2\}$	b) $\{-3, -1, 1, 3\}$	c) $\{-2, 2\}$	d) $\{-1, 0, 1\}$	e) $\{-5, 5\}$	f) $\{0\}$
-------------------	-----------------------	----------------	-------------------	----------------	------------

$$R = \int_0^1 A \cos^2(20\pi t) dt = \frac{A}{2} \int_0^1 (1 - \cos(40\pi t)) dt = \frac{A}{2} \text{ at the end of } T_s \text{ s.}$$

Hence the correlator outputs at measurement instants are expected to be $\{\frac{-3}{2}, \frac{-1}{2}, \frac{1}{2}, \frac{3}{2}\}$. Hence the ML threshold between them should be $\{-1, 0, 1\}$.

11. Why is Gray coding commonly used when mapping bits onto a QAM constellation?

a) It minimizes the average transmitted power for a given constellation.
b) It increases the minimum Euclidean distance between adjacent symbols.
c) It allows coherent detection with a simpler receiver architecture.
d) It reduces the bandwidth required to transmit the same bit rate.
e) It eliminates the need for carrier phase synchronization.
f) It reduces the average number of bit errors caused by a symbol error.

12. A single integrator receiver is used for receiving signals of binary antipodal rectangular pulses of $\mp A$ with $T_b=100\text{ms}$. Noise pdf at the integrator output is given as $f_x(x) = e^{-|x|}/2$. What should be the minimum value of A such that $p_e \leq 0.01$?

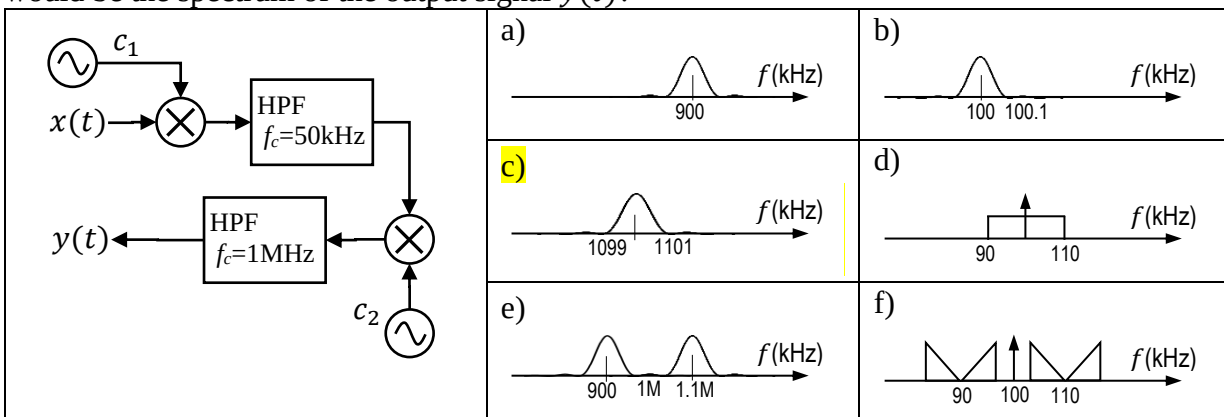
a) 39.1	b) 9.78	c) 19.6	d) 46.1	e) 78.2	f) 391
----------------	---------	---------	---------	---------	--------

$R_b = AT_b = A \times 0.1$. $p_e = \int_{R_b}^{\infty} f_x(x) dx = \frac{1}{2} \int_{0.1A}^{\infty} e^{-|x|} dx = -\frac{1}{2} e^{-x} \Big|_{0.1A}^{\infty} = \frac{1}{2} e^{-0.1A}$. For $p_e \leq 0.01$, $\frac{1}{2} e^{-0.1A} \leq 0.01$. This gives $A \geq -\frac{\ln(0.02)}{0.1} \cong 39.1$.

13. Why must the receiver know the PN sequence in a DSSS communication system?

a) To improve the signal-to-noise ratio without despreading.
b) To determine the carrier frequency of the transmitter.
c) To recover the transmitted signal by despreading.
d) To eliminate additive white Gaussian noise.
e) To increase the transmitted data rate.
f) To reduce the occupied bandwidth of the transmitted signal.

14. A random binary rectangular pulse train $x(t)$ (baseband signal) with symbol rate $T_s = 1\text{ms}$ is applied to the following system where c_1 and c_2 are 100 kHz and 1 MHz sinusoidal signals respectively. What would be the spectrum of the output signal $y(t)$?



By multiplying c_1 we upconvert the signal to 100kHz. First HPF do not have any measurable effect as the signals bandwidth is only 2 kHz. Multiplying this with 1 MHz carrier, we will have two sincs at 900 kHz and 1.1 MHz. Second HPF filter removes the mirror at 900 kHz. What remains is a sinc at 1.1 MHz with null-to-null bandwidth of 2 kHz.

15. Why does applying the same lossless compression algorithm a second time usually provide little or no additional compression?

a) The second compression increases the transmission bandwidth instead of reducing the file size.
b) The second compression converts the compressed data into an encrypted bit stream.
c) The first compression increases the entropy of the original information source.
d) The second compression must use a different codebook to reduce the data further.
e) The compression algorithm cannot process data that has already been compressed.
f) The first compression has already removed most of the statistical redundancy in the data.

16. A baseband OFDM system has one of its subcarriers at 27 kHz. The useful OFDM symbol duration is 1/3 ms. Which of the following could be another subcarrier frequency in this OFDM system?

a) 11 kHz.
b) 12 kHz.
c) 32 kHz.
d) 38 kHz.
e) 4 kHz.
f) 40 kHz.

$f_s = \frac{1}{T_s} = \frac{1}{\frac{1}{3}ms} = 3 \text{ kHz}$. So the subcarrier frequencies must be integer multiples of 3 kHz. There is only one in the choices given: 12 kHz.

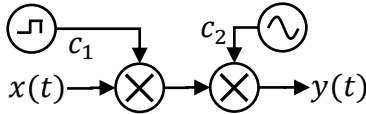
17. An OFDM system employs 64 subcarriers each using BPSK, QPSK or 64-QAM. DC subcarrier is reserved for pilot transmission and carries no user data. The symbol duration is 4 μs (no CE). What is null-to-null bandwidth?

a) 64 MHz	b) 15.75 MHz	c) 250 kHz	d) 8.25 MHz	e) 16.25 MHz	f) 16 MHz
-----------	--------------	------------	-------------	--------------	-----------

$\frac{1}{4\mu s} = 250 \text{ kHz}$ (distance between carriers)

$(64 + 1) \times 250k = 16.25 \text{ MHz}$

18. $x(t)$ random binary rectangular pulse train is first multiplied by SSRG[10,3] m-sequence pn-generator output (c_1) and frequency upconverted to 5 GHz by multiplying the result with 5 GHz sinusoidal carrier (c_2) as shown. Rate of the $x(t)$ signal is 50 kbps, and the chip rate of the pn-generator is 2 sequence-period per bit. What is the null-to-null bandwidth of the resulting signal $y(t)$?

	a) 204.6 MHz	b) 50 MHz	c) 200 MHz
	d) 102.4 MHz	e) 102.1 MHz	f) 1 GHz

$L = 2^{10} - 1 = 1023$. Two sequence per bit is given, so chip-rate is $50 \text{ kbps} \times 2046 = 102.3 \text{ Mchips/s}$. Therefore, null-to-null bandwidth is 204.6 MHz, independent from the carrier frequency.

19. A FHSS system employing 4-FSK uses a hop rate of 10 kHz (10000 hops/s). Assuming one hop per symbol, determine the bit rate.

a) 10 kbps	b) 16 kbps	c) 20 kbps	d) 2 kbps	e) 28 kbps	f) 32 kbps
------------	------------	------------	-----------	------------	------------

4-FSK means 2 bits/symbol. $10000 \text{ hops/s} \times 2 = 20 \text{ kbps}$

20. An OFDM system employs 64 subcarriers. DC subcarrier is reserved for pilot transmission and carries no user data. 10 of the subchannels use 64-QAM and the rest use QPSK. The symbol duration is 4 μs (no CE). What is the bit-rate?

a) 24.8 Mbps	b) 32 Mbps	c) 15.5 Mbps	d) 124 Mbps	e) 62 Mbps	f) 31 Mbps
--------------	------------	--------------	-------------	------------	------------

$64-1=63$ subcarriers carry data. 64-QAM carries 6 bits per symbol, whilst QPSK has 2 bits per symbol.

System has $6 \times 10 + 2 \times 53 = 166$ bits per OFDM symbol. Hence, $\frac{166 \text{ bits}}{4\mu s} = 41.5 \text{ Mbps}$

(unfortunately, it seems that I forgot to change the selections accordingly, therefore everyone gets 5 points here)