SIGNALS AND SYSTEMS

Lecture 1

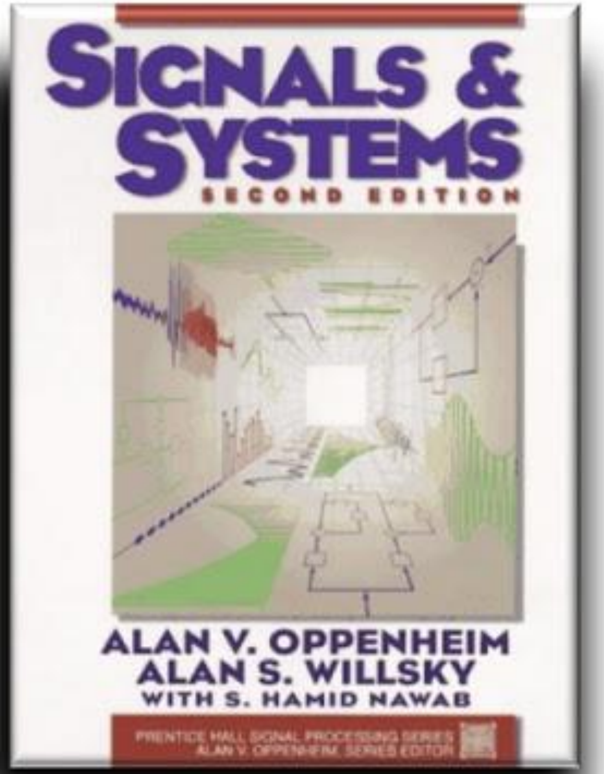
Course Textbook

Signals & Systems (Second Edition) → Text book

by

Alan V. Oppenheim, Alan S. Willsky,

S. Hamid Nawab



Other reference textbooks

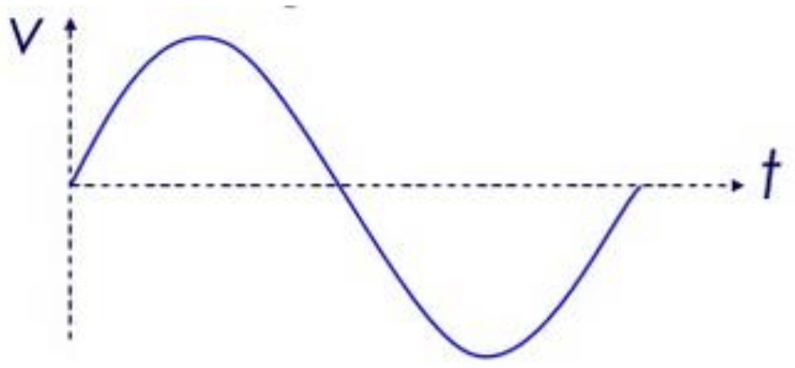
- S. Haykin and B. Van Veen, Signals and Systems, John Wiley & Sons, Inc., 2003, 2nd edition.
- L. Balmer, Signals and Systems, Prentice-Hall, Inc., New Jersey, 1997.

Brief content of the course

- Introduction to Signals and Systems,
- Linear Time Invariant Systems,
- Fourier Series Representation of Periodic Signals,
- The Continuous-Time Fourier Transform,
- The Discrete-Time Fourier Transform,
- Time and Frequency Characterization of Signals and Systems,
- Sampling,
- Z-Transform.

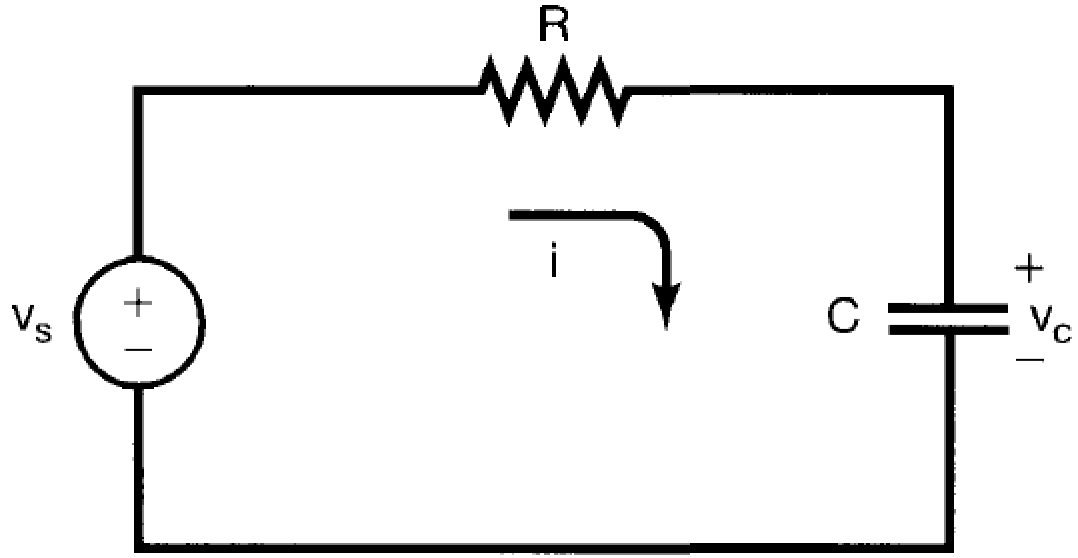
Continuous-Time and Discrete-Time Signals

- Signals describe a wide variety of physical phenomena
- A signal is a physical quantity (for example voltage) dependent on another independent quantity (for example time)
- In this course we will deal with signals varying with time.



Examples to signals

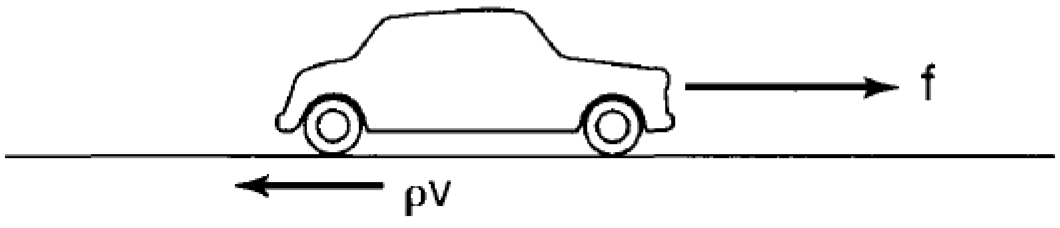
- Variation over time in the source and capacitor voltages, V_s and V_c, with respect to time are examples of signals



A simple *RC* circuit with source voltage v_s and capacitor voltage v_c.

Examples to signals

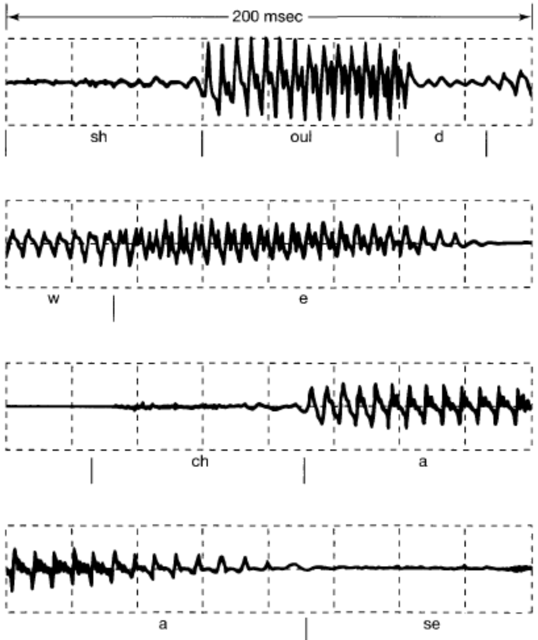
- The variations over time of the applied force f and the resulting automobile velocity v are signals



An automobile responding to an applied force f from the engine and to a retarding frictional force ρv proportional to the automobile's velocity v.

Examples to signals

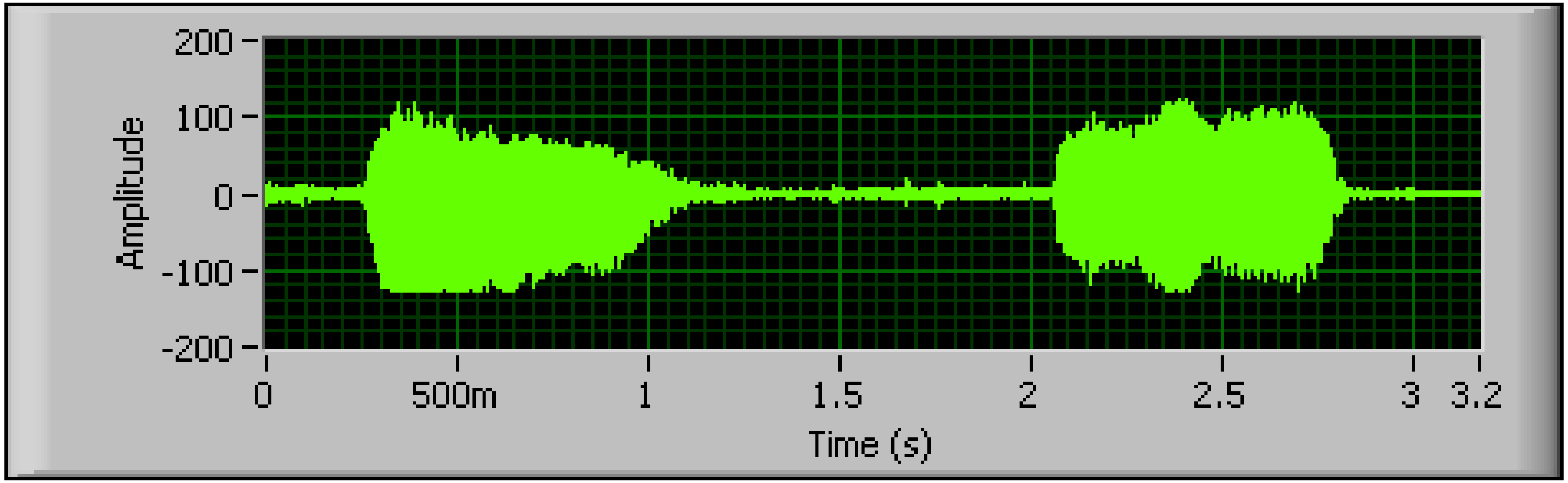
- Consider the human vocal mechanism, which produces speech by creating fluctuations in acoustic pressure.
- Different sounds correspond to different patterns in the variations of acoustic pressure, and the human vocal system produces intelligible speech by generating particular sequences of these patterns.



Should we chase?

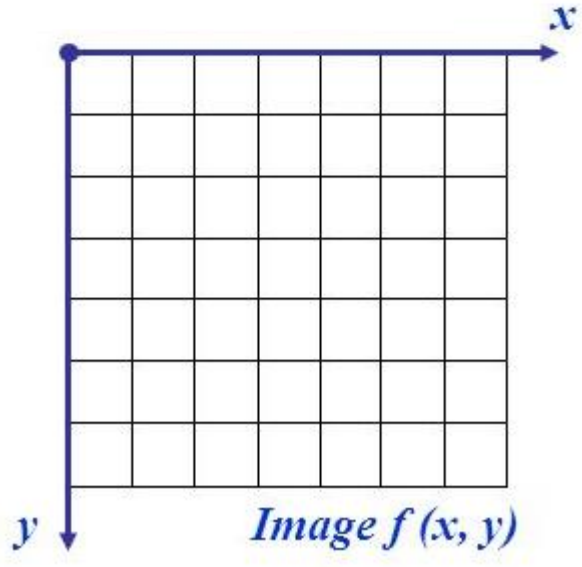
Mathematical Representations of Signals

- Signals are represented mathematically as functions of one or more independent variables.
- For example, a speech signal can be represented mathematically by acoustic pressure as a function of time.



Mathematical Representations of Signals

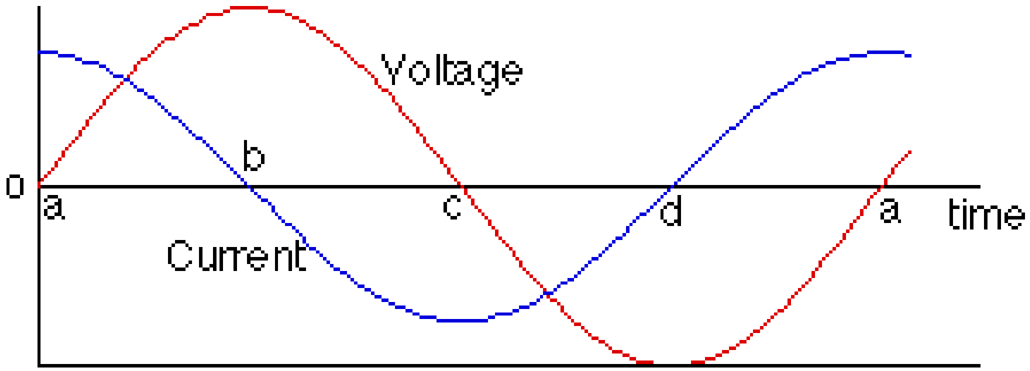
- A picture can be represented by brightness (or color) as a function of two spatial variables.



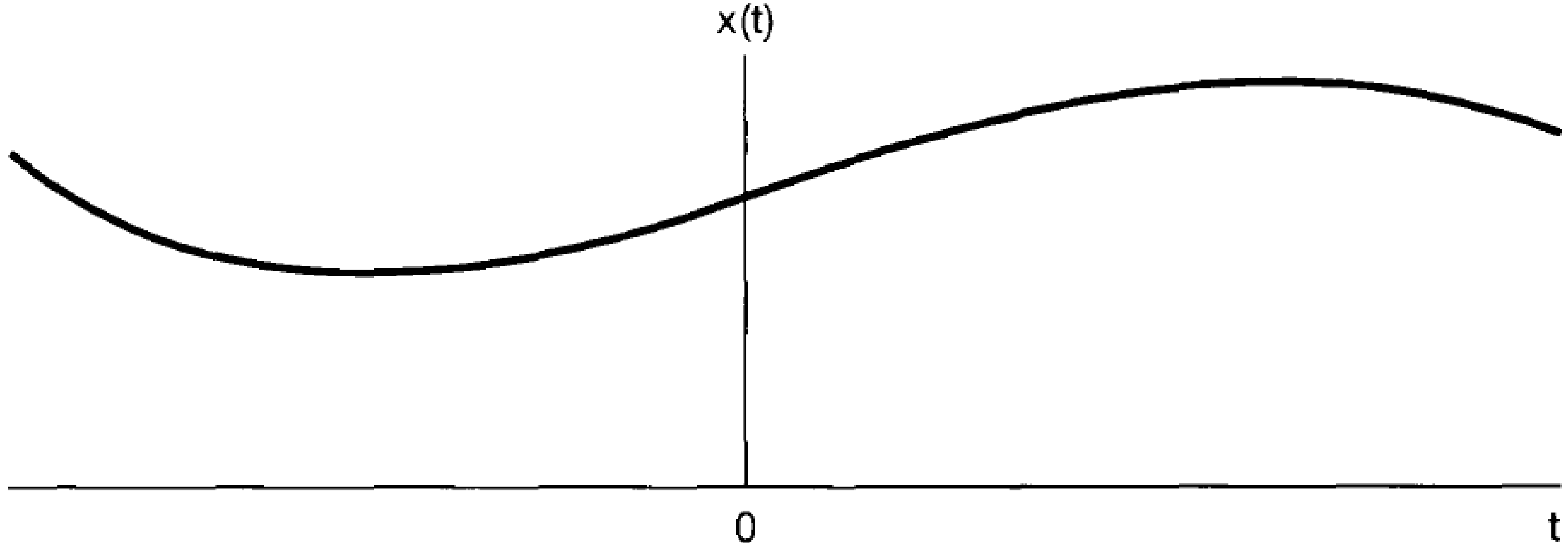
- In this course, we will focus on signals involving a single independent variable.
- For convenience, we will generally refer to the independent variable **as time**.

Continuous-time Signals

- For continuous-time signals the independent variable is continuous, and thus these signals are defined for a continuum of values.
- Example: A speech signal as a function of time
- Example: Voltage and current on a capacitor



Continuous-time Signals

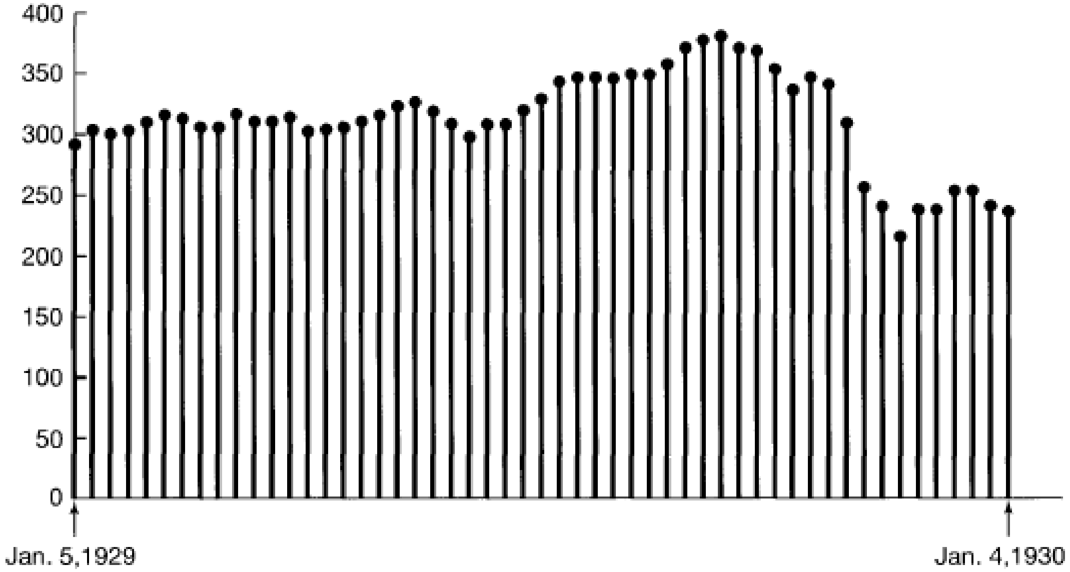


We will represent continuous-time signals as a function of **the continuous variable t**

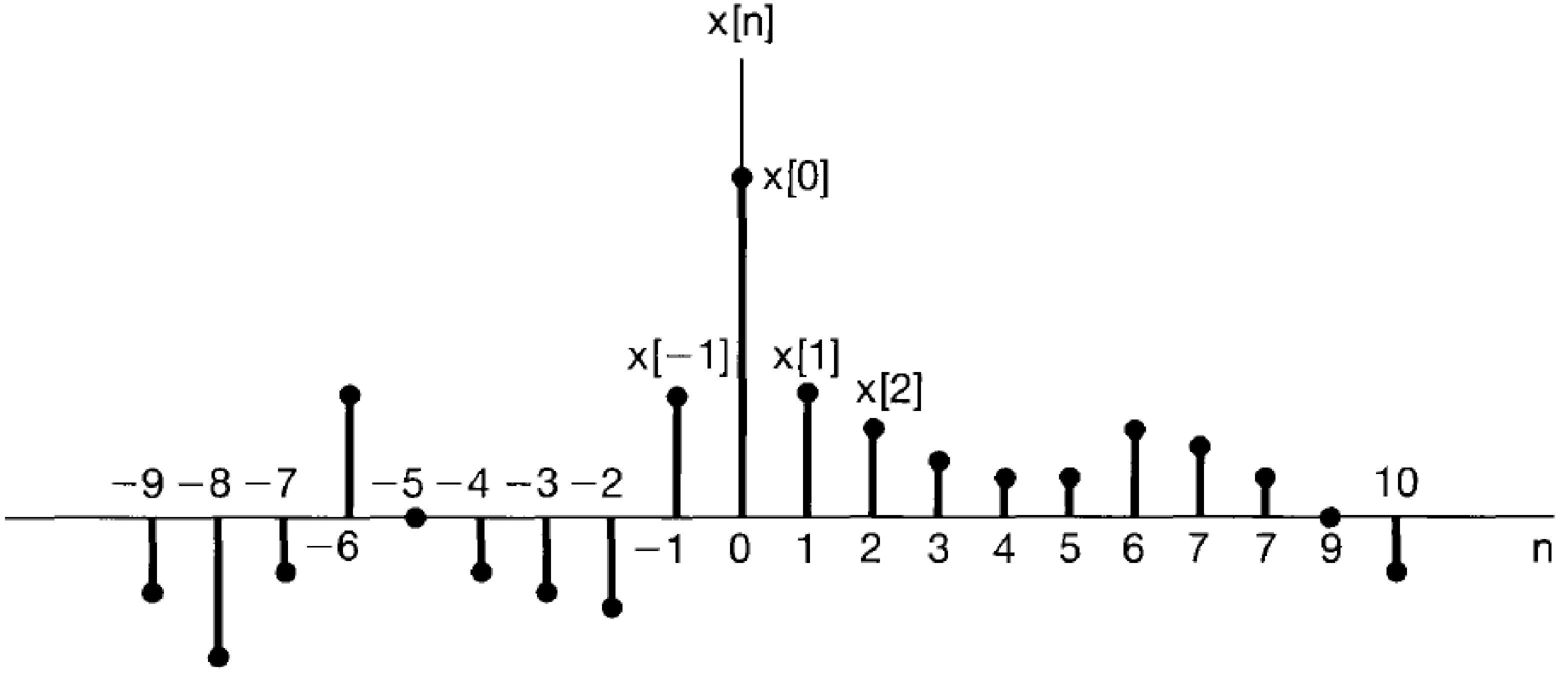
x(t)

Discrete-time Signals

- Discrete-time signals are defined only at discrete times
- For these signals, the independent variable takes on only a discrete set of values.
- Example: Weekly stock-market index



Discrete-time Signals

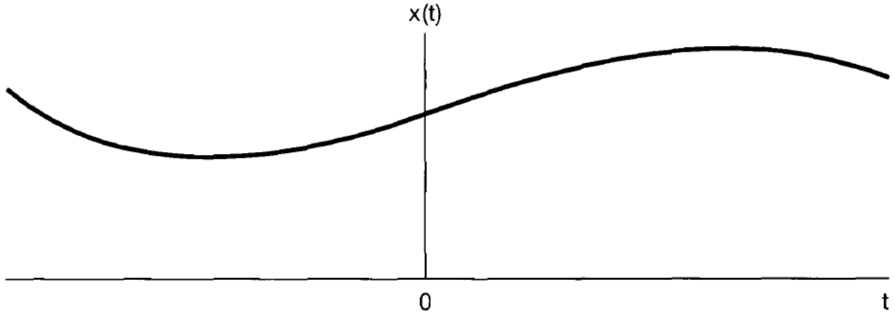


We will represent a discrete-time signal as a function of the **integer variable n**
x[n]
A discrete-time signal is also called **a sequence**.
The independent variable n is also referred to as the **index** of the sequence

Continuous-time Signals

x(t)

For continuous-time signals we will enclose the independent variable in parentheses (·).

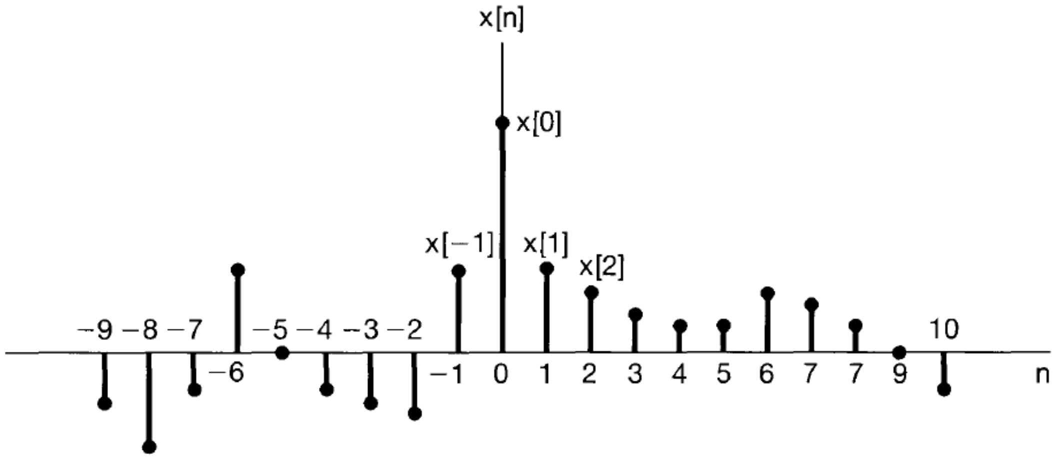


Discrete-time Signals

x[n]

For discrete-time signals we will use brackets [·] to enclose the independent variable.

The independent variable n is always an integer



Signal Energy and Power

- In many applications, the signals we consider are directly related to physical quantities capturing power and energy in a physical system. (Example: Electrical Systems)
- In many systems, the power and energy in signals over an infinite time interval are important quantities.

Signal Energy

Total energy of a continuous-time signal over the time interval $t_1 \leq t \leq t_2$

$$\int_{t_1}^{t_2} |x(t)|^2 dt,$$

Total energy of a discrete-time signal over the time interval $n_1 \leq n \leq n_2$

$$\sum_{n=n_1}^{n_2} |x[n]|^2$$

Total Signal Energy

Total energy of a continuous-time signal:

$$E_\infty \triangleq \lim_{T \to \infty} \int_{-T}^{T} |x(t)|^2 \, dt = \int_{-\infty}^{+\infty} |x(t)|^2 \, dt$$

Total energy of a discrete-time signal:

$$E_\infty \triangleq \lim_{N \to \infty} \sum_{n=-N}^{+N} |x[n]|^2 = \sum_{n=-\infty}^{+\infty} |x[n]|^2$$

Signal Energy

$$E_\infty \triangleq \lim_{T \to \infty} \int_{-T}^{T} |x(t)|^2 \, dt = \int_{-\infty}^{+\infty} |x(t)|^2 \, dt$$

$$E_\infty \triangleq \lim_{N \to \infty} \sum_{n=-N}^{+N} |x[n]|^2 = \sum_{n=-\infty}^{+\infty} |x[n]|^2$$

Note that for some signals the integral or the sum might not converge,
e.g., if *x(t)* or *x[n]* equals a nonzero constant value for all time.
Such signals have infinite energy.

Signal Power

Time-averaged power over an infinite interval for a continuous-time signal:

$$P_\infty \triangleq \lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} |x(t)|^2\, dt$$

Time-averaged power over an infinite interval for a discrete-time signal:

$$P_\infty \triangleq \lim_{N\to\infty} \frac{1}{2N+1}\sum_{n=-N}^{+N} |x[n]|^2$$

Energy signals, Power Signals

Three classes of signals:

- Class 1: signals with finite total energy, $E_\infty < \infty$ and zero average power, (Energy Signal)

$$P_\infty = \lim_{T\to\infty} \frac{E_\infty}{2T} = 0$$

- Class 2: with finite average power P_∞. If $P_\infty > 0$, then $E_\infty = \infty$. An example is the signal $x[n] = 4$, it has infinite energy, but has an average power of P_∞=16. (Power Signal)

Class 3: signals for which neither P_∞ and E_∞ are finite. An example of this signal is $x(t) = t$.

Transformations of the independent variable

We will study the following transformations of the independent variable.

- Time shift
- Time reversal
- Time scaling

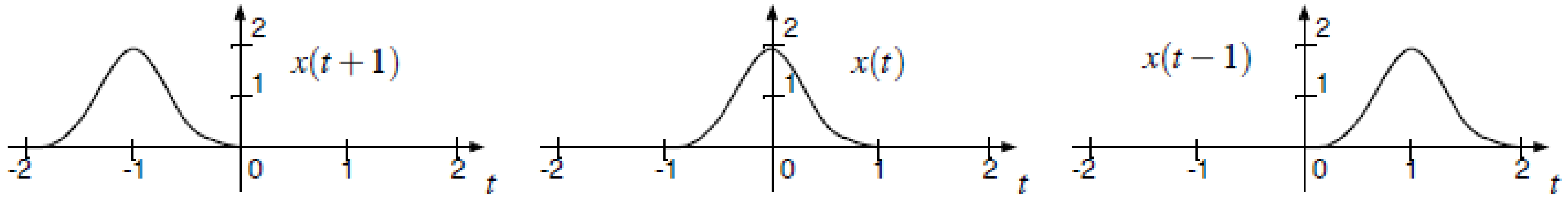
Transformations of the independent variable

Time shift

Continuous-time signals

For a continuous-time signal $x(t)$, and a time $t_1 > 0$,

- Replacing t with $t - t_1$ gives a *delayed* signal $x(t - t_1)$
- Replacing t with $t + t_1$ gives an *advanced* signal $x(t + t_1)$



Shift to the left
Advanced signal

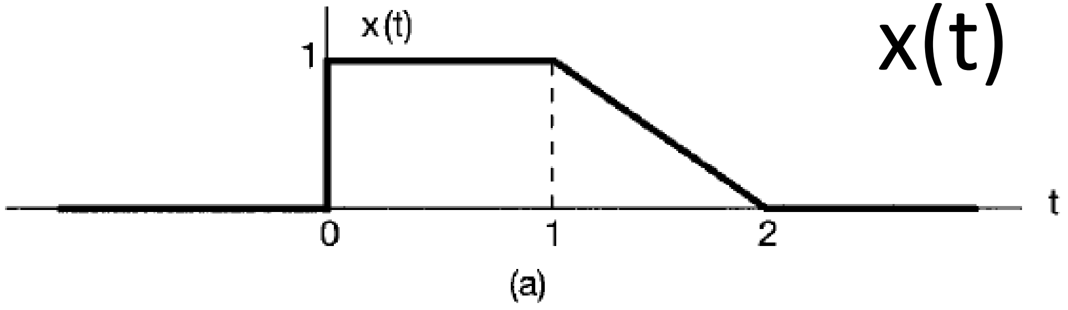
Shift to the right
Delayed signal

Transformations of the independent variable

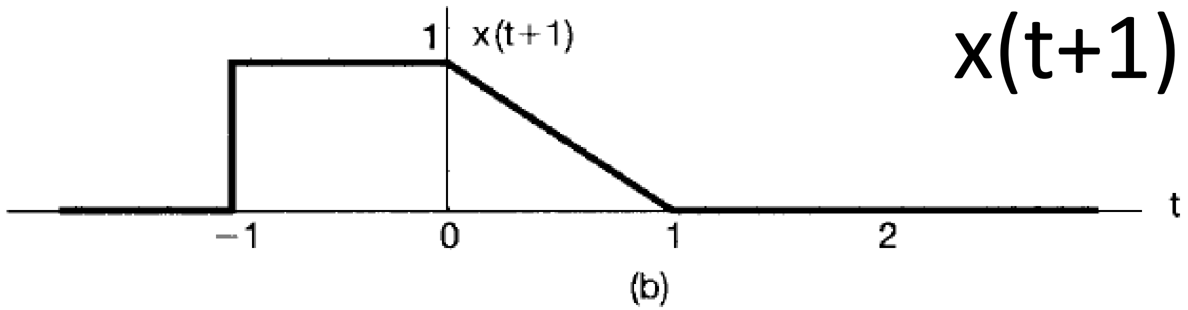
Time shift

Continuous-time signals

Example:



x(t)



x(t+1)

Shift to the left

Advance the signal in time.

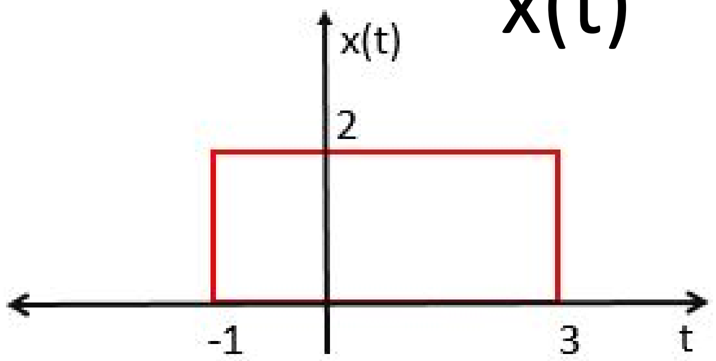
Transformations of the independent variable

Time shift

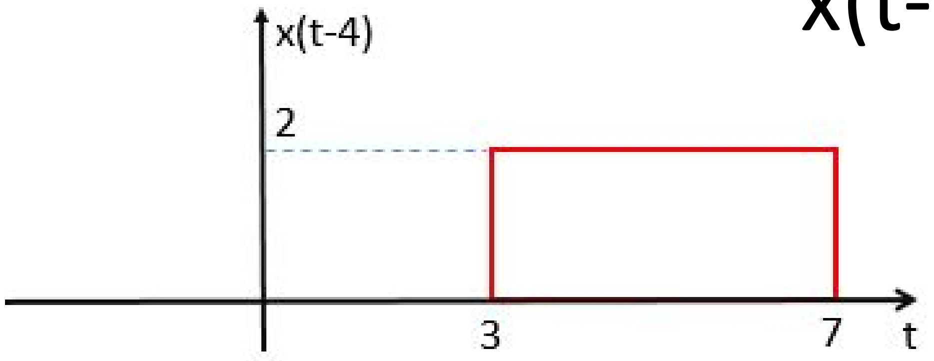
Continuous-time signals

Example:

x(t)



x(t-4)



Shift to the right

Delay the signal in time.

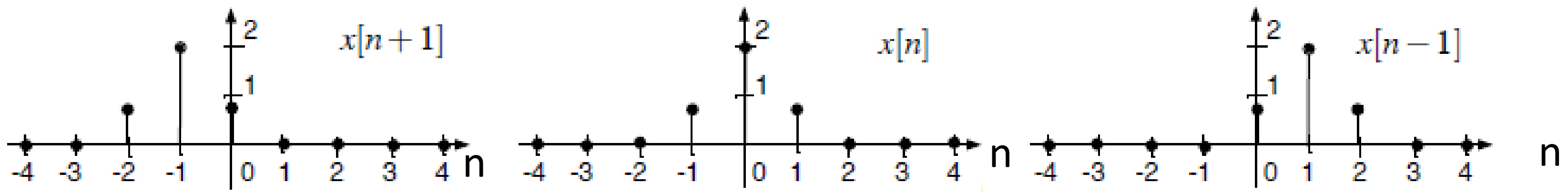
Transformations of the independent variable

Time shift

Discrete-time signals

For a discrete time signal $x[n]$, and an integer $n_1 > 0$

- $x[n - n_1]$ is a delayed signal.
- $x[n + n_1]$ is an advanced signal.
- The delay or advance is an integer number of sample times.

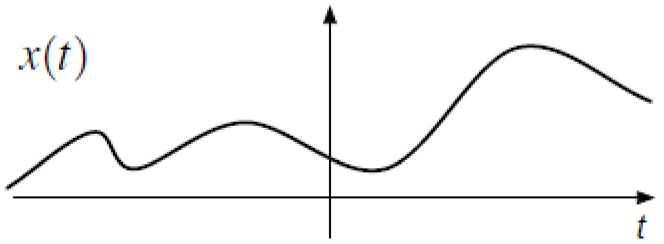


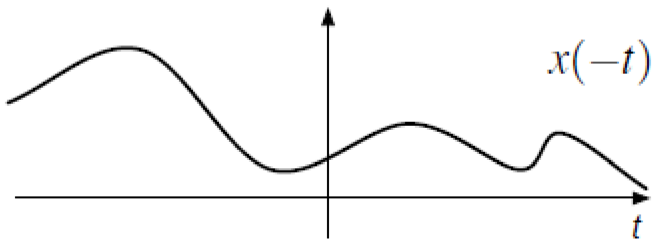
Shift to the left
Advanced signal

Shift to the right
Delayed signal

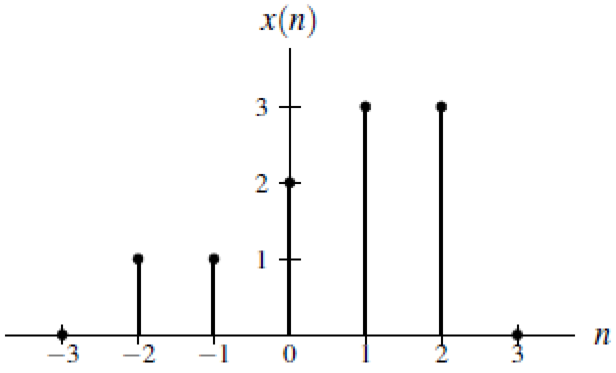
Transformations of the independent variable
Time reversal

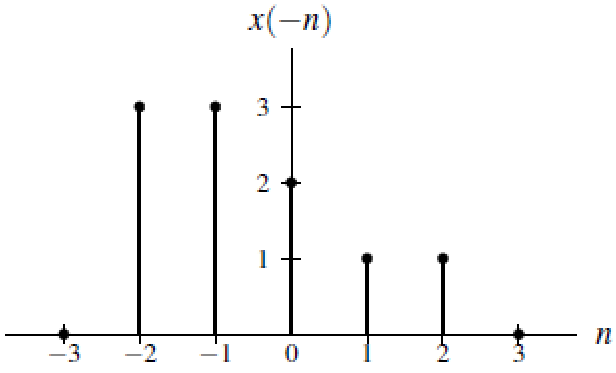
- Continuous time: replace t with $-t$, time reversed signal is $x(-t)$





- Discrete time: replace n with $-n$, time reversed signal is $x[-n]$.



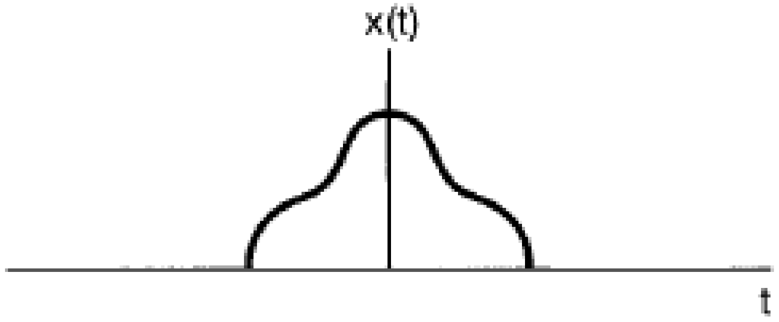


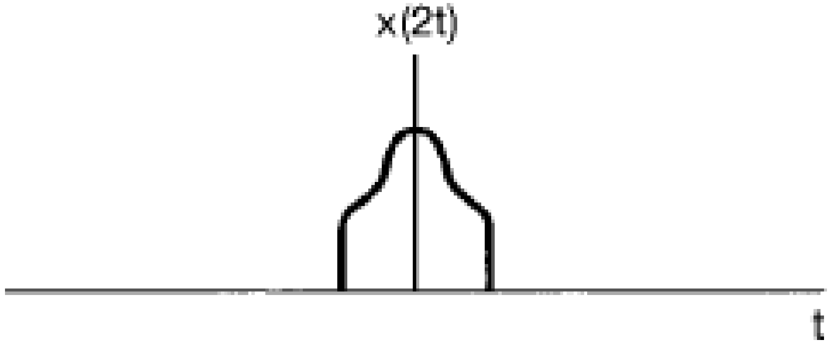
Transformations of the independent variable

Time scaling

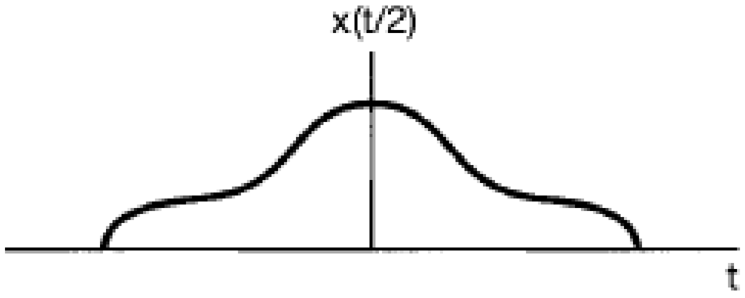
Continuous-time signals

A signal $x(t)$ is scaled in time by multiplying the time variable by a positive constant b, to produce $x(bt)$. A positive factor of b either expands $(0 < b < 1)$ or compresses $(b > 1)$ the signal in time.





Compressed in time



Expanded in time

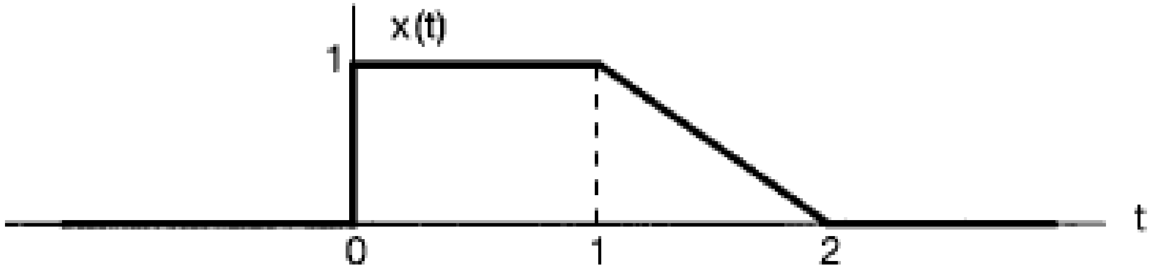
Transformations of the independent variable

Time scaling

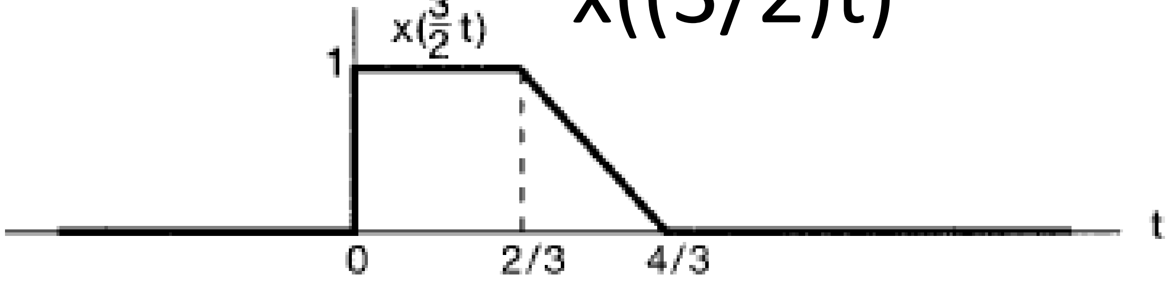
Continuous-time signals

Example:

x(t)



x((3/2)t)



Compression in time

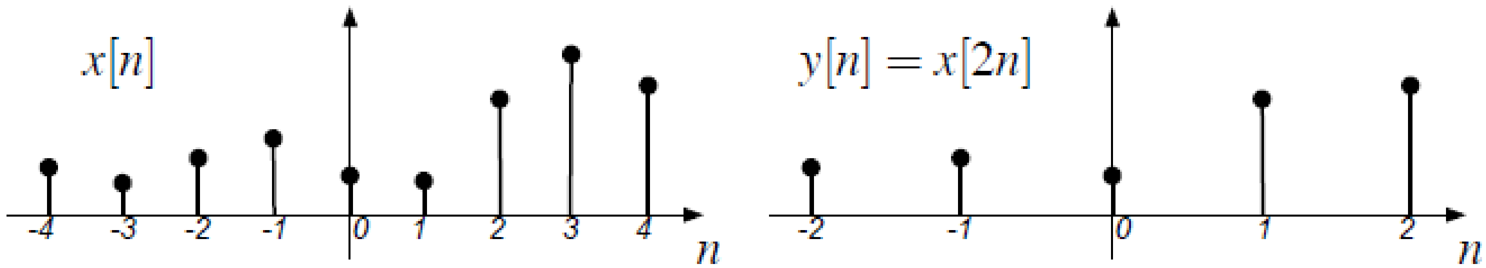
Transformations of the independent variable

Time scaling

Discrete-time signals

The discrete-time sequence $x[n]$ is *compressed* in time by multiplying the index n by an integer k, to produce the time-scaled sequence $x[nk]$.

- This extracts every k^{th} sample of $x[n]$.
- Intermediate samples are lost.
- The sequence is shorter.



Called *downsampling*, or *decimation*.

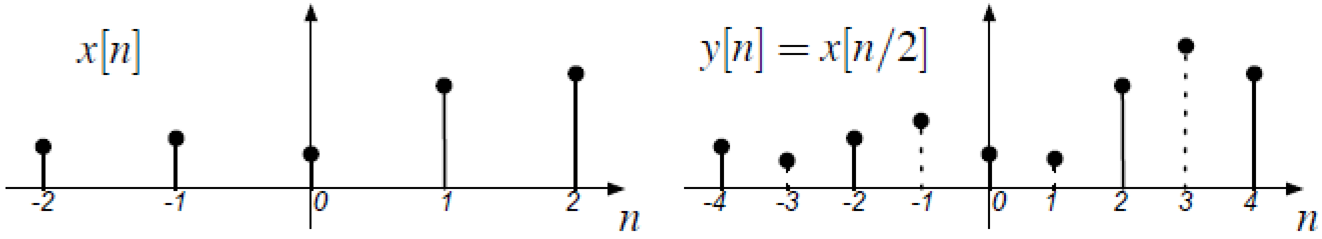
Transformations of the independent variable

Time scaling

Discrete-time signals

The discrete-time sequence $x[n]$ is *expanded* in time by dividing the index n by an integer m, to produce the time-scaled sequence $x[n/m]$.

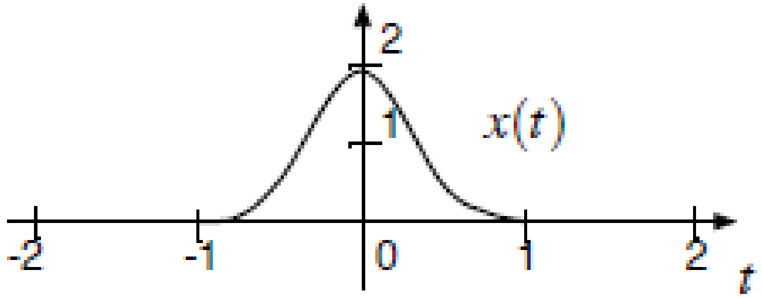
- This specifies every m^{th} sample.
- The intermediate samples must be synthesized (set to zero, or interpolated).
- The sequence is longer.



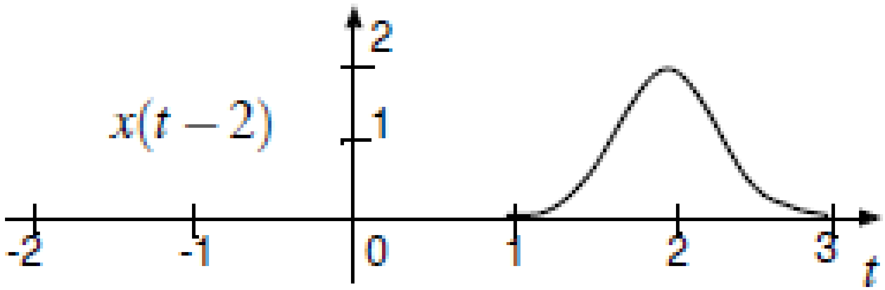
Called *upsampling*, or *interpolation*.

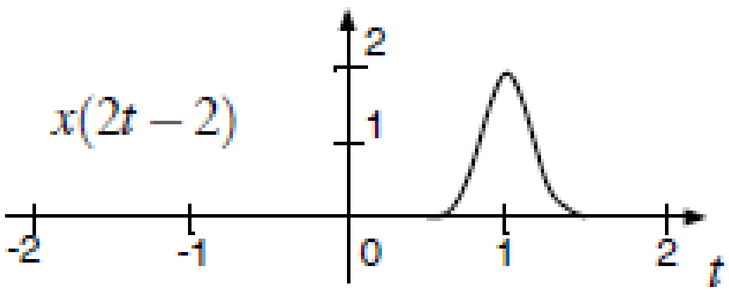
Transformations of the independent variable
Combinations

Example



Given x(t), sketch x(2t-2)





First shift to the right by 2

Then scale by 2

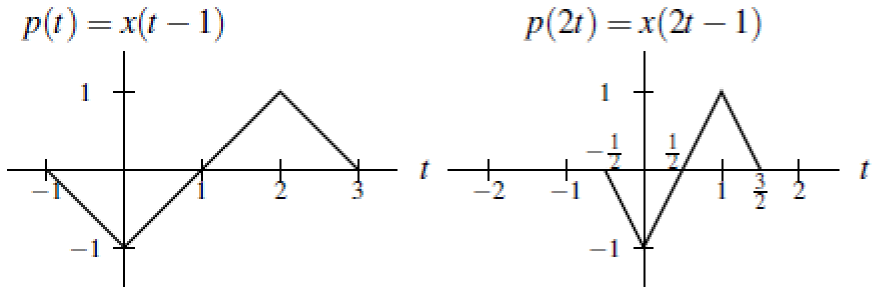
Transformations of the independent variable

Combinations

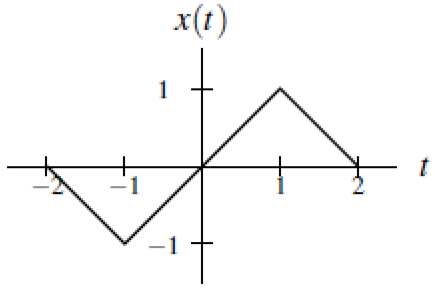
Example

$x(2t - 1)$

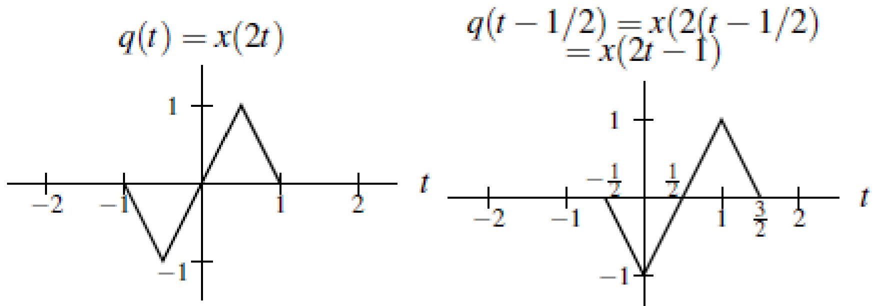
time shift by 1 and then time scale by 2



Given $x(t)$ as shown below, find $x(2t-1)$.



time scale by 2 and then time shift by $\frac{1}{2}$



$x(2(t - 1/2))$

Periodic Signals
Continuous-time periodic signals

- A periodic continuous-time signal x(t) has the property that there is a positive value of T for which

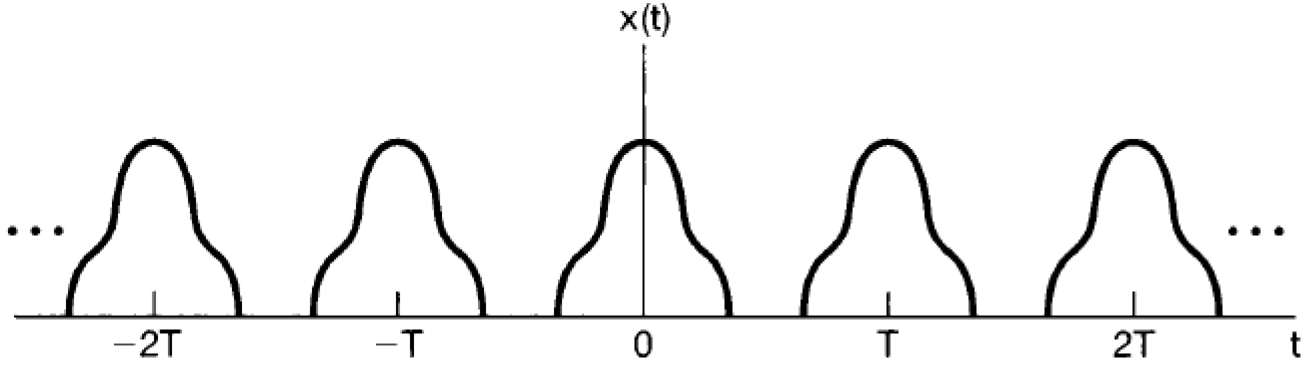
$$x(t) = x(t + T)$$

for all values of t.

- The fundamental period T_o of x(t) is the smallest positive value of T for which the following equation holds:

$$x(t) = x(t + T)$$

Example



This signal is periodic with T, 2T, 3T, etc.
The smallest is T.
The fundamental period of this signal is T.

Periodic Signals
Discrete-time periodic signals

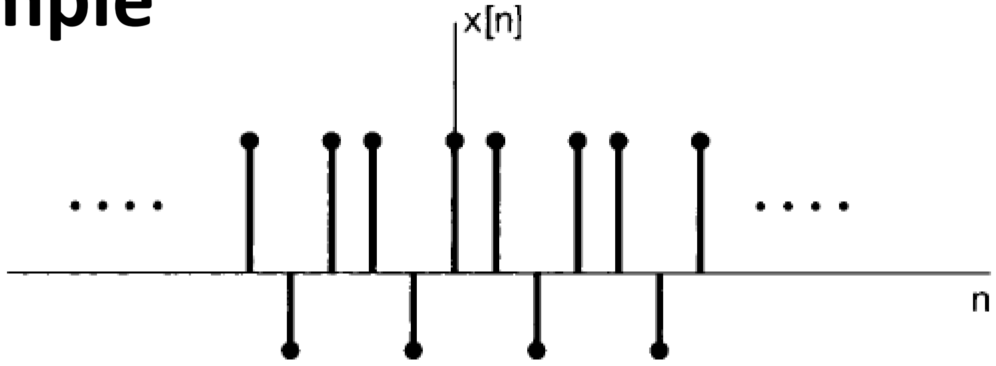
- A discrete-time signal x[n] is periodic with period N, where N is a positive integer, if it is unchanged by a time shift of N, i.e., if

$$x[n] = x[n + N]$$

for all n.

- The fundamental period N_0 is the smallest positive value of N for which the above equation holds.

Example



A discrete-time periodic signal with fundamental period $N_0 = 3$

Even and Odd Signals

- A property related to the symmetry of a signal under time-reversal
- A signal x(t) or x[n] is referred to as an even signal if it is identical to its time-reversed counterpart, i.e., with its reflection about the origin.
- In continuous time a signal is even if

$$x(-t) = x(t)$$

- A discrete-time signal is even if

$$x[-n] = x[n]$$

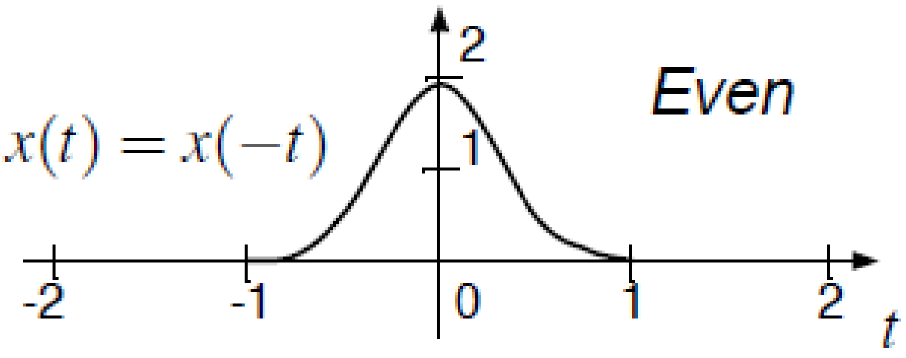
- A signal is referred to as odd if

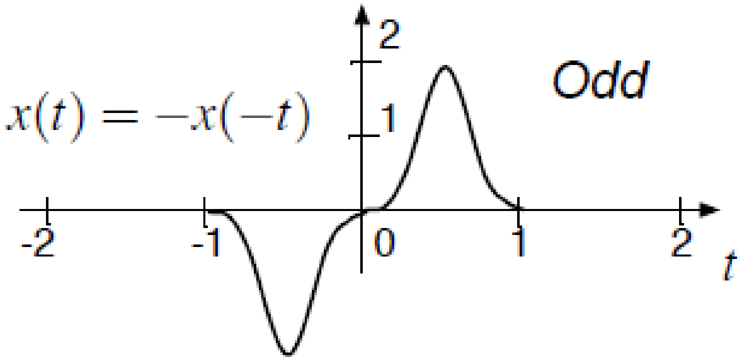
$$x(-t) = -x(t)$$

$$x[-n] = -x[n]$$

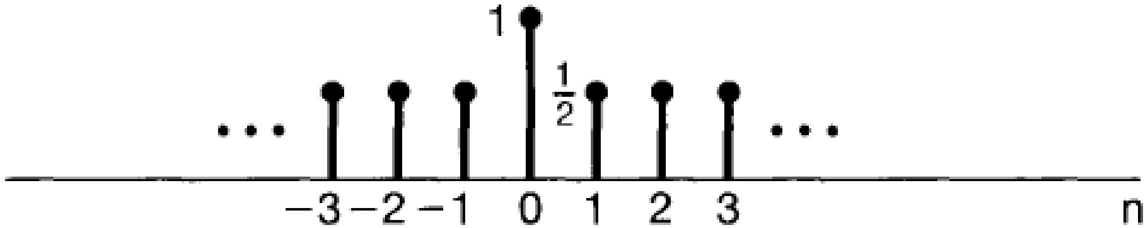
An odd signal must necessarily be 0 at $t = 0$ or $n = 0$

Even and Odd Signals

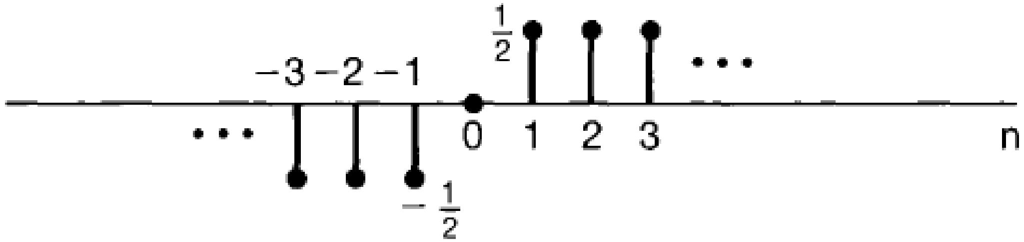




x[- n] = x[n] *Even*



x[- n] = - x[n] *Odd*



Even and Odd Parts of a signal

Any signal can be decomposed into even and odd components

$$
\begin{aligned}
x_e(t) &= \frac{1}{2}\left[x(t) + x(-t)\right] \\
x_o(t) &= \frac{1}{2}\left[x(t) - x(-t)\right].
\end{aligned}
$$

Check that

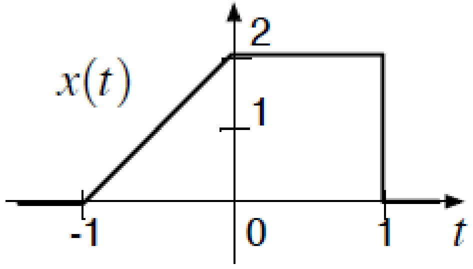
$$
\begin{aligned}
x_e(t) &= x_e(-t), \\
x_o(t) &= -x_o(-t),
\end{aligned}
$$

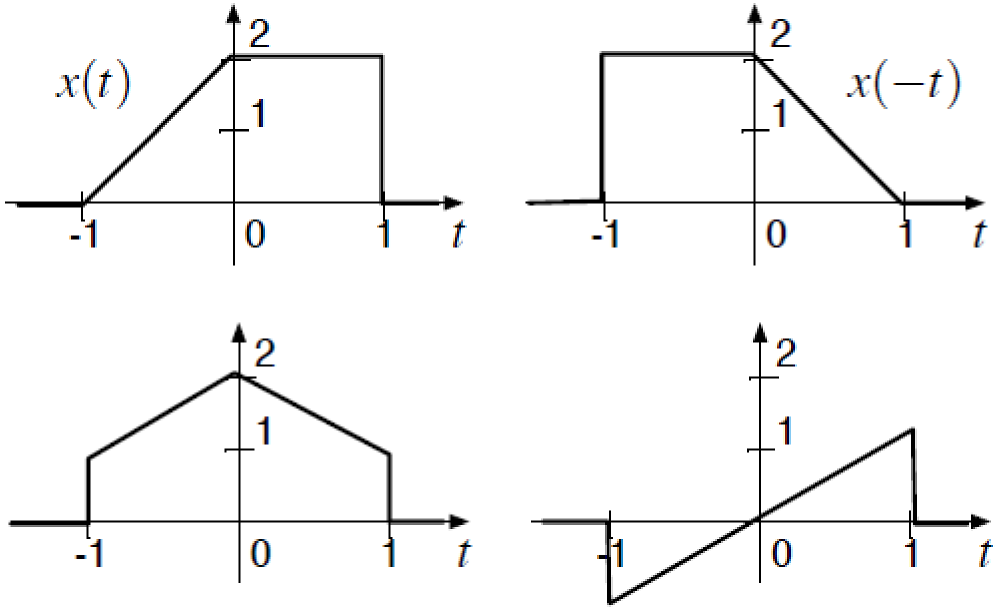
and that

$$x_e(t) + x_o(t) = x(t).$$

Even and Odd Parts of a signal

Example





$$x_e(t) = \frac{1}{2}[x(t) + x(-t)] \qquad x_o(t) = \frac{1}{2}[x(t) - x(-t)]$$