No : Answer **Name : Solution**

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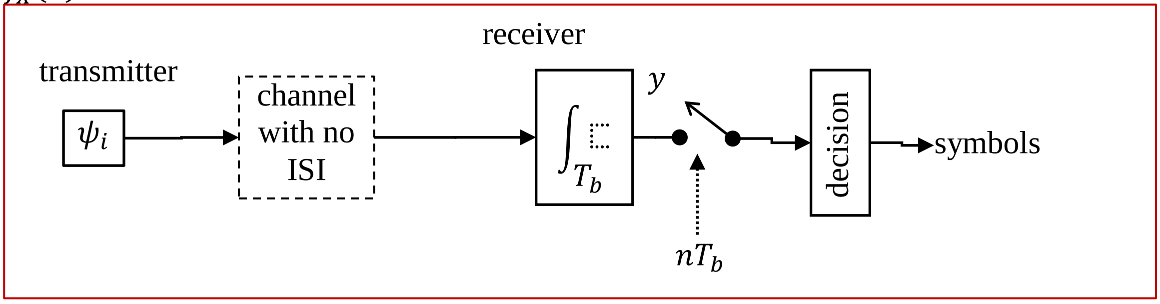
1. A baseband binary communication system is given below. The waveforms representing binary symbols are $\psi_i = \mp A$ for a symbol period T_b and 0 elsewhere. The receiver is in sync with the received signal. The pdf of the noise component at the integrator output is given as $f_X(x) = 5000e^{-10000|x|}$.



a) Find the signal part of the integrator output at the decision instant.

b) Find A for $p_e \leq 0.01$ at R=1 kbits/s.

Answer:

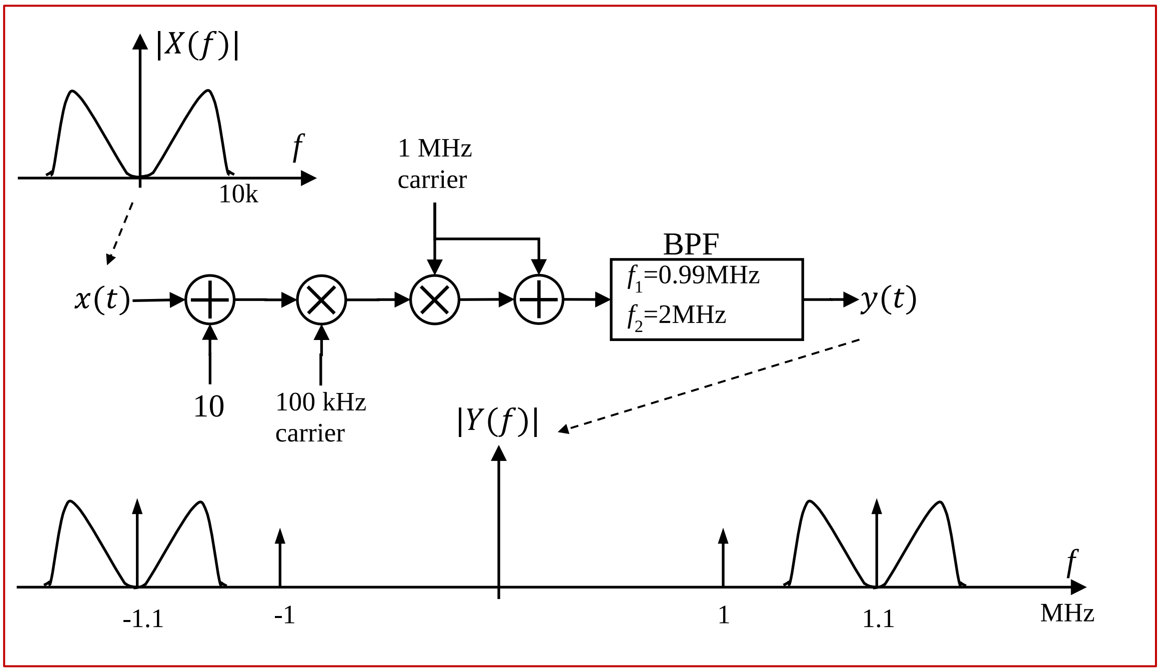
a) $y(nT_b) = \int_0^{T_b} A dt = AT_b$ (10p)

b) $p_e = 5000 \int_{AT_b}^{\infty} e^{-10000x}\, dx = -\frac{1}{2}e^{-10000x}\Big|_{AT_b}^{\infty} = \frac{1}{2}e^{-10000AT_b}$

for $p_e \leq 0.01$ and T_b=1/1000, $\frac{1}{2}e^{-10000AT_b} \leq 0.01 \Rightarrow \frac{1}{2}e^{-10A} \leq 0.01$,
$A \geq -0.1 \times \ln(0.02)$, $A \geq \sim 0.3912$ (15p)

In case you incorrectly find $y(nT_b) = A^2T_b$, you get 0p from that part. But if you used that result correctly in the second part and find $A \geq \sim 0.6255$, you get 10p. All other solutions are invalid and get you 0p.

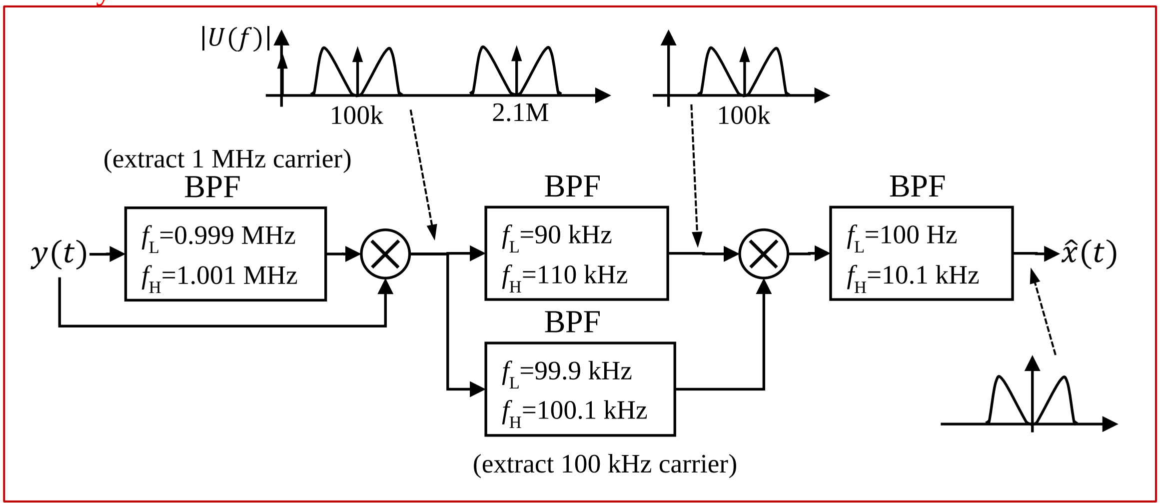
2. Baseband input signal whose spectrum is given as $X(f)$, is applied to the following system in order to obtain output spectrum which is also given.



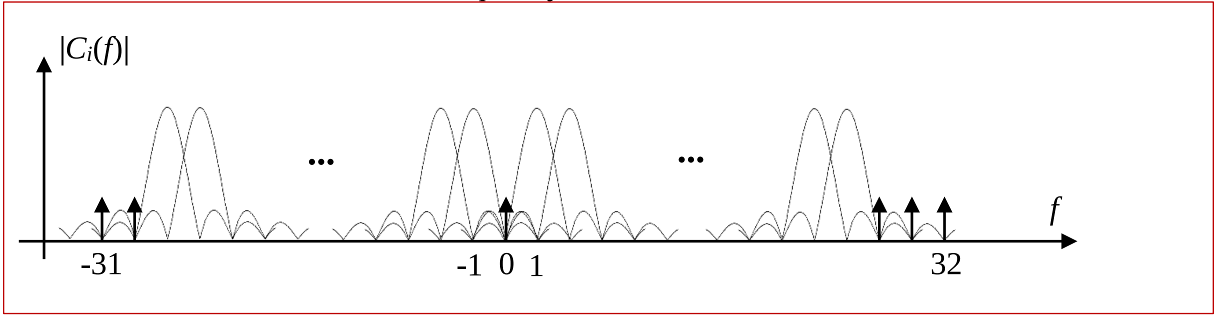
Determine and draw the system required to frequency down-convert $y(t)$ to 100 kHz first and demodulate afterwards to obtain $x(t)$ back.

Answer:

1. Extract 1 MHz carrier using a sharp filter.
2. Multiply it with the received signal to obtain $u(t)$ ($|U(f)|$ is shown below).
3. Extract 100 kHz carrier. Remove components outside 90-110 kHz range.
4. Do synchronous demodulation.



3. The following figure illustrates and OFDM communication spectrum where *sinc* shaped sub-carrier spectrums depict one of BPSK, QPSK, 16QAM or 64QAM and short arrows show null-carriers. Numbers below the frequency axis are sub-carrier indexes.

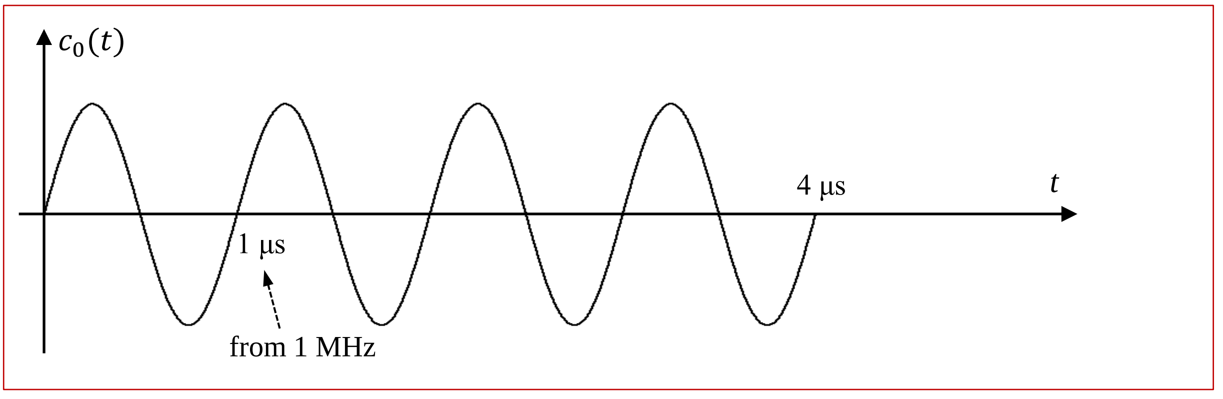


a) Calculate the bit rate when T_s=1 ms, half the data channels employ 64QAM, two data-channels are used as pilot carriers and the remaining use BPSK to transmit data.
b) Assume that 0 indexed carrier is at 1 MHz and it is not null, draw that carrier for 4 μs (for any desired phase and amplitude).
c) Calculate the null-to-null bandwidth of the OFDM signal.

Answer:

a) #data-channels = 64-6=58, #64QAMchannels=58/2=29
#pilotcarriers=2, #BPSKchannels=58/2-2=27
bitsperOFDMsymbol=29x6+27=201 bits/sym
T_s=1 ms means R_s=1/10^{-3}=1 ksym/s, hence R_b=1k x 201=201 kbits/s

b) For the 0 indexed carrier, the number of periods per OFDM symbol duration is 1000 kHz / 1 ms = 1000. The number of periods of that carrier within 4 μs is 0.004 x 1000=4. That is 4 μs corresponds to 4 periods of that carrier.



c) Including the DC carrier at the center, we have 59 sub-carriers between nulls at the edges. Spacing between sub-carriers are 1/1 ms = 1 kHz. Therefore the null-to-null bandwidth is (59+1)x1kHz = 60 kHz.

4. A single-bit ECC is given by the generator matrix $G_{ns} = \begin{bmatrix} 101011 \\ 111000 \\ 001101 \end{bmatrix}$.

a) Find a systematic generator matrix G.
b) Calculate the minimum Hamming distance d_{min} for both G and G_{ns}.
c) Find the output of the decoder with G when the received codeword is 001100.

Answer:

We have -seemingly- linearly independent 3 rows, meaning that it is a (6,3) code. We will confirm this by generating a systematic matrix.
a) Systematic generator matrix can be obtained by elementary row/column operations. Adding first row onto the second row and afterwards adding third row onto the first row, we get the systematic generator matrix $G = \begin{bmatrix} 100110 \\ 010011 \\ 001101 \end{bmatrix}$. Since the rank is 3 we confirm that it is a (6,3) code.

b) The number of parity bits (3) hints that d_{min}=3. Because for a linear code the codeword 000000 is one of the codewords in the code, and the distance between 000000 and any selected row is 3. Surest way to confirm this is to generate all codewords and search for the minimum distance between codewords;
For the information words {000,001,010,011,100,101,110,111}, the codewords are found to be {000000,001101,010011,011110,100110,101011,110101,111000}.
An exhaustive search in the code reveals that d_{min}=3.
Since we obtained G from G_{ns} using row/column operations, d_{min}=3 for G_{ns} too.

c) We have already found all the codewords. The closest codeword to the received word is 001101 with d=1, and the output of the decoder shall be 001. One may also construct a syndrome vector and correct the erroneous bit accordingly, if there is one.