1. Consider an M-PSK system with in-phase carrier $\cos \left(6 \times 10^{4} \pi t\right)$ and symbol rate 10 $\mathrm{ksym} / \mathrm{s}$. Given the constellation diagram and a period of the waveform representing the symbol 00 , which of the following can be the full waveform representing the symbol 01 ?


Soln: The number of carrier periods in a symbol period ( 0.1 ms ) is $3(30 \mathrm{kHz} / 10 \mathrm{ksym} / \mathrm{s})$. That leaves us with 3 choices. Since the waveform representing 01 is $90^{\circ}$ ahead of the waveform representing 00 the answer is as marked.
2. Entropy of a source given as $z=\left\{0.5, p_{1}, p_{2}, p_{3}\right\}$ is found to be equal to the average code length when Huffman or Shannon-Fano method is applied for dictionary generation. What would be $p_{3}$ when $z$ is assumed to be ordered?
a) 0.125
b) 0.1667
c) 0.25
d) 0.75
e) $0.0625 \quad$ f) 0.5

Soln: In order to have $H(z)=L_{\text {avg }}, I_{i}$ must be integers for all $i$, like $I_{0}=-\log _{2}(0.5)=1$ given. The other possible $I_{i}$ for a 4 symbol source are 2 and 3 , corresponding to 0.25 and 0.125 respectively. Hence, when ordered from higher to lower $p_{i}, z$ must be $z=\{0.5,0.25,0.125,0.125\}$ meaning that $p_{3}=0.125$.
3. What would be the null-to-null bandwidth of the resulting signal when $10 \mathrm{kbits} / \mathrm{s}$ random binary rectangular pulse train is applied to DSSS using SSRG[7,3,2,1] (two pn-sequence period per bit) and frequency up-converted to 1 GHz using QPSK?
a) 20 kHz
b) 1.02 GHz
c) 1.27 MHz
d) $127 \mathrm{kHz} \quad$ e) 2.54 MHz
f) 2.54 kHz

Soln: Symbols of QPSK are 2 bits each. pn-sequence period is $2^{7}-1=127$ chips. Two pn-seqperiod has 254 chips. BW of baseband pulse train is $10 \mathrm{k} \times 254=2.54 \mathrm{M}$. When up-converted using QPSK, the BW will be $2 \times \mathrm{BW}_{\text {baseband }} / \#$ cipspersymbol $=2.54 \mathrm{MHz}$.
4. A $10 \mathrm{ksym} / \mathrm{s}$ random binary rectangular pulse train is applied to DSSS using SSRG[10,3] (two pn-sequence period per symbol). What would be the appropriate clock frequency for the pn -sequence generator?
a) 1024 kHz
b) 1023 kHz
c) 1.023 MHz
d) 20.46 MHz
e) 2048 kHz f) 10.23 MHz

Soln: $2^{10}-1=1023$ chips. 2 pn periods per bit means 2046 chips per bit. $2046 \times 10 \mathrm{k}=20.46 \mathrm{M}$.
5. Given the magnitude spectrum of baseband $x(t)$, what would be the spectrum of $y(t)$ ?


##  <br>  <br> d) <br>  <br> b) <br>  <br> e) <br> 

Soln:

6. Inputs to the following binary receiver are $\psi(t)= \pm 10$ and the symbol duration is 1 ms .

Noise pdf at the output of the integrator is $f_{x}=50-2500|x|$ for $|x|<0.02$ and zero elsewhere. What is the probability of decision error?

a) 1.25
b) 0.125
c) 0.0125
d) 0.01
e) 0.02 f) 0.25

Soln: $E_{b}=\int_{0}^{T_{b}=0.001} 10 d t=0.01 . p_{e}=\int_{0.01}^{\infty} f_{x}(x) d x=\int_{0.01}^{0.02}(50-2500 x) d x$
$p_{e}=\left[50 x-2500 x^{2} / 2\right]_{0.01}^{0.02}=0.125$. Or, one can find the area (error) by looking at the triangle area that $f_{x}$ implies.

7. What would the matched filter response be like, for the given rectangular pulse?

a)
c)
b)

d)

e)

f)

$\psi(t)$ is symmetric, therefore causal $h(t)=\psi(t)$. Hence, $y(t)=\int_{-\infty}^{\infty} \psi(\tau) h(t-\tau) d \tau$ is also symmetric. Within $t=(0,0.1) y_{1}(t)=\int_{0}^{t} d \tau=t$ and within $t=(0.1,0.2) y_{2}(t)=$ $\int_{t}^{0.2} d \tau=\left.t\right|_{t} ^{0.2}=0.2-t . \therefore$ it is a triangular pulse from 0 to 0.2 having max value at 0.1 .
8. Given the following periodic waveform $x(t)$ and its Fourier Series magnitude graph, what would be the period $T$ of $x(t)$ ?

a) $1 / 3$
b) $1 / 4$
c) $1 / 9$
d) 9
e) $2 / 9 \quad$ f) 3

Soln: $f_{0}=3$ (from the FS graph). $T=1 / f_{0}=1 / 3$
9. $x(t)=1+\cos (1000 \pi t+\pi / 4)$ is applied as input to the LP filter given below. What is the average/expected value of the output signal?

a) $\sqrt{3}$
b) $1 / 3$
c) 1
d) $1 / 2$
e) 3
f) 0

Soln: Average/expected value is the DC value at the output. Since it is a linear system (transfer function is given), $\mathrm{DC}_{\text {output }}=\mathrm{DC}_{\text {input }} \times \mathrm{DC}_{\text {response }}$, meaning that the average value is 1 $\mathrm{x} 1=1$.
10. An OFDM communication system employs 64 subcarriers. 7 of these subcarriers are null and 3 are used as pilot carriers for synchronization. 10 subcarriers employ QPSK and the rest use 64-QAM for data transmission. What is the physical bit-rate when subcarrier spacing is 100 kHz and CP is not used?
a) 24.4 Mbps
b) 224 kbps
c) 32.4 Mbps
d) 802.4 kbps
e) 28.4 Mbps f) 284 kbps

Soln: We have 64-7-3=54 data subcarriers 10 of which uses QPSK (2 bits per symbol). The remaining 44 subchannels carry 6 bits/symbol each. $10 \times 2+44 \times 6=284$ bits per ofdm-symbol is transmitted. Subcarrier spacing of 100 kHz says that the symbol rate is $100 \mathrm{ksym} / \mathrm{s}$.
Therefore, 100k x $284=28.4 \mathrm{M}$.
11. A $(6,3)$ single-bit ECC system uses the systematic generator matrix $G=\left[\begin{array}{l}100110 \\ 010011 \\ 001101\end{array}\right]$. What should be the output of the decoder when the block code 010010 is received?

| a) 0010 | b) 010 | c) 110 | d) 1101 | e) 000 | f) 100 |
| :--- | :--- | :--- | :--- | :--- | :--- |

Soln: Since the syndrome vector has only 1 non-zero bit, indicating a parity bit is in error, the decoded output should be 010 (no error in information bits). (note: one may also construct entire code table and find the nearest code).
12. What is the minimum Hamming distance for the ECC code given by $G=\left[\begin{array}{l}100110 \\ 010101 \\ 001011\end{array}\right]$ ?
a) 2.5
b) 2
c) 1
d) 4
e) 2.6
f) 3

Soln: Since the code is linear \& systematic, 000000 is in the code too. For such codes $d_{\min }=$ $w_{\min }$. Since $w_{\min }=3$ (the minimum number of non-zero bits) $d_{\min }$ is also 3 .
13. A Shannon-Fano dictionary is given as $C=\{0,10,110,11100,11001,11110,11111\}$ for 7 symbols. Which of the codes is incorrect?
a) 11110
b) 11001
c) 10
d) 110
e) $11100 \quad$ f) 11111

Soln: 11001 cannot be correct because it has a prefix of 110 which is also in the code.
14. A quadrature modulator is fed with $1+\mathrm{j}$. What would be the modulated signal output?


Soln: multipliers of in-phase (I) and quadrature phase (Q) carriers are 1 and 1 respectively. $\cos ()+.\sin ()=.\operatorname{cosi}\left(.+\frac{\pi}{2}\right)$, that is $+\frac{\pi}{2}$ phased carrier with the same frequency.
15. A squarer circuit used in carrier synchronization is fed with a BPSK signal as shown. What is magnitude spectrum of the output signal?

a)

b)
-

d)

e)

f)


Soln: Output of a squarer is DC plus a sinusoidal with twice the frequency of input carrier. Since the carrier frequency is $3 / T_{b}$ (from the figure), output sinusoidal has the frequency of $6 / T_{b}$.
16. A DSB-suppressed-carrier signal is obtained by modulating a carrier with frequency $f_{c}>200 \mathrm{kHz}$ with a tone signal. The signal is measured with a spectrum analyzer as shown. What is the frequency of the tone signal?


Soln: carrier frequency is $(304-296) / 2=300 \mathrm{kHz} . \therefore 304 \mathrm{k}-300 \mathrm{k}=4 \mathrm{kHz}$
17. An 8-FSK system sub-carriers are arranged as shown. All sub-carriers are passed through an FHSS system with 8 hopping frequencies whose first carriers are given as $f_{0}+n \times 8 \times 10^{6}$ Hz where $n$ is a pseudo-random number between 0 and 7 . Assuming that the hop rate is $1 \mathrm{Mhops} / \mathrm{s}$ and the number of symbols per hop is 1 , what is the bit rate?

|  | a) 3 Mbps | b) 24 Mbps |
| :--- | :--- | :--- |
|  | c) 8 Mbps | d) 21 Mbps |



Soln: Hops/s x bitsperhop=bits/s $=1 \mathrm{M} \times 3=3 \mathrm{M}$
18. A binary-PSK system uses signals given by $\psi_{i}(t)= \pm 2 \sin (4000 \pi t)$. Since $T_{b}=1 \mathrm{~ms}$ and symbol probabilities are equal, what is the average energy per symbol?
a) $1 / 2$
b) 1
c) 0
d) $\pi / 2$
e) $8 \pi$
f) $4 \pi$

Soln: $E=\int_{0}^{T_{b}}|\psi(t)|^{2} d t=2 \times 10^{-3}$ (this question is removed from the exam because it appears that I changed the question into energy per symbol question but forget the change the choices from power to energy and the choices do not have the correct answer. Hence, all get +5 points from this question)
19. An 8 Mbps binary baseband communication system uses rectangular pulses of $\pm 2$. What is the signal power?
a) 4
b) 32
c) $1 / 2$
d) 16
e) 1
f) 24

Soln: $P=\frac{1}{T} \int_{0}^{T}|x(t)|^{2} d t=4$
20. An OFDM communication system with 54 Mbps physical bit rate is working at 2.4 GHz ISM band with 64 carriers. What is the actual bit rate when $25 \%$ of the capacity is used for channel coding?
a) 72 Mbps
b) 40.5 Mbps
c) 13.5 Mbps
d) 54 Mbps
e) $64 \mathrm{Mbps} \quad$ f) 67.5 Mbps

