No : Answers **Name : Solutions**

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I handed over all smart devices (mobile phone, smartwatch, earphones, etc.) to the exam supervisor before the exam. I take full responsibility if any are found on me.	Sign here

1. Determine if $x(t) = Acos(\frac{2\pi t}{T} + \varphi)$ and $y(t) = Bcos(\frac{2\pi mt}{T} + \theta)$ are orthogonal over $(0,T)$ interval, where A, B, φ.and θ are arbitrary constants and $m = 1,2, \dots$ is an integer.

$R = \int_0^T Acos\left(\frac{2\pi t}{T} + \varphi\right) Bcos\left(\frac{2\pi mt}{T} + \theta\right) dt$ (use $cos(a)cos(b) = \frac{1}{2}cos(a+b) + \frac{1}{2}cos(a-b)$)

$= \frac{AB}{2}\int_0^T cos\left(\frac{2\pi t}{T} + \varphi + \frac{2\pi mt}{T} + \theta\right) dt + \frac{AB}{2}\int_0^T cos(\frac{2\pi t}{T} + \varphi - \frac{2\pi mt}{T} + \theta)dt$

$= \frac{AB}{2}\int_0^T cos\left(\frac{2\pi(1+m)t}{T} + \varphi + \theta\right) dt + \frac{AB}{2}\int_0^T cos(\frac{2\pi(1-m)t}{T} + \varphi - \theta)dt$

$= \left[\frac{ABT}{4\pi(1+m)} sin\left(\frac{2\pi(1+m)t}{T} + \alpha\right) + \frac{ABT}{4\pi(m-1)} sin\left(\frac{2\pi(m-1)t}{T} + \beta\right)\right]_0^T$

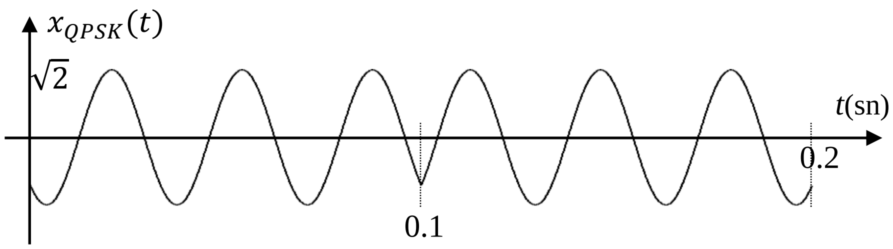
$= \frac{ABT}{4\pi(1+m)}\left(sin(2\pi(1+m) + \alpha) - sin(\alpha)\right) + \frac{ABT}{4\pi(m-1)}\left(sin(2\pi(m-1) + \beta) - sin(\beta)\right)$

$= \frac{ABT}{4\pi(1+m)}\left(sin(\alpha) - sin(\alpha)\right) + \frac{ABT}{4\pi(m-1)}\left(sin(\beta) - sin(\beta)\right) = 0?$

$R = 0$ for all integer values of $m > 1$; $x(t)$ and $y(t)$ are orthogonal. For m=1, $|\varphi - \theta| = \frac{\pi}{2}$ must be satisfied in order to have orthogonality.

2. Given the IQ-table representing a QPSK constellation diagram, in-phase carrier $c(t) = cos(60\pi t)$ and symbol rate of 10 sym/s, draw the QPSK signal for stream 1101 within 2 symbol period, marking relevant numbers on the graph (draw like an engineer!).

sym	i	q
00	1	1
01	-1	1
11	-1	-1
10	1	-1



3. A memoryless source is given with symbol probabilities of z={0.1, 0.14, 0.2, 0.12, 0.12, 0.02, 0.1, 0.08, 0.07, 0.05}. Estimate the possible minimum and maximum lengths of the codewords in Huffman dictionary.

$I_{min} = -log_2(0.2) \cong 2.322$ bits, hence we can have 2 bits as minimum codeword length.
$I_{max} = -log_2(0.02) \cong 5.644$ bits, hence we can have 6 bits as maximum codeword length.
Or, one may find codebook first to see min/max codeword lengths.

4. Outputs of two pn-sequence generators are modulo summed to generate another sequence. What is the length of the new sequence when SSRG[5,3] and SSRG[6,1] run with the same clock signal.

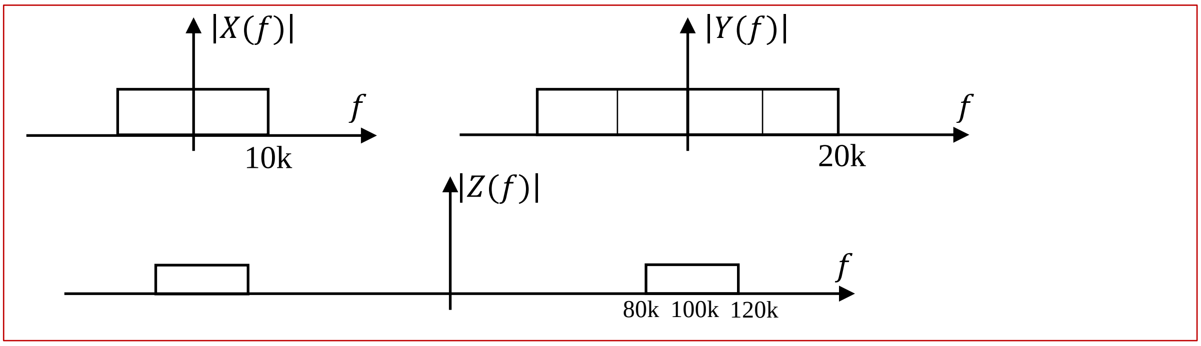
We should find smallest common period for $N_1 = 31$ and $N_2 = 63$. That is, we find lowest common multiplier.
$gcd(63,31) = gcd(32,31) = gcd(1,31) = gcd(1,30) = \ldots = gcd(1,1) = 1$

$lcm(63,31) = 63 \times \frac{31}{\gcd(63,31)} = 1953$

5. Calculate the entropy of the memoryless source given with symbol probabilities z={0.5, 0.25, 0.125, 0.125}. Calculate the average code length when the output is compressed using Shannon-Fano dictionary method.

$H(z) = -\sum_{i=0}^{3} p_i log_2(p_i) = -0.5 \times log_2(0.5) - 0.25 \times log_2(0.25) - 0.25 \times log_2(0.125)$
$H(z) = 1.75$
Using Shannon-Fano method of variable length code generation, we find
$L = \{1,2,3,3\}$. Hence $L_{avg} = \sum_{i=0}^{3} p_i l_i = 0.5 \times 1 + 0.25 \times 2 + 0.25 \times 3 = 1.75$.
This is the same as entropy, because self information of all symbols are integers.

6. A baseband signal with bandwidth of 10 kHz AM modulates a carrier with f_c=10 kHz. The resulting signal is multiplied with another carrier with f_2=100 kHz. What is the bandwidth of the resulting final signal?



The bandwidth of the resulting signal is 40 kHz.

7. Calculate the output noise power of a filter given as $|H(f)| = \begin{cases} 1 - 0.5|f| & ,|f| < 2 \\ 0 & ,otherwise \end{cases}$, when two sided input noise power spectral density is 1 μW/Hz.

$P_\eta = \int_{-\infty}^{\infty} N_0 |H(f)|^2 df = 2\int_0^2 10^{-6} \left(\frac{f}{2}\right)^2 df = \frac{10^{-6}}{6} f^3 \Big|_0^2 = \frac{4}{3}\ \mu W$
(you get the same grade if you inadverdently calculate for one sided density)

8. Calculate the output signal power of a filter given as $|H(f)| = \begin{cases} 1 - 0.5|f| & ,|f| < 2 \\ 0 & ,otherwise \end{cases}$, when input signal is given as $x(t) = 2cos(2\pi t + \pi/4)$+1.

Part of the signal is 1 Hz sinusoidal; the output amplitude is halved as $H(1)$=0.5, giving $y(t) = cos(2\pi t + \varphi)$ from which we calculate the power as $P_1 = 0.5\ W$. We also have a dc component whose power is $P_2 = 1\ W$. Therefore, $P_x = 1.5\ W$.

9. An integrator receiver expects the 4-ary PAM pulses of amplitudes $\psi_i(t) = \{-4, -2, 2, 4\}$. Find optimal threshold values at decision instants, assuming symmetric noise pdf and equal symbol probabilities.

$R_2 = \int_0^{T_s} 2dt = 2T_s \implies R_{-2} = -2T_s,\ R_4 = \int_0^{T_s} 4dt = 4T_s \implies R_{-4} = -4T_s.$
The ML decision thresholds are then found as $\{-3T_s, 0, 3T_s\}$.

10. In an OFDM system with 4 μs symbol duration (no CP), 11 band-edge carriers are null, 1 is used for RF carrier synchronization, and the rest are used to transmit data using either BPSK, QPSK or 64-QAM. Determine the number of sub-carriers needed (FFT-size) in order to achieve 200 Mbps.
$6 \times (N_c - 12)/4 \times 10^{-6} \geq 200 \times 10^6$
$N_c \geq 200 \times 10^6 \times 4 \times 10^{-6} /6 + 12$
$N_c \geq 145.3.$