Eskişehir Osmangazi University, Faculty of Engineering and Architecture Department of Electrical Engineering \& Electronics, "Communications" Homework 10.04.2022

The following quarternary $(\mathrm{M}=4)$ baseband communication system and one of the waveforms representing one of the symbols are given.

a) Find remaining waveforms that are either orthogonal or make up an antipodal pair with the given. That is, you are expected to find $\psi_{1}, \psi_{2}$ and $\psi_{3}$. All waveforms should be either orthogonal to each other or pairwise antipodal and orthogonal to other pair(s). Hint: There is no single/unique solution. Everyone can find different waveforms.
b) Determine correlator outputs at the decision instants for all waveforms.

Turn in:
Answers sheets (either printed or clearly written on A4) must be returned to my office (B305). Slide them in under my office door by 15.4.2022 Friday 12:00. No electronic forms will be accepted. No infringed / plagiarised answers will be evaluated. No late turn-ins will be accepted. Unreadeable sheets will not be evaluated.

## Solution:

Given that $\psi_{0}(t)=\left\{\begin{array}{ll}t & 0<t<T_{s} \\ 0 & \text { otherwise }\end{array}\right.$, we can choose $\psi_{1}(t)=-\psi_{0}(t)$ so that we have an antipodal pair.
We can choose to have other pair to be antipodal too. Let,
$0=\int_{0}^{T s} \psi_{0}(t) \psi_{2}(t) d t$
so that $\psi_{0}(t)$ and $\psi_{2}(t)$ are orthogonal to each other. It is obvious that we are not limited to a limited number of solutions with this orthogonality definition. Let us chose $\psi_{2}(t)$ to be composed of two rectangular pulses whose magnitudes are yet to be calculated.
$\psi_{2}(t)= \begin{cases}a, & 0<t<\frac{T_{s}}{2} \\ b, & \frac{T_{s}}{2}<t<T_{s} \\ 0, & \text { otherwise }\end{cases}$
$R_{x, 21}=\int_{0}^{T s / 2} a t d t=\frac{a T_{s}^{2}}{8}$ and $R_{x, 22}=\int_{T s / 2}^{T s} b t d t=\frac{b}{2}\left(T_{s}^{2}-\frac{1}{4} T_{s}^{2}\right)=\frac{3 b}{8} T_{s}^{2}$.
We need to have $R_{x, 21}=-R_{x, 22}$ so that they will cancel out. Hence,
$\frac{a T_{s}^{2}}{8}=\frac{-3 b}{8} T_{s}^{2}$ gives us $a=-3 b$. Example: $b=-1$ and $a=3$.


Note that this solution is just an example out of infinite number of possible solutions.
Verify;
$\int_{0}^{T s} \psi_{0}(t) \psi_{2}(t) d t=\int_{0}^{T s / 2} 3 t d t-\int_{T s / 2}^{T s} t d t=0$ (orthogonal)

