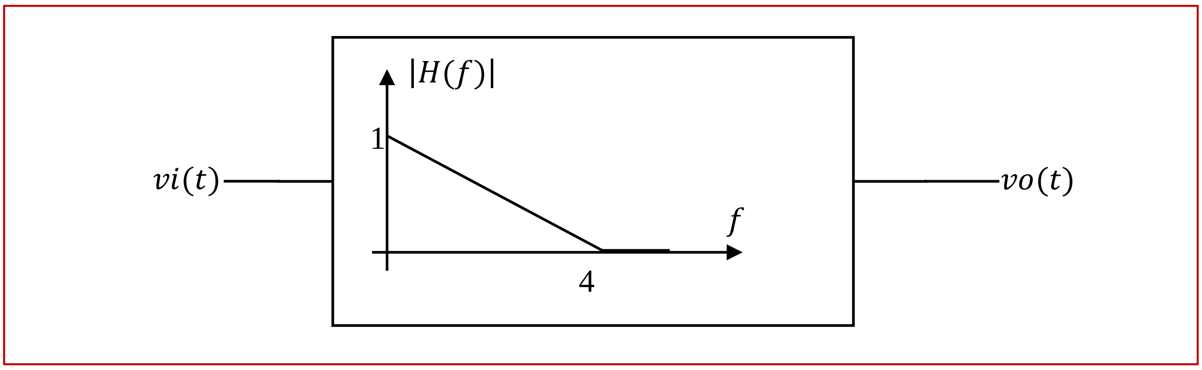
**No :** **Name :**
**Eskişehir Osmangazi University, Faculty of Engineering and Architecture**
**Dept. of Electrical Engineering & Electronics,** "*Communications*" SingleCourse 14.09.2023

**1.** The input to the following filter is $vi(t) = x(t) + \eta(t)$ where $\eta(t)$ is white noise with spectral density of 0.01 W/Hz. The signal part is $x(t) = cos(4\pi t) + sin(8\pi t)$. Calculate the SNR at the output, ignoring the distortion on the signal part.



Answer:

$x(t)$ has two sinusoidal components; 2 Hz and 4 Hz. However, the response of the filter to 4 Hz sinusoidal is zero, therefore it will not be seen at the output.

Note: Filters' alterations on the signal components, including phase and amplitude, can falsely be called as distortions. However, in signal processing (especially audio processing) introduction of previously nonexistent components are actually called distortion. These effects are created by nonlinear responses of the filters/systems. We usually assume that the filters are linear and no such effects exist when doing linear analysis. Therefore, rejection (or attenuation) of some sinusoidal components originally in the input signal is not considered as distortion. When the system is linear, no additional frequency components will be seen at the output. Only the components at the input will be seen, altered or not.

We will see only 2 Hz sinusoidal at the output with possible phase and amplitude change. At 2 Hz, the filter's gain is 0.5. Since the phase information is not given we may assume an arbitrary phase or 0 as well. The power is independent of the phase anyway. So the signal output is $y(t) = 0.5cos(4\pi t + \varphi)$, where $\varphi$ is an arbitrary phase value.

Signal power at the output is then $P_s = \frac{V_p{}^2}{2} = (0.5)^2/2$ W. One may also calculate the power using $P = \frac{1}{T}\int_{\alpha}^{\alpha+T}|y(t)|^2 dt$ (energy per unit time), only to find the same power value.

Noise power at the output will be calculated by integrating psd over the whole range;

$P_\eta = \int_{-\infty}^{\infty} G_{\eta o}(f)df$ where $G_{\eta o}(f) = G_{\eta i}(f)|H(f)|^2$ and $H(f) = 1 - f/4$ (0<t<4) (from the figure). One may also calculate this using $H(f) = f/4$ because of the constant input noise psd.

$$P_\eta = \int_0^4 0.01 \times \left(\frac{f}{4}\right)^2 df = 0.01 \times \frac{f^3}{48}\Big]_0^4 \cong 0.013 \text{ W.}$$

The SNR is then

$$\frac{P_s}{P_\eta} = 0.125/0.013 = 9.377 \cong 9.721 \text{ dB.}$$

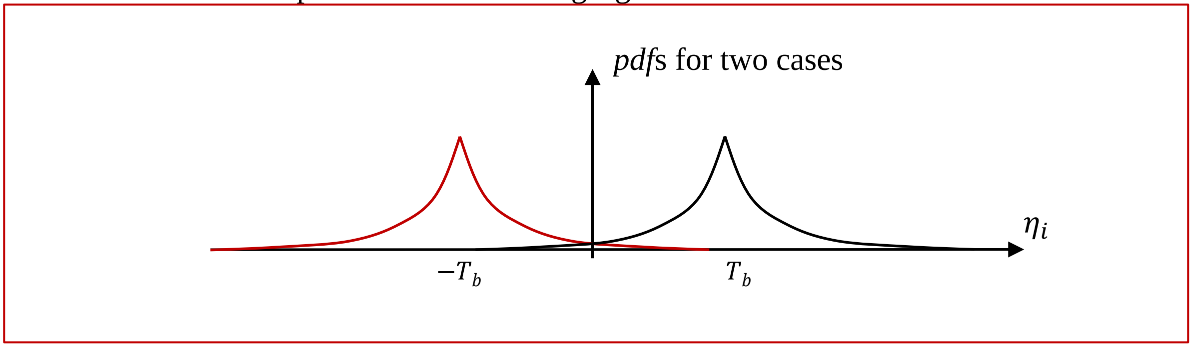
**2.** A correlator receiver is designed to detect antipodal waveforms given as $x(t) = \mp 1$ received from the noisy transmission channel at every $T_b$. Pdf of the noise part at the correlator output is given as $f_\eta = \frac{1}{2}e^{-|\eta|}$. Assume that the receiver is in sync with the received signal.

a) Determine the probability of bit detection error in the system.

b) What is the maximum bit rate when the desired $p_e \leq$0.01.

Answer:

We need to calculate the signal energy at the output of the correlator at the decision instant. We assume that, since the waveforms are rectangular antipodal, the receiver does not generate $\psi(t) = 1$ and there is no multiplier (i.e. it is just an integrator). $E_b = \int_0^{T_b} dt = T_b$. The decision situation is depicted in the following figure.



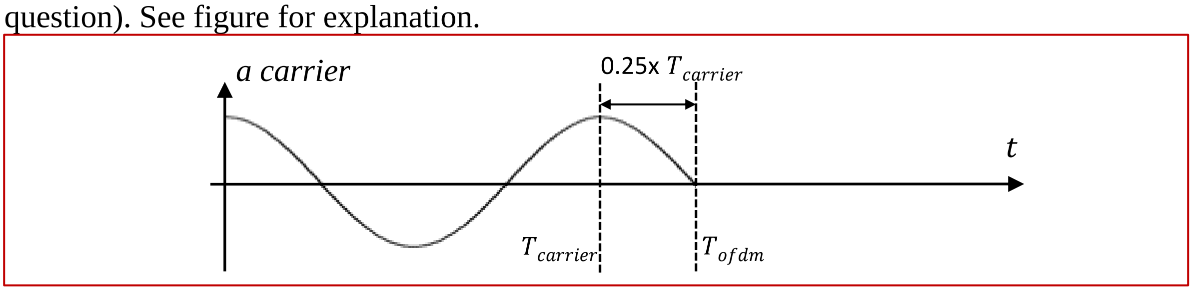
a) $p_e = \int_{T_b}^{\infty} f_\eta d\eta$ = $p_e = \int_{T_b}^{\infty} \frac{1}{2}e^{-\eta} d\eta$ = $-\frac{1}{2}e^{-\eta}]_{T_b}^{\infty}$ = $\frac{1}{2}e^{-T_b}$.

b) 0.01=$\frac{1}{2}e^{-T_b}$ ⇒ $T_b = -ln(0.02) \cong$ 3.912 s. ⇒ $R_b = 1/T_b$ = 0.256 bps.

**3.** An OFDM communication system comprises 64 sub-channels 14 of which are used for pilot carriers and null carriers for interference protection. All channels must fit into 20 MHz band, null-to-null. A CP of 25% is employed.
a) Determine the symbol rate.
b) Determine the bit rate when half the data sub-channels employ 16-QAM and the rest use QPSK.

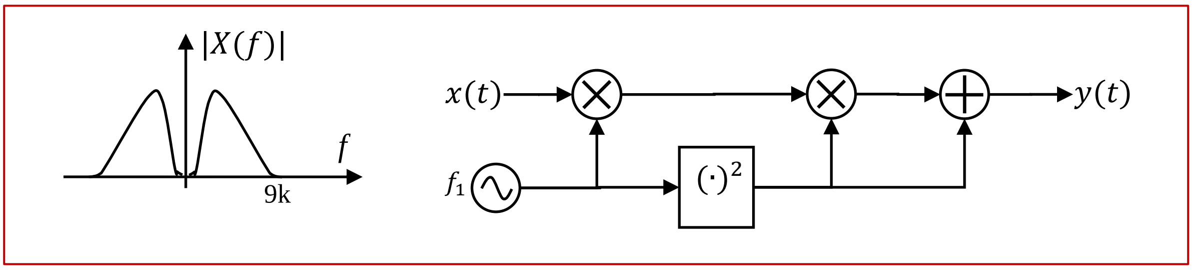
Answer:
We have 64-14 = 50 data sub-channels with $f_d$ = 20 MHz / 65 ≅ 307.692 kHz sub-carrier spacing.
a) Therefore, the symbol rate is $R_{sym}$ = 307.692 ksym/s, independent of CP.
b) For 50/2=25 16-QAM sub-channels the total number of bits per symbol is 25x4=100. The total number of bits transmitted per OFDM symbol for 25 QPSK channels is 25x2=50. Therefore, the number of bits transmitted per OFDM symbol duration is 100+50=150. Since the symbol rate is 307.692 ksym/s, the bit rate is 307.692k x150 ≅ 46.154 Mbits/s.

Note that any passband OFDM schema, the sub-carriers must start at an integer multiple of 307.692x1.25=384.615 kHz in order for passband orthogonality (this is not asked in the question). See figure for explanation.



However, in OFDM systems, it is only required to have orthogonality between carriers for the duration of the lowest carrier frequency (not for ofdm symbol duration) when the signal is frequency-down-converted to baseband (because of FFT). That is, they do not need to be orthogonal in passband. Even though the shape of the spectrum is preserved, the carriers may not be orthogonal for neither the lowest carrier period nor the CP expanded symbol period.

**4.** A baseband message signal is frequency up-converted using the following system for transmission. Assume $f_1$>10kHz.



Draw the two sided spectrum of the output ($|Y(f)|$). Mark all significant frequencies and values.

Answer: