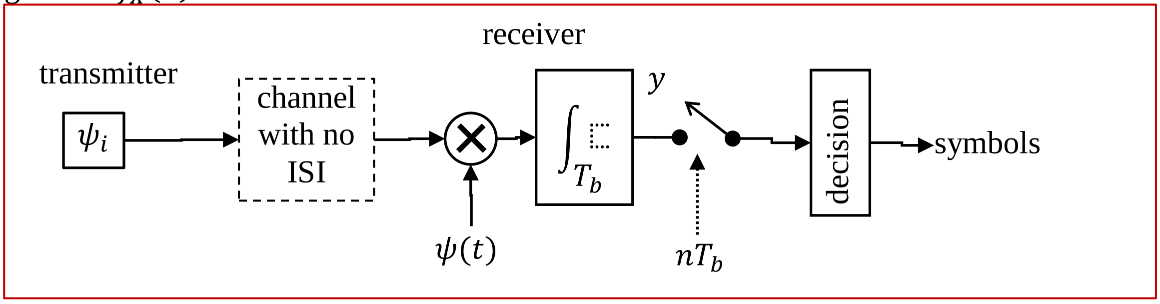
No : Answer **Name :** Solution

Eskişehir Osmangazi University, Faculty of Engineering and Architecture

Department of Electrical Engineering & Electronics, "*Communications*" Midterm 22.04.2024

1. A baseband binary communication system is given below. The waveforms representing binary symbols are $\psi_i = \mp sin(\pi t/T_b)$ for a symbol period T_b and 0 elsewhere. The receiver is in sync with the received signal. The pdf of the noise component at the correlator output is given as $f_X(x) = 500e^{-1000|x|}$.



Find the maximum bit rate achievable for $p_e \leq 0.01$

Answer:

Let $\psi(t) = sin(\pi t/T_b)$.

$$y(nT_b) = \int_0^{T_b} \psi_i(t)\psi(t)dt = \mp \int_0^{T_b} sin(\pi t/T_b)\, sin(\pi t/T_b)\, dt = \mp \int_0^{T_b} sin^2(\pi t/T_b)\, dt$$

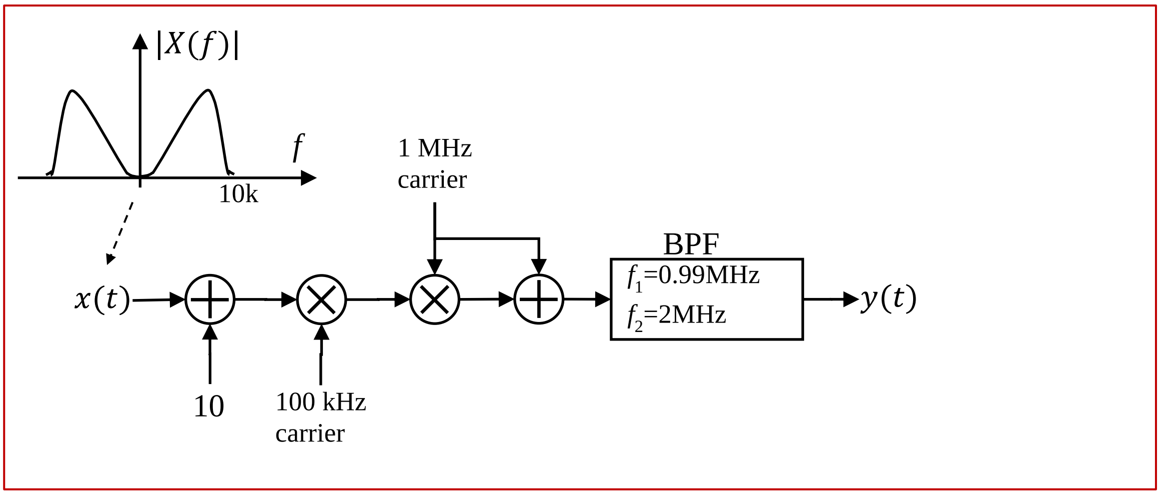
$$= \mp \frac{1}{2}\int_0^{T_b}(1 - cos(2\pi t/T_b))dt = \mp \frac{1}{2}\left[t - \frac{T_b}{2\pi}\sin(2\pi t/T_b)\right]_0^{T_b} = \mp \frac{T_b}{2} = \mp E_b$$

$$p_e = 500 \int_{\frac{T_b}{2}}^{\infty} e^{-1000x}\, dx = -\frac{1}{2}e^{-1000x}\Big|_{\frac{T_b}{2}}^{\infty} = \frac{1}{2}e^{-500T_b}$$

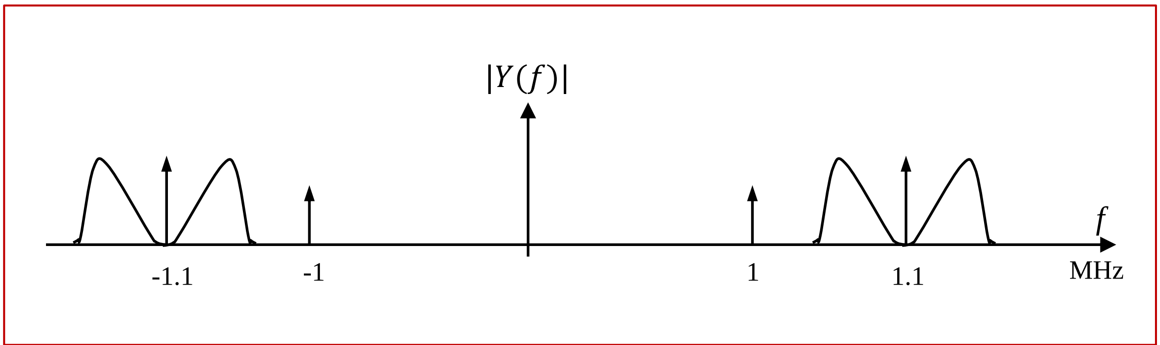
for $p_e \leq 0.01$, $\frac{1}{2}e^{-500T_b} \leq 0.01 \Rightarrow T_b \geq -\ln(0.02)/500$, $T_b \geq \sim 7.824$ ms

$$R_{max} = \frac{1}{T_{bmin}} = 127.8 \text{ bps}$$

2. Given the spectrum of the baseband input signal, estimate and draw the output spectrum of the following system.



Answer:



3. Let the compression ratio is defined as the ratio of the average numbers of bits per symbols compressed data and the uncompressed data ($R_{comp} = L_{avg-compressed}/L_{avg-uncompressed}$). Calculate the compression ratios (using Huffman's dictionary preparation method) for the second and third extensions of the binary source given by $z = \{0.8, 0.2\}$.

Answer:

Second extension of the source is S_2={00,01,10,11} with $z_2 = \{0.64, 0.16, 0.16, 0.04\}$. Huffman dictionary would have $L_2 = \{1,2,3,3\}$. And
$L_{2avg} = \sum_i p_i l_i$=0.64+0.16x2+0.16x3+0.04x3=1.56 bits/symbol. Hence, the ratio is
$R_2 = \frac{1.56}{2} = 0.78$.

Third extension source is S_3={000,001,010,011,100,101,110,111} with $z_2 = \{0.512,0.128,0.128,0.032,0.128,0.032,0.032, 0.008\}$. Code lengths in the dictionary are calculated as $L_3 = \{1,3,3,5,3,5,5,5\}$. Therefore,
$L_{3avg} = \sum_i p_i l_i$=0.512+0.128x3+0.128x3+0.032x5+0.128x3+0.032x5+0.032x5+0.008x5
L_{3avg} =2.184 bits/symbol
Ratio is then $R_3 = \frac{2.184}{3} = 0.728$.