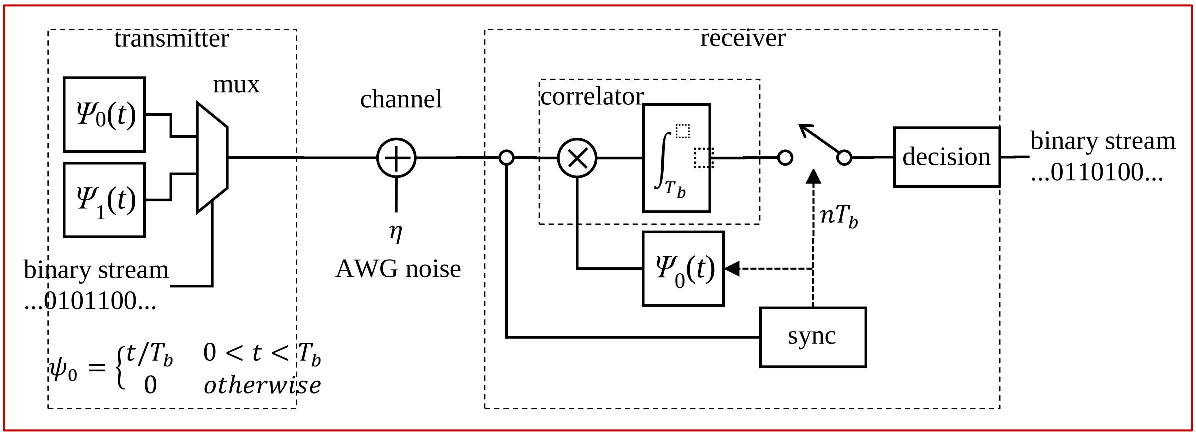
No : Answers **Name : Solutions**

Eskişehir Osmangazi University, Faculty of Engineering and Architecture

Department of Electrical Engineering & Electronics, "*Communications*" Midterm 18.04.2022

1. The following binary baseband communication system is given. ψ_i are antipodal and $\psi_0 = \begin{cases} t/T_b & 0 < t < T_b \\ 0 & otherwise \end{cases}$. Psd of AWG Noise added in the channel is N_0=10^{-7} W/Hz.



a) Determine the probability of bit error when the bitrate is 1 Mbps.

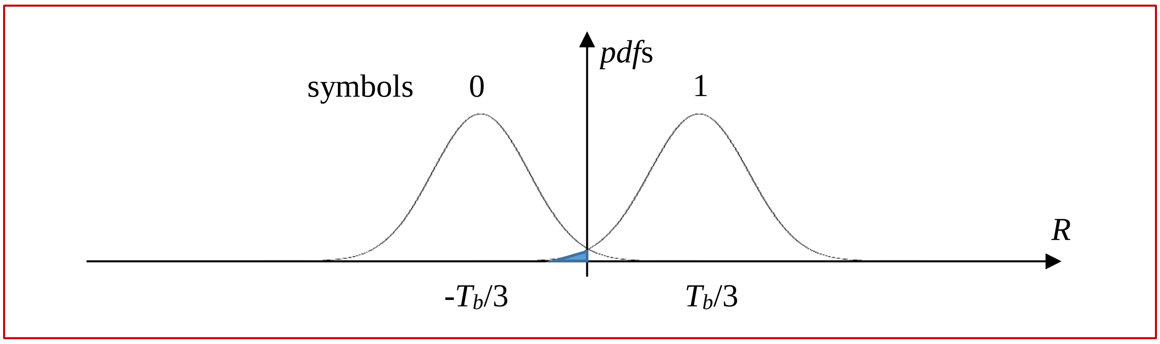
b) Determine the probability of bit error when the bitrate is 2 Mbps.

Solution:

Signal part of the correlator output at the decision instants is

$$E_b = \pm \int_0^{T_b} (\psi_0)^2 dt = \pm \int_0^{T_b} \frac{t^2}{T_b^2} dt = \pm \frac{1}{T_b^2} \frac{t^3}{3} \Big]_0^{T_b} = \pm \frac{T_b}{3}$$

Therefore the probabilistic situation will be as shown



The probability of bit error is then $p_e = Q\left(\sqrt{\frac{2E_b}{N_0}}\right) = \left(\sqrt{\frac{2T_b}{3N_0}}\right)$

a) For $T_b = 1 \times 10^{-6}$, $p_e = Q\left(\sqrt{\frac{2\times10^{-6}}{3\times10^{-7}}}\right) \cong Q(2.582)$. From the $Q(x)$ table, we find that p_e ≅0.002364

b) $p_e = Q\left(\sqrt{\frac{1\times10^{-6}}{3\times10^{-7}}}\right) \cong Q(1.8257)$. p_e ≅0.034

2. A memoryless binary source generates the symbols A={0,1} with probabilities z={0.8,0.2}. It seems that the entropy of the source allows a compression ratio (output bits per input bit) as low as R_C=0.74. Find out and show how it can be done.

Solution:

$H(z) = -\sum_{i=0}^{1} p_i \, log_2(p_i) \cong 0.8 \times 0.3219 + 0.2 \times 2.3219 \cong 0.722$

Let us look into the second extension as the original one does not allow compression.

B_2={00,01,10,11} and u_2={0.64,0.16,0.16,0.04}

Huffman codes are C_2={0,10,110,111} and $L_{2,avg} = \sum_{i=0}^{3} p_i \, l_{2,i}$=1.56

R_{C2}=1.56/2=0.78

Third extension;

B_3={000,001,010,011,100,101,110,111} and

u_3={0.512, 0.128, 0.128, 0.032, 0.128, 0.032, 0.032, 0.008}

Huffman codes are C_3={0,100,101,11000,111,11001,11010,11011}

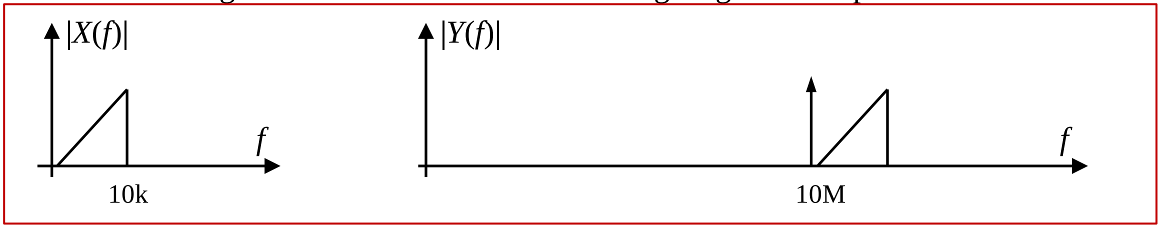
$L_{3,avg} = \sum_{i=0}^{7} p_i \, l_{3,i}$=0.512+0.128x3+0.128x3+0.032x5+0.128x3+0.032x5+0.032x5+0.008x5

$L_{3,avg} = 2.184$

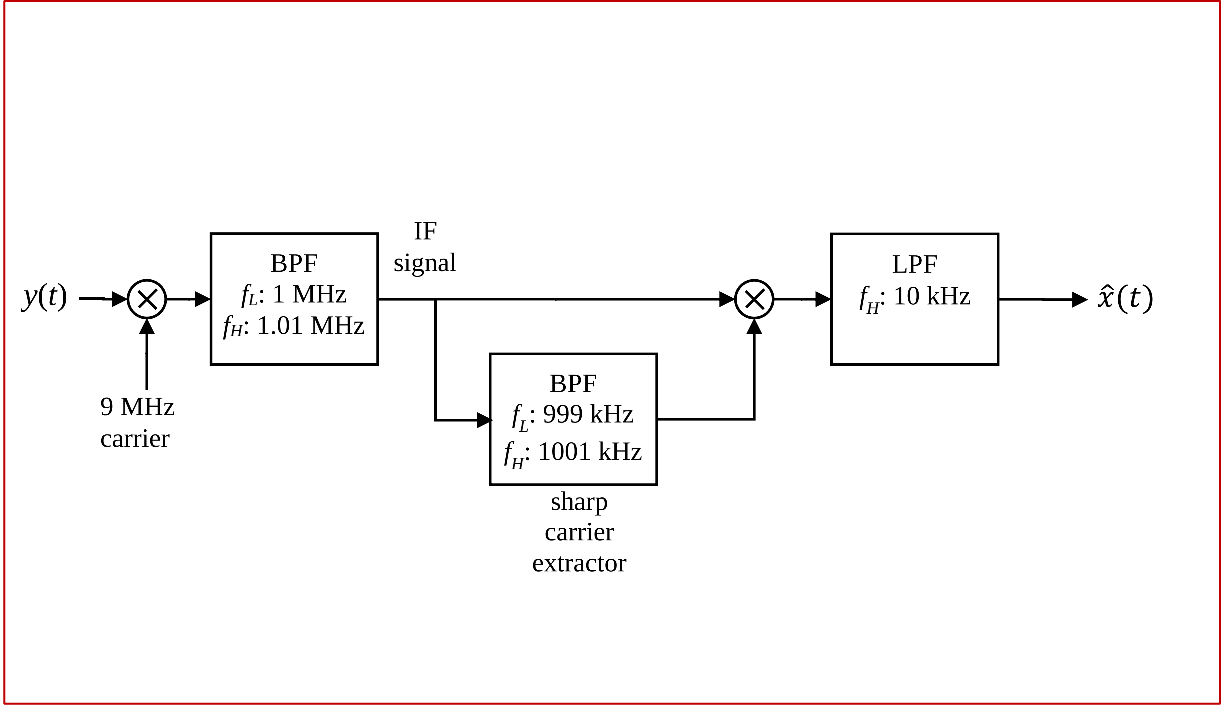
R_{C3}=2.184/3=0.728

This is better than the requirement. But still have room for better compression since the entropy is 0.722.

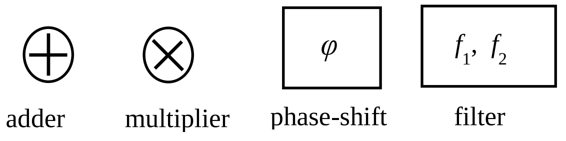
3. A voice signal with 10 kHz bandwidth USB-AM modulates a 10 MHz carrier. The carrier is added to the signal too. as shown in the following single-sided spectrums.



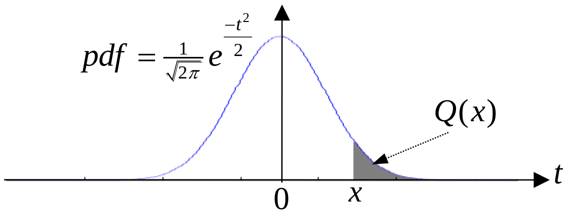
For demodulation of this signal, the signal is first downconverted to 1 MHz IF (intermediate frequency), as shown in the following figure.



Complete the demodulator block diagram using the following circuit blocks:



$Q(x)$ Function



$$Q(x) = \frac{1}{\sqrt{2\pi}}\int_x^{\infty} e^{\frac{-t^2}{2}}\,dt\,, \quad erf(x) = \frac{2}{\sqrt{\pi}}\int_0^x e^{-u^2}\,du\,, \quad Q(x) = \frac{1}{2} - \frac{1}{2}\,erf\left(\frac{x}{\sqrt{2}}\right)$$

Example: Given x=2.575, which equals to 2.5+0.075, we take the number at the crossection of column 2.5 and row 075. That is Q(2.575)= 0.00501200.

Example: Given $Q(x)$= 0.0378, the nearest number in the table is found to be 03794894. From column and row headers, x is found as x=1.5+0.275=1.775.

x	1.	1.5	2.	2.5	3.	3.5	4.	4.5	5.
000	15865525	06680720	02275013	00620967	00134990	00023263	00003167	00000340	00000029
025	15268159	06362955	02143368	00578491	00124317	00021174	00002849	00000302	00000025
050	14685906	06057076	02018222	00538615	00114421	00019262	00002561	00000268	00000022
075	14118736	05762822	01899327	00501200	00105251	00017511	00002301	00000238	00000019
100	13566606	05479929	01786442	00466119	00096760	00015911	00002066	00000211	00000017
125	13029452	05208128	01679331	00433245	00088903	00014448	00001854	00000187	00000015
150	12507194	04947147	01577761	00402459	00081635	00013112	00001662	00000166	00000013
175	11999736	04696712	01481506	00373646	00074918	00011892	00001490	00000147	00000011
200	11506967	04456546	01390345	00346697	00068714	00010780	00001335	00000130	00000010
225	11028761	04226374	01304062	00321507	00062986	00009766	00001195	00000115	00000009
250	10564977	04005916	01222447	00297976	00057703	00008842	00001069	00000102	00000008
275	10115462	03794894	01145296	00276009	00052831	00008000	00000956	00000090	00000007
300	09680048	03593032	01072411	00255513	00048342	00007235	00000854	00000079	00000006
325	09258558	03400051	01003598	00236403	00044209	00006539	00000763	00000070	00000005
350	08850799	03215677	00938671	00218596	00040406	00005906	00000681	00000062	00000004
375	08456572	03039636	00877448	00202014	00036908	00005331	00000607	00000054	00000004
400	08075666	02871656	00819754	00186581	00033693	00004810	00000541	00000048	00000003
425	07707860	02711468	00765419	00172228	00030740	00004336	00000482	00000042	00000003
450	07352926	02558806	00714281	00158887	00028029	00003908	00000429	00000037	00000003
475	07010627	02413407	00666181	00146494	00025543	00003519	00000382	00000033	00000002