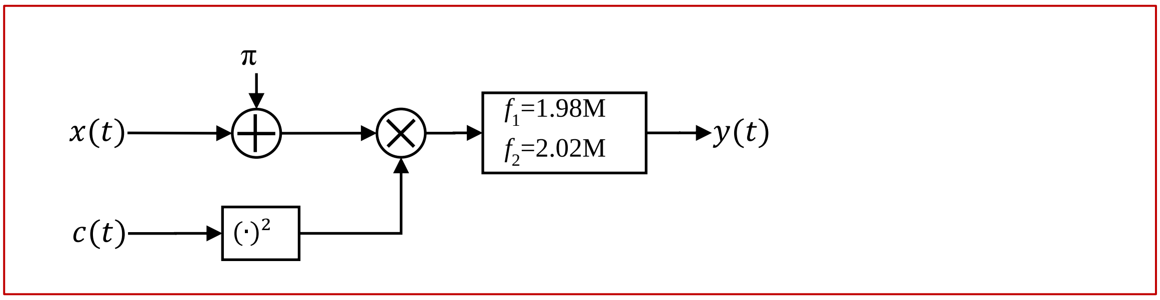
No : Answer **Name : Solution**

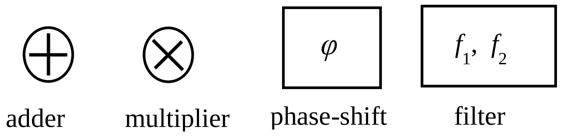
Eskişehir Osmangazi University, Faculty of Engineering and Architecture

Department of Electrical Engineering & Electronics, "*Communications*" Midterm 24.03.2025

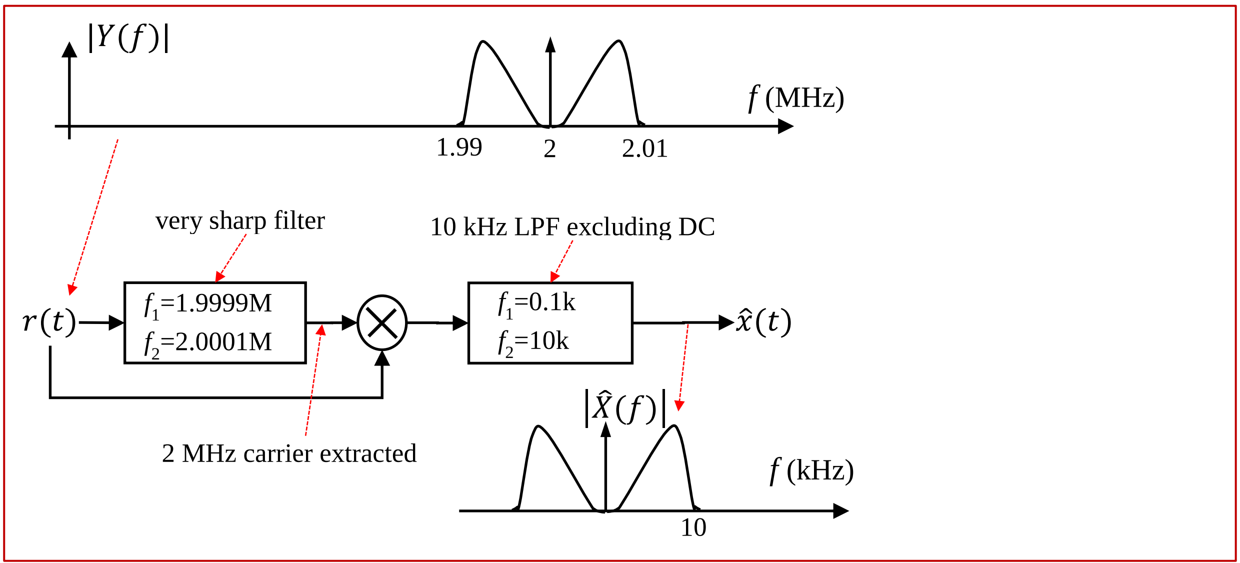
1. A voice signal $x(t)$ occupying 0.1kHz-10kHz band is applied to the following system where $c(t)$ is a 1 MHz carrier. The resulting signal $y(t)$ is amplified and sent to an appropriate antenna.



Draw the block diagram for conceptual demodulation of the received signal. Use only the blocks given below.

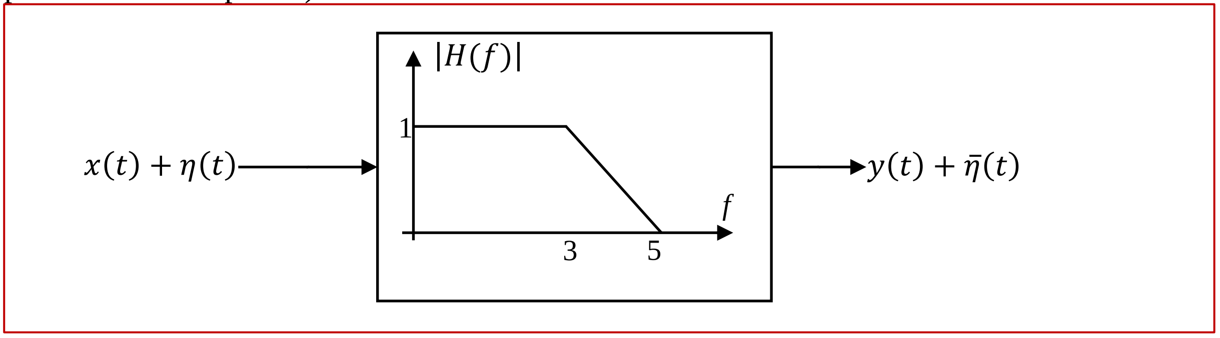


A solution:



Notes: π is just a DC value added onto the voice signal, resulting a recoverable carrier at the carrier frequency (2 MHz). At the receiver, we extract this carrier using a sharp filter and use it for synchronous demodulation.

2. The following filter is fed with a signal given as $x(t) = cos(8\pi t) + 2sin(4\pi t + \pi/8)$. There is also $\eta(t)$, AWGN with N_0=1 μW/Hz, at the input. Noise is uncorrelated (with the signal). Calculate the output SNR (Signal to Noise Ratio is calculated as the ratio of signal-power and noise-power).



Answer:

Just by looking at the system transfer function, we can say that the signal part of the output would be something like $y(t) = 0.5cos(8\pi t + \varphi_1) + 2sin(4\pi t + \varphi_2)$ where φs are unknown (and not needed for calculations) phases of the corresponding sine-waves.

Powers of these sinusoids are calculated using either $P_{s1} = \frac{1}{T}\int_0^T |x_1(t)|^2 dt$ or $P_{s1} = V_{p1}^2/2$ (of course with signal-power assumptions). Hence,

$P_s = P_{s1} + P_{s2} = 0.125 + 1 = 1.625$ W.

Noise power, on the other hand, is

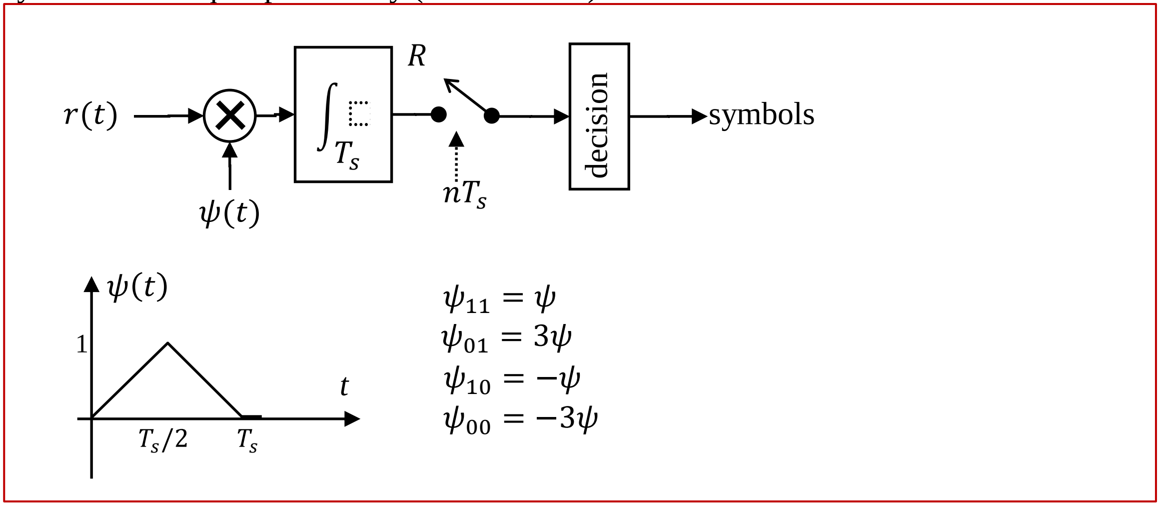
$P_{\eta o} = \int_0^3 N_0 df + \int_3^5 |H(f)|^2 N_0 df = 3N_0 + N_0 \int_0^2 |f/2|^2 df$ (since N_0 is constant)

$N_0 \int_0^2 |f/2|^2 df = \frac{N_0}{4}\int_0^2 f^2 df = \frac{N_0}{4}\frac{f^3}{3}\Big|_0^2 = \frac{N_0}{12}8 = \frac{2}{3}N_0$

$P_{\eta o} = 3N_0 + \frac{2}{3}N_0 = \frac{11}{3}N_0 = \frac{11}{3}$ μW.

The SNR is then 1.625 W / $\frac{11}{3}$ μW ≈ 443182 ≈ 56.5 dB

3. The following correlator and four possible input waveforms are given (in 4-ary PAM system). Determine the estimated thresholds when noise pdf is symmetric and all four symbols have equal probability (ML criterion).



Answer:

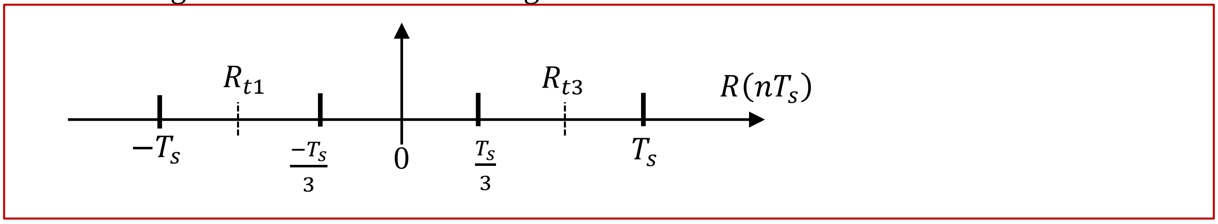
Let $a_i = \{a_{11}, a_{01}, a_{10}, a_{00}\} = \{1,3,-1,-3\}$

$R_{a_i}(nT_s) = \int_0^{T_s} \psi(t)\psi_{a_i}(t)\, dt = a_i \int_0^{T_s} \psi^2(t)dt$, when $\psi_{a_i} = a_i\psi$.

$R_{a_i}(nT_s) = 2a_i \int_0^{T_s/2} \left(\frac{2t}{T_s}\right)^2 dt = \frac{8a_i}{T_s^2}\int_0^{T_s/2} t^2 dt = \frac{8a_i t^3}{3T_s^2}\Big|_0^{T_s/2} = \frac{8a_i T_s^3}{3T_s^2 2^3} = \frac{a_i T_s}{3}$

$R_{a_{11}=1}(nT_s) = \frac{T_s}{3}$, $R_{a_{01}=3}(nT_s) = T_s$, $R_{a_{10}=-1}(nT_s) = \frac{-T_s}{3}$, $R_{a_{00}=-3}(nT_s) = -T_s$

The following resultant constellation diagram



suggests that the thresholds should be at the midpoints of the constellation points, and are $R_{t1} = \frac{-2T_s}{3}$, $R_{t2} = 0$, $R_{t3} = \frac{2T_s}{3}$

4. Find the dictionary using Shannon-Fano method and encode the stream "000111001100000010101111000000" for the second extension of the source (2 bits/sym).

Answer:

Counting the occurrences of 2-bit symbols we find that $p_i = \{8/15, 4/15, 2/15, 1/15\}$ for the symbols $A = \{00, 11, 10, 01\}$. Applying the Shannon-Fano division algorithm on this ordered set

S_i	$15p_i$			
00	8	0		
11	4	1	0	
10	2	1	1	0
01	1	1	1	1

we get $C = \{0, 10, 110, 111\}$ as the code. Encoding the given stream according to the dictionary found, we obtain

"00 01 11 00 11 00 00 00 10 10 11 11 00 00 00" => "0 111 10 0 10 0 0 0 110 110 10 10 0 0 0"

That is, "011110010000110110101000".