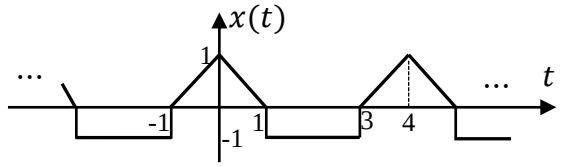
No : Answers **Name : Solutions**

Eskişehir Osmangazi University, Faculty of Engineering and Architecture

Department of Electrical Engineering & Electronics, "*Signals&Systems*" Midterm 13.11.2026

I handed over all smart devices (mobile phone, smartwatch, earphones, etc.) to the exam supervisor before the exam. I take full responsibility if any are found on me. *Exam duration:90 minutes. No books, notes.*	Sign here

1. A periodic signal $x(t)$ is given below. Determine the energy in one period and power of the signal.



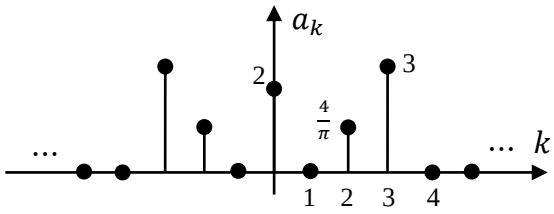
Solution: using the symmetry of triangular region

$E_T = \int_T |x(t)|^2 dt = 2\int_0^1 |1-t|^2 dt + \int_0^2 |-1|^2 dt$

$E_T = 2\int_0^1 t^2 dt + 2 = 2\left[\frac{1}{3}t^3\right]_0^1 + 2 = \frac{2}{3} + 2 = \frac{8}{3}$

$P_x = \frac{E_T}{T} = \frac{8/3}{4} = \frac{2}{3}$

2. A Fourier Series magnitude spectrum is given in the figure below. Write down one possible real periodic signal corresponding to this spectrum.



Solution: A valid solution would be $x(t) = 2 + \frac{8}{\pi}cos(4\pi t/T) + 6sin(6\pi t/T)$.

or $x(t) = 2 + \frac{8}{\pi}cos(2\omega_0 t) + 6sin(3\omega_0 t)$

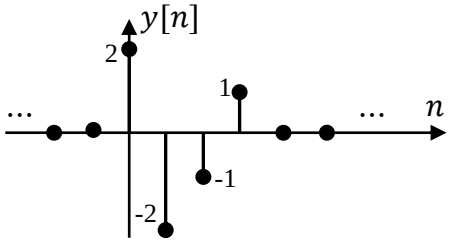
or $x(t) = 2 + \frac{4}{\pi}\left(e^{j2\omega_0 t} + e^{-j2\omega_0 t}\right) + 3\left(e^{j3\omega_0 t} - e^{-j3\omega_0 t}\right)$ or … with any value of T.

3. A discrete-time LTI system has an impulse response $h[n] = \delta[n] - \delta[n-1]$. If its input is $x[n] = 2\delta[n] - \delta[n-2]$, determine the output $y[n]$.

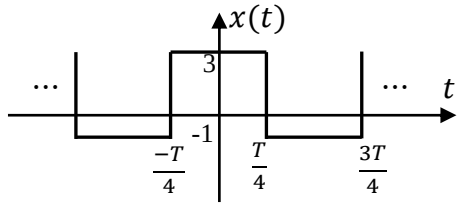
Solution: $y[n] = x[n] * h[n] = (2\delta[n] - \delta[n-2]) * (\delta[n] - \delta[n-1])$. Using sifting property of impulse function, we have

$y[n] = 2\delta[n] - \delta[n-2] - 2\delta[n-1] + \delta[n-3]$, or

$y[n] = 2\delta[n] - 2\delta[n-1] - \delta[n-2] + \delta[n-3]$.



4. Determine/Sketch the magnitude spectrum of the complex exponential Fourier series coefficients a_k of the periodic waveform shown below.



Solution: $x(t)$ is an even function, therefore we'll find only coefficients of cosine terms. a_0 is the average; a_0=(3-1)/2=1.

$$a_k = \frac{1}{T}\int_{T} x(t)e^{-jk\omega_0 t}dt = \frac{1}{T}\left[-\int_{-\frac{T}{2}}^{-\frac{T}{4}} e^{-jk\omega_0 t}dt + 3\int_{-T/4}^{T/4} e^{-jk\omega_0 t}dt - \int_{\frac{T}{4}}^{\frac{T}{2}} e^{-jk\omega_0 t}dt\right]$$

$$a_k = \frac{1}{-jk\omega_0 T}\left[-e^{-jk\omega_0 t}\Big|_{-\frac{T}{2}}^{-\frac{T}{4}} + 3e^{-jk\omega_0 t}\Big|_{-\frac{T}{4}}^{\frac{T}{4}} - e^{-jk\omega_0 t}\Big|_{\frac{T}{4}}^{\frac{T}{2}}\right]$$

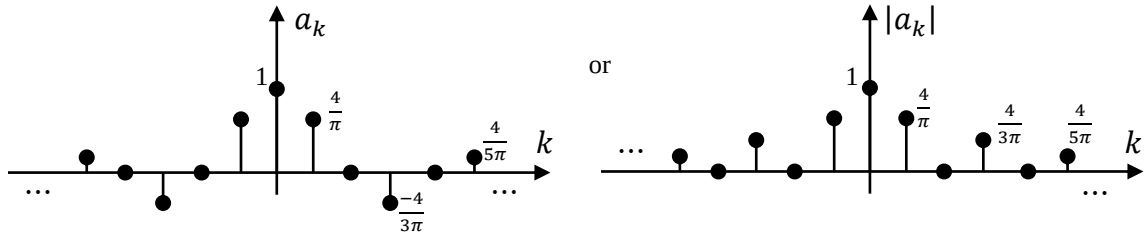
$$a_k = \frac{1}{-jk\omega_0 T}\left[-e^{\frac{jk\omega_0 T}{4}} + e^{\frac{jk\omega_0 T}{2}} + 3e^{\frac{-jk\omega_0 T}{4}} - 3e^{\frac{jk\omega_0 T}{4}} - e^{\frac{-jk\omega_0 T}{2}} + e^{\frac{-jk\omega_0 T}{4}}\right]$$

Since $\omega_0 = 2\pi/T$, $a_k = \frac{1}{-jk2\pi}\left[-e^{\frac{jk\pi}{2}} + e^{jk\pi} + 3e^{\frac{-jk\pi}{2}} - 3e^{\frac{jk\pi}{2}} - e^{-jk\pi} + e^{\frac{-jk\pi}{2}}\right]$

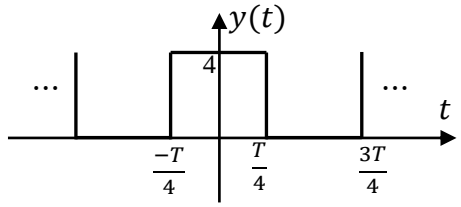
Since $e^{jk\pi} = e^{-jk\pi} = (-1)^k$, these terms cancel out. Therefore,

$a_k = \frac{1}{-jk2\pi}\left[4e^{\frac{-jk\pi}{2}} - 4e^{\frac{jk\pi}{2}}\right]$, and using Euler's identity $e^{j\alpha} - e^{-j\alpha} = 2jsin(\alpha)$, we have

$a_k = \frac{4}{k\pi}sin(\frac{k\pi}{2})$, that is, 0s between alternating + and – values, and $a_k = a_{-k}$.



Another way to calculate; Let $x(t) = y(t) - 1$, where $y(t)$ is drawn below,



so that integration would be easier.

$b_k = \frac{1}{T}\int_{-T/4}^{T/4} 4e^{-jk\omega_0 t}dt$

$b_k = \frac{-4}{jk\omega_0 T}e^{-jk\omega_0 t}\Big|_{-\frac{T}{4}}^{\frac{T}{4}}$, and replace $\omega_0 = 2\pi/T$.

$b_k = \frac{-4}{jk2\pi}\left[e^{-jk2\pi t/T}\right]_{-T/4}^{T/4}$

$b_k = \frac{-2}{jk\pi}\left[e^{-jk\pi/2} - e^{jk\pi/2}\right] = \frac{2}{jk\pi}\left[e^{jk\pi/2} - e^{-jk\pi/2}\right] = \frac{4}{k\pi}sin(\frac{k\pi}{2})$ (for $k \neq 0$).

We can use different interval for integration too;

$$a_k = \frac{1}{T}\left[3\int_{-\frac{T}{4}}^{\frac{T}{4}} e^{-jk\omega_0 t}dt - \int_{\frac{T}{4}}^{\frac{3T}{4}} e^{-jk\omega_0 t}dt\right] = \frac{1}{-jk\omega_0 T}\left[3e^{-jk\omega_0 t}\Big|_{-\frac{T}{4}}^{\frac{T}{4}} - e^{-jk\omega_0 t}\Big|_{\frac{T}{4}}^{\frac{3T}{4}}\right]$$

$a_k = \frac{1}{-jk2\pi}\left[3e^{-\frac{jk\pi}{2}} - 3e^{\frac{jk\pi}{2}} - e^{-\frac{jk3\pi}{2}} + e^{\frac{jk\pi}{2}}\right]$. Since $e^{-\frac{jk3\pi}{2}} = e^{\frac{jk\pi}{2}}$. we have

$a_k = \frac{1}{-jk2\pi}\left[2e^{\frac{-jk\pi}{2}} - 2e^{\frac{jk\pi}{2}}\right] = \frac{4}{k\pi}sin(\frac{k\pi}{2})$.