SEMI-SUPERVISED DISTANCE METRIC LEARNING FOR VISUAL OBJECT CLASSIFICATION

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Abstract: This paper describes a semi-supervised distance metric learning algorithm which uses pairwise equivalence (similarity and dissimilarity) constraints to discover the desired groups within high-dimensional data. As opposed to the traditional full rank distance metric learning algorithms, the proposed method can learn nonsquare projection matrices that yield low rank distance metrics. This brings additional benefits such as visualization of data samples and reducing the storage cost, and it is more robust to overfitting since the number of estimated parameters is greatly reduced. Our method works in both the input and kernel induced-feature space, and the distance metric is found by a gradient descent procedure that involves an eigen-decomposition in each step. Experimental results on high-dimensional visual object classification problems show that the computed distance metric improves the performance of the subsequent clustering algorithm.

1 INTRODUCTION

Learning distance metrics is very important for various vision applications such as object classification, image retrieval, and video retrieval (Chen et al., 2005; Cevikalp et al., 2008; Hertz et al., 2003; Hadsell et al., 2006), and this task is much easier when the target values (labels) associated to the data samples are available. However, in many vision applications, there is a lack of labeled data since obtaining labels is a costly procedure as it often requires human effort. On the other hand, in some applications, side information - given in the form of pairwise equivalence (similarity and dissimilarity) constraints between points - is available without or with less extra cost. For instance, faces extracted from successive video frames in roughly the same location can be assumed to represent the same person, whereas faces extracted in different locations in the same frame cannot be the same person. Side information may also come from human feedback, often at a substantially lower cost than explicit labeled data. Our motivation in this study is that using side information effectively in metric learning can bridge the semantic gaps between the low-level image feature representations and high-level semantic concepts in many visual applications, which enables us to select our preferred characteristics for distinction. A typical example is organizing image galleries in accordance to the personal preferences. For example, one may want to group the images as outdoors or indoors. Similarly, we may want to group face images by race or gender. In most of these cases, typical distance functions employed in vision community such as Euclidean distance or Gaussian kernels do not give satisfactory results.

Recently, learning distance metrics from side information has been actively studied in machine learning. Existing distance metric learning methods revise the original distance metric to accommodate the pairwise equivalence constraints and then a clustering algorithm with the learned distance metric is used to partition data to discover the desired groups within data. In (Xing et al., 2003), a full pseudo distance metric, which is parameterized by positive semi-definite matrices, is learned by means of convex programming using side information. The metric is learned via an iterative procedure that involves projection and eigen-decomposition in each step. Relevant Component analysis (RCA) (Bar-Hillel et al., 2003) is introduced as an alternative to this method. But it can exploit only similarity constraints. (Kwok and Tsang, 2003) formulate a metric learning problem that uses side information in a quadratic optimization scheme. Using the kernel trick, the method is also extended to the nonlinear case. Although the authors claim that the learned metric is a pseudo-metric, there is no guarantee that the resulting distance metric yields a positive semi-definite matrix. (Shalev-Shwartz et al.,

2004) proposed a sophisticated online distance metric learning algorithm that uses side information. The method incorporates the large margin concept and the distance metric is modified based on two successive projections involving an eigen-decompsoition. Note that all semi-supervised distance metric learning algorithms mentioned above attempt to learn full rank distance metrics. In addition to these methods, there are some hybrid algorithms that unify clustering and metric learning into a unique framework (Bilenko et al., 2004).

In this paper we are interested in semi-supervised visual object classification problems. In these tasks, the quality of the results heavily relies on the chosen image representations and the distance metric used to compare data samples. The imagery data samples are typically represented by pixel intensities, multi-dimensional multi-resolution histograms or more sophisticated "bag-of-features" based representations using patch-based shape, texture and color features. Unfortunately, these representations usually tend to be high-dimensional and most of the distance metric learning techniques fail in these situations. This is due to the fact that most dimensions in high-dimensional spaces do not carry information about class labels. Furthermore, learning an effective full rank distance metric by using side information cannot be carried out in such high-dimensional spaces since the number of parameters to be estimated is related to the square of the dimensionality and there is insufficient side information to obtain accurate estimates (Cevikalp et al., 2008). A typical solution to this problem is to project the data onto a lower-dimensional space and then learn a suitable metric in the resulting low-dimensional space. There is a large number of dimensionality reduction methods in the literature (Goldberger et al., 2004; Globerson and Roweis, 2005; Torresani and Lee, 2006; Turk and Pentland, 1991). But most of them cannot be used in our case since they are supervised methods that require explicit class labels. On the other hand, relying on an unsupervised dimensionality reduction method is also problematic since important discriminatory information may be lost during a completely unsupervised dimensionality reduction. A better approach would be to use a semi-supervised dimensionality reduction method to find a low-dimensional embedding satisfying the pairwise equivalence constraints as in (Cevikalp et al., 2008). In this paper we propose such an algorithm that works in both the input and kernel induced-feature space. In contrast to the traditional full rank distance metric learning methods, the proposed method allows us to learn nonsquare projection matrices that yield low rank pseudo metrics. This brings additional benefits such as visualization of data samples and reducing the storage cost and it is more robust to overfitting since the number of estimated parameters is greatly reduced. The proposed method bears similarity to the semi-supervised dimension reduction method introduced in (Cevikalp et al., 2008), but it does not assume that samples in a sufficiently small neighborhood tend to have same label. Instead we focus on improving the local margin (separation).

The remainder of the paper is organized as follows: In Section 2, we introduce the proposed method and extend it to the nonlinear case. Section 3 describes the data sets and experimental results. Finally, we present conclusions in Section 4.

2 METHOD

2.1 Problem Setting

Let $\mathbf{x}_i \in \mathbb{R}^d$, $i = 1,\ldots,n$, denote the samples in the training set. We are given a set of equivalence constraints in the form of similar and dissimilar pairs. Let S be the set of similar pairs

$$S = \left\{(\mathbf{x}_i, \mathbf{x}_j) | \mathbf{x}_i \text{ and } \mathbf{x}_j \text{ belong to the same class}\right\}$$

and let D be the set of dissimilar pairs

$$D = \left\{(\mathbf{x}_i, \mathbf{x}_j) | \mathbf{x}_i \text{ and } \mathbf{x}_j \text{ belong to different classes}\right\}.$$

Assuming consistency of the constraints, the constraint sets can be augmented using transitivity and entailment properties as in (Basu et al., 2004).

Our objective is to find a pseudo-metric that satisfies the equivalence constraints and at the same time reflects the true underlying relationships imposed by such constraints. We focus on pseudo-metrics of the form

$$d_{\mathbf{A}}(\mathbf{x}_i, \mathbf{x}_j) = ||\mathbf{x}_i - \mathbf{x}_j||_{\mathbf{A}} = \sqrt{(\mathbf{x}_i - \mathbf{x}_j)^\top \mathbf{A} (\mathbf{x}_i - \mathbf{x}_j)}, \quad (1)$$

where $\mathbf{A} \geq \mathbf{0}$ is a symmetric positive semi-definite matrix. In this case there exists a rectangular projection matrix $\mathbf{W}$ of size $q \times d$ $(q \leq d)$ satisfying $\mathbf{A} = \mathbf{W}^\top \mathbf{W}$ such that

$$||\mathbf{x}_i - \mathbf{x}_j||^2_{\mathbf{A}} = ||\mathbf{W}\mathbf{x}_i - \mathbf{W}\mathbf{x}_j||^2. \quad (2)$$

From this point of view the distance between two points under metric $\mathbf{A}$ can be interpreted as linear projection of the samples by $\mathbf{W}$ followed by Euclidean distance in the projected space. As a result, optimizing with respect to $\mathbf{W}$ rather than $\mathbf{A}$ allows us to reduce the dimensionality of the data and find low rank distance metrics. In the following, we will first show how to find a (potentially) full rank distance metric $\mathbf{A}$ using side information and then extend the idea to allow low rank metrics.

2.2 Learning Full Rank Distance Metrics

Intuitively, the learned distance metric must pull similar pairs closer and push the dissimilar pairs apart. Additionally, it should generalize well to unseen data. To this end, we minimize the following differentiable cost function defined based on sigmoids

$$J(\mathbf{A}) = \frac{1}{N}\sum_{i,j\in S}\frac{1}{1+\exp[-\beta(||\mathbf{x}_i-\mathbf{x}_j||^2_{\mathbf{A}}-t_s)]} + \frac{1}{M}\sum_{i,j\in D}\frac{1}{1+\exp[\beta(||\mathbf{x}_i-\mathbf{x}_j||^2_{\mathbf{A}}-t_d)]}, \quad (3)$$

where N is the number of similar pairs, M is the number of dissimilar pairs, β is a design parameter that controls the slope of the sigmoid functions, and t_s and t_d are the selected thresholds. This cost function has two competing terms as illustrated in Fig. 1. The first term encourages pulling similar points closer, and the second term penalizes small distances between dissimilar pairs. The dissimilar pairs which are closer to each other contribute more to the loss function than the ones which are further from each other for well chosen β (In fact if the dissimilar pairs are too far from each other they do not contribute to the los function at all). Therefore, just as in the Support Vector Machine's hinge loss, the second term of the above loss function is only triggered by dissimilar pairs in the vicinity of decision boundary which participate in shaping the inter-class decision boundaries. From a dimensionality reduction point of view, this can be thought as paying more attention to the displacement vectors between the dissimilar pairs where classes approach each other since these are good candidates for discriminant directions preserving inter-class separability. Although recent supervised distance learning techniques take the margin concept into consideration during learning (Torresani and Lee, 2006; Weinberger et al., 2005), this issue is largely ignored in semi-supervised distance metric learning methods (Kwok and Tsang, 2003; Xing et al., 2003).

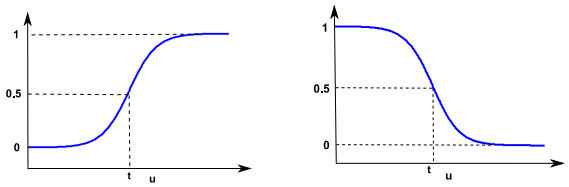
It should be noted that we need at least one active dissimilar sample pair (the closer dissimilar samples contributing to the lost function) since simply minimizing the above loss function over the set of all similar pairs leads to a trivial solution. Therefore including dissimilar pairs is crucial in our method[1] .We would like to find a positive semi-definite distance matrix that minimizes the above criterion. To do so, we can apply a gradient descent based approach. Let $u = (\mathbf{x}_i-\mathbf{x}_j)^\top\mathbf{A}(\mathbf{x}_i-\mathbf{x}_j)$ and $\mathbf{dx}_{ij} = (\mathbf{x}_i-\mathbf{x}_j)$. Differentiating $J(\mathbf{A})$ with respect to the distance matrix $\mathbf{A}$ gives the following gradient for the update rule

$$\frac{\partial J(\mathbf{A})}{\partial \mathbf{A}} = \frac{1}{N}\sum_{i,j\in S}\frac{\beta\exp[-\beta(u-t_s)]}{(1+\exp[-\beta(u-t_s)])^2}\mathbf{dx}_{ij}\mathbf{dx}_{ij}^\top - \frac{1}{M}\sum_{i,j\in D}\frac{\beta\exp[\beta(u-t_d)]}{(1+\exp[\beta(u-t_d)])^2}\mathbf{dx}_{ij}\mathbf{dx}_{ij}^\top. \quad (4)$$

To optimize the cost function we iteratively take a small step in the direction of the negative of this gradient. However, this updating rule does not guarantee positive semi-definiteness on matrix $\mathbf{A}$. To do so, the matrix $\mathbf{A}$ must be projected onto the positive semi-definite cone at each iteration. This projection is performed by taking the eigen-decomposition of the computed distance matrix and removing the components with negative eigenvalues if exist any. At the end, the resulting distance matrix is shaped mostly by the displacement vectors between closer dissimilar pairs and the displacement vectors between far-away similar pairs. The algorithm is summarized below:

Initialization: Initialize $\mathbf{A}_0$ to some positive definite matrix.
Iterate: Do the following steps until convergence:

- Set $\widetilde{\mathbf{A}}_{t+1} = \mathbf{A}_t - \eta\frac{\partial J(\mathbf{A})}{\partial \mathbf{A}}$.
- Apply eigen-decomposition to $\widetilde{\mathbf{A}}_{t+1}$ and reconstruct it using positive eigenvalues and corresponding eigenvectors $\mathbf{A}_{t+1} = \sum_k \lambda_k \mathbf{e}_k\mathbf{e}_k^\top$.



(a) (b)

Figure 1: Visualization of sigmoidal functions used in optimization. The first function (a) handles similar pairs, and it takes higher values as the distances between similar pairs increase. The second function (b) is used with dissimilar pairs and it takes higher values if the distances between dissimilar pairs are smaller than the selected threshold.

2.3 Learning Low Rank Distance Metrics

As we mentioned earlier, distance between two samples under positive semi-definite distance matrix $\mathbf{A}$

[1] If the dissimilarity information is not available, we need an additional constraint such as $\sum_{i,j}|\mathbf{A}_{ij}| > 0$ in order to avoid a trivial solution. But, we will not consider this case here since dissimilarity information is available in most applications.

can be interpreted as linear projection of the samples followed by Euclidean distance in the projected space, i.e., $d_{\mathbf{A}}(\mathbf{x}_i - \mathbf{x}_j) = ||\mathbf{W}\mathbf{x}_i - \mathbf{W}\mathbf{x}_j||$. Therefore low rank distance metrics satisfying equivalence constraints also allow low-dimensional projections which reduce the dimensionality of the original input space. Reducing the dimensionality offers several advantages: First, projection of samples onto a lower-dimensional space reduces the storage requirements. Secondly, projections onto 2 or 3-dimensional space allow us visualization of data, so we can devise an interactive constraint selection tool and verify the effects of our selections visually.

Unfortunately, optimization of $J(\mathbf{A})$ subject to rank-constraints on $\mathbf{A}$ is not convex and difficult to solve (Globerson and Roweis, 2005; Torresani and Lee, 2006). One way to obtain a low rank distance matrix is to solve for full rank matrix $\mathbf{A}$ using the algorithm described earlier, and then obtain a low rank projection by using its leading eigevalues and corresponding eigenvectors as in (Globerson and Roweis, 2005). A more elaborate way to obtain low rank distance matrix is to formulate the optimization problem with respect to nonsingular projection matrix $\mathbf{W}$ of size $q \times d$ rather than $\mathbf{A}$. Here $q \leq d$ represents the desired rank of the distance matrix. This formulation is more efficient and robust to overfitting since the number of unknown parameters (elements of $\mathbf{W}$) is significantly reduced. The rank of the resulting distance matrix $\mathbf{A}$ is at most q since the equation $\mathbf{A} = \mathbf{W}^\top \mathbf{W}$ holds and the projected samples $\mathbf{W}\mathbf{x}_i$ lie in $\mathbb{R}^q$.

Our original cost function can be written in terms of $\mathbf{W}$ as

$$J(\mathbf{W}) = \frac{1}{N} \sum_{i,j \in S} \frac{1}{1+\exp[-\beta(||\mathbf{W}\mathbf{x}_i - \mathbf{W}\mathbf{x}_j||^2 - t_s)]} + \frac{1}{M} \sum_{i,j \in D} \frac{1}{1+\exp[\beta(||\mathbf{W}\mathbf{x}_i - \mathbf{W}\mathbf{x}_j||^2 - t_d)]}, \quad (5)$$

Now let $u = (\mathbf{x}_i - \mathbf{x}_j)^\top \mathbf{W}^\top \mathbf{W}(\mathbf{x}_i - \mathbf{x}_j)$. If we differentiate $J(\mathbf{W})$ with respect to $\mathbf{W}$, we obtain

$$\frac{\partial J(\mathbf{W})}{\partial \mathbf{W}} = \frac{2\mathbf{W}}{N} \sum_{i,j \in S} \frac{\beta \exp[-\beta(u - t_s)]}{(1+\exp[-\beta(u - t_s)])^2} \mathbf{d}\mathbf{x}_{ij} \mathbf{d}\mathbf{x}_{ij}^\top - \frac{2\mathbf{W}}{M} \sum_{i,j \in D} \frac{\beta \exp[\beta(u - t_d)]}{(1+\exp[\beta(u - t_d)])^2} \mathbf{d}\mathbf{x}_{ij} \mathbf{d}\mathbf{x}_{ij}^\top. \quad (6)$$

As in the first case we have to ensure that the resulting distance matrix is positive semi-definite. To this end, we construct $\mathbf{A}$ from $\mathbf{W}$ and apply eigendecomposition on $\mathbf{A}$. This computation can be efficiently done by performing a thin singular value decomposition on $\mathbf{W}$ instead of performing a full eigendecomposition on $\mathbf{A}$. After removing the negative eigenvalues and corresponding eigenvectors we reconstruct the projection matrix as

$$\mathbf{W} = \Lambda^{1/2} \mathbf{E}, \quad (7)$$

where Λ is a diagonal matrix of nonzero eigenvalues of positive semi-definite matrix $\mathbf{A}$, and $\mathbf{E}$ is the matrix whose columns are the corresponding eigenvectors. The algorithm is summarized as follows:

Initialization: Initialize $\mathbf{W}_0$ to some rectangular matrix such that $\mathbf{W}_0^\top \mathbf{W}_0$ is positive semi-definite.
Iterate: Do the following steps until convergence:

- Set $\widetilde{\mathbf{W}}_{t+1} = \mathbf{W}_t - \eta \frac{\partial J(\mathbf{W})}{\partial \mathbf{W}}$.
- Construct $\widetilde{\mathbf{A}}_{t+1} = \widetilde{\mathbf{W}}_{t+1} \widetilde{\mathbf{W}}_{t+1}^\top$ and apply eigendecomposition to $\widetilde{\mathbf{A}}_{t+1}$ and reconstruct it using positive eigenvalues and corresponding eigenvectors $\widetilde{\mathbf{A}}_{t+1} = \sum_k \lambda_k \mathbf{e}_k \mathbf{e}_k^\top$.
- Reconstruct the projection matrix as $\mathbf{W}_{t+1} = \Lambda_{t+1}^{1/2} \mathbf{E}_{t+1}$.

3 EXTENSIONS TO NONLINEAR CASES

Here we consider the case where the data samples are mapped into a higher-dimensional feature space and the distance metric is sought in this space. We restrict our analysis to nonlinear mappings $\phi : \mathbb{R}^d \rightarrow \mathcal{F}$ where the dot products in the mapped space can be obtained by using a kernel function such that $< \phi(\mathbf{x}_i), \phi(\mathbf{x}_j) >= k(\mathbf{x}_i, \mathbf{x}_j)$ for some kernel $k(.,.)$.

Let $\Phi = [\phi(\mathbf{x}_1) \ \ldots \ \phi(\mathbf{x}_n)]$ denote the matrix whose columns are the mapped samples in $\mathcal{F}$. We define $\mathbf{k}_\mathbf{x} = \Phi^\top \phi(\mathbf{x}) = [k(\mathbf{x}_i, \mathbf{x}]_{i=1}^n$ as $n \times 1$ kernel vector of $\mathbf{x}$ against training samples. As in Kernel Principal Components Analysis (Scholkopf et al., 1998), we consider parametrizations of $\mathbf{W}$ of the form $\mathbf{W} = \Omega \Phi^\top$, where $\Omega \in \mathbb{R}^{q \times n}$ is some matrix allowing to write $\mathbf{W}$ as a linear combinations of the mapped samples. In this setting, the distance matrix $\mathbf{A}$ can be written as

$$\mathbf{A} = \mathbf{W}^\top \mathbf{W} = \Phi \Omega^\top \Omega \Phi^\top. \quad (8)$$

By defining the positive semi-definite matrix as $\widehat{\mathbf{A}} = \Omega^\top \Omega$, the original problem can be converted into looking for a positive semi-definite matrix $\widehat{\mathbf{A}}$ since the distance in the mapped space under the distance matrix $\mathbf{A}$ can be written as

$$(\mathbf{k}_{\mathbf{x}_i} - \mathbf{k}_{\mathbf{x}_j})^\top \widehat{\mathbf{A}} (\mathbf{k}_{\mathbf{x}_i} - \mathbf{k}_{\mathbf{x}_j}) = \hat{\mathbf{d}\mathbf{x}}_{ij}^\top \Phi \Omega^\top \Omega \Phi^\top \hat{\mathbf{d}\mathbf{x}}_{ij}, \quad (9)$$

where $\hat{\mathbf{d}\mathbf{x}}_{ij} = \phi(\mathbf{x}_i) - \phi(\mathbf{x}_j)$. As can be seen in the equation above, the distance between two samples

in the mapped space depends only on dot products which are computed in the original input space. This is equivalent to transformation of the input data into n-dimensional feature space through $\Phi^\top \phi(\mathbf{x}_i)$ followed by the distance metric learning in the transformed space. Thus, by using the proposed algorithms described earlier, we can search a full rank matrix $\widehat{\mathbf{A}}$ or low-dimensional projection matrix Ω in the transformed kernel feature space.

Nonlinear distance metric learning is not very useful for visual object classification tasks since the original input space is already high-dimensional. But, it may be useful for some other applications where the input space is typically low-dimensional and finding a distance metric satisfying all pairwise equivalence constraints is not be feasible, e.g., exclusive-or problem.

4 EXPERIMENTS

We perform experiments on three different computer vision applications and attempt to discover the desired unknown groups in these. The proposed Semi-Supervised Distance Metric Learning (SSDML) algorithm is compared to the full rank distance metric learning algorithm followed by dimensionality reduction and the Constrained Locality Preserving Projection (CLPP) method of (Cevikalp et al., 2008). The k-means and spectral clustering methods are used as clustering algorithm with the learned distance metric, and pairwise F-measure is used to evaluate the clustering results based on the underlying classes. The pairwise F-measure is the harmonic mean of the pairwise precision and recall measures. To demonstrate the effect of using different number of equivalence constraints, we gradually increased the number of similar and dissimilar pairs. In all visual object classification experiments, constraints are uniformly random selected from all possible constraints induced by the true data labels of the training data, and clustering performance is measured using only the test data. We used the same value for both thresholds t_s and t_d, and it is chosen to be $0.1\mu_S$, where μ_S is the averages of distances between similar pairs under the initial distance metric.

4.1 Experiments on Gender Database

Here we demonstrate how the proposed method can be used to organize image galleries in accordance to the personal preferences. In these applications we determine a characteristic for distinction and group images based on this selection. In our case we group images by gender and use the gender recognition database used in (Villegas and Paredes, 2008). This database consist of 1892 images (946 males and 946 females) coming from the following databases: AR, BANCA, Caltech Frontal face, Essex Collection of Facial Images, FERET, FRGC version 2, Georgia Tech and XM2VTS. Only the first frontal image of each individual was taken, however because all of the databases have more male subjects than females, the same number of images is taken for both male and female subjects. All images are cropped based on the eye coordinates and resized to 32×40 yielding a 1280-dimensional input space. Then, images are converted to gray-scale followed by histogram equalization. Some samples are shown in Fig. 2.

Figure 2: Some male and female samples from Gender database.

We used 50% of the images as training data and the remaining for testing. The dimensionality $d = 1280$ of the input space is too high, thus we learned a projection matrix of size $10 \times d$ yielding a low rank distance matrix. Since we cannot directly apply the other full rank distance metric learning techniques in this high-dimensional space, we first applied dimensionality reduction methods, Principal Component analysis (PCA) and Locality Preserving Projections (LPP) (He and Niyogi, 2003), to the high-dimensional data, and learned a distance metric in the reduced space. The size of the reduced space is chosen such that 99% of the overall energy (sum of the eigenvalues) is retained. To learn the distance metric in the reduced space, we used the method proposed in (Kwok and Tsang, 2003). The reported clustering performances are averages over 10 random test/training splits.

Clustering accuracies as a function of constraints are shown in Fig. 3. As can be seen, the proposed method outperforms competing methods for both k-means and spectral clustering in all cases. PCA followed by the distance metric learning comes the second and LPP followed by the distance metric learning performs the worst. CLPP method yields similar accuracies to LPP followed by the distance metric learning. The poor performance of CLPP suggests

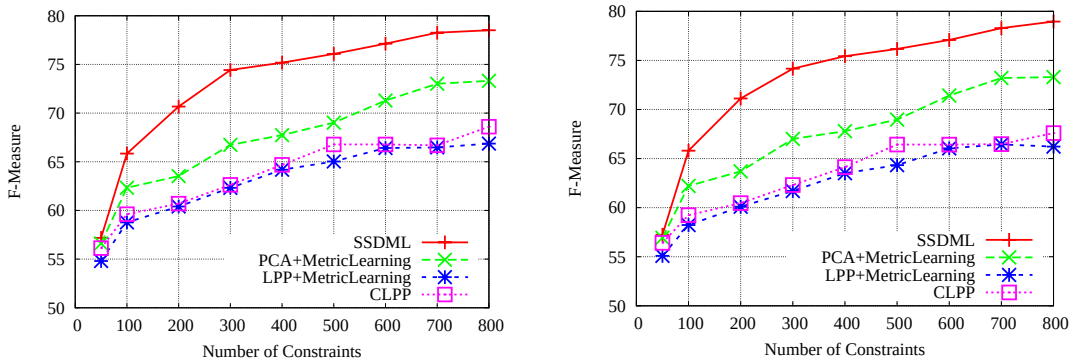


Figure 3: F-measure as a function of number of constraints for (a) k-means clustering, (b) spectral clustering on Gender database.

that the samples coming from male and female subjects in small neighborhoods do not have the same label. Both clustering algorihtms, k-means and spectral clustering, yield similar results.

4.2 Experiments on Birds Database

The Birds database (Lazebnik et al., 2005) contains six categories, each having 100 images. It is a challenging database since the birds appear against highly cluttered backgrounds and images have large intra-class, scale, and viewpoint variability. We used a "bag of features" representation for the images as they are too diverse to allow simple geometric alignment of their objects. In this method, patches are sampled from the image at many different positions and scales, either densely, randomly or based on the output of some kind of salient region detector. Here we used a dense grid of patches. Each patch was described using the robust visual descriptor SIFT assignment against a 2000 word visual dictionary learned from the complete set of training patches. The dimensionality of the input space is still high, thus we learned a non-square projection matrix with rank 10 and we reduced the dimensionality before applying the full distance metric learning technique as in the first experiment. We used 50% of the images as training data and remaining for testing. Results are again averages over 10 random test/training splits.

Results are shown in Fig. 4. Initially, PCA followed by the full rank distance metric learning performs better than the proposed method. As the number of the constraints increases, the proposed method takes the lead and outperforms competing methods. CLPP comes the second and LPP followed by the distance metric learning again performs the worst. This time, k-means clustering yields better results than spectral clustering.

4.3 Image Segmentation Applications

We also tested proposed method on image segmentation applications where the dimensionality of the sample space is relatively small compared to the visual object classification problems. We experimented with images chosen from the Berkeley Segmentation dataset[2]. Centered at every pixel in each image we extracted a 20×20 pixel image patch for which we computed the robust hue descriptor of (van de Weijer and Schmid, 2006). This process yields a 36-dimensional feature vector which is a histogram over hue values observed in the patch, where each observed hue value is weighted by its saturation. We compared our proposed method to the image segmentation based on Normalized Cuts (NCuts) (Shi and Malik, 2000). The Heat kernel function using Euclidean distance is used as kernel in NCuts segmentation. As in (Cevikalp et al., 2008), we set the number of clusters to two, one cluster for the background and another for the object of interest.

The pairwise equivalence constraints are chosen from the samples corresponding to pixels shown with magenta and cyan in the second row of Fig. 5. We first segmented the original images without any supervision using NCuts algorithm. Then, we used the proposed method with the selected constraints to learn a projection matrix $\mathbf{W}$ with rank 10 and then used NCuts segmentation in the learned space. As can be seen in the figures, simple used added equivalence constraints can improve the segmentations.

[2]Available at `http://www.eecs.berkeley.edu/Research/Projects/CS/vision/grouping/segbench/`

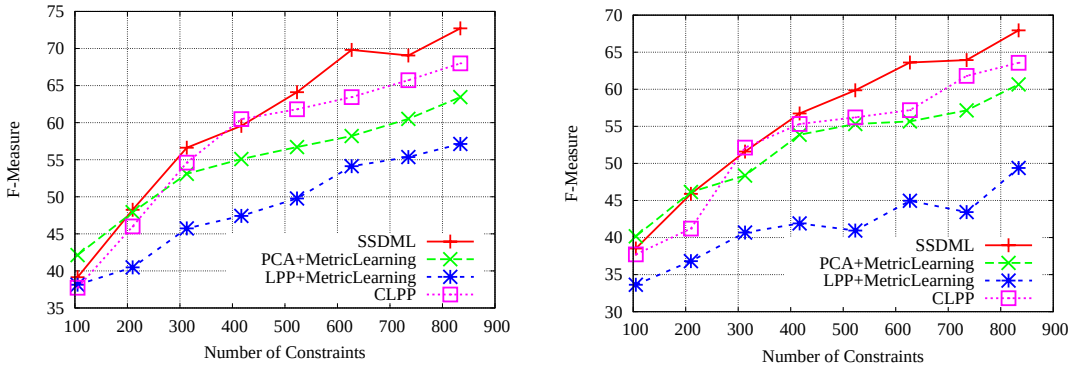


Figure 4: F-measure as a function of number of constraints for (a) k-means clustering, (b) spectral clustering on Birds database.

5 SUMMARY AND CONCLUSION

In this paper we proposed a semi-supervised distance metric learning method, which uses pairwise equivalence constraints to discover the desired groups in high-dimensional data. The method works in both the input and kernel induced-feature space and it can learn nonsquare projection matrices that yield low rank distance metrics. The optimization procedure involves minimizing two terms defined based on sigmoids. The first term encourages pulling similar sample pairs closer while the second term maximizes the local margin. The solution is found by a gradient descent procedure that involves an eigendecomposition.

Experimental results show that the proposed method increases performance of subsequent clustering and classification algorithms. Moreover, it yields better results than methods applying unsupervised dimensionality reduction followed by full rank metric learning.

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Figure 5: Original images (top row), pixels used for equivalence constraints (second row), segmentation results without constraints (third row), and segmentation results using constraints (bottom row). Figure is best viewed in color.

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